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# Practical Comprehensive Arithmetic

BY
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Vice-Principal of Peirce School

FIFTH EDITION

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# PREFACE

IN the preparation of this book, great care and attention have been given to the logical relation of the science, and its adaptation to practical affairs; to definiteness and exactness in principles and rules; to brevity and clearness in the presentation of the subject matter; and to modern progressive methods.

The aim has been to present the truths and principles of a practical science in a practical, business-like way, and to develop them in a natural, simple, effective manner, with a variety of well-selected problems of a character suited to students of varied ability.

The features of this book are:

1—The logical development of NUMBER.
   (*a*) The properties of number.
   (*b*) The relation of a quantity to a fixed unit of measure.
   (*c*) The mechanical and reasoning processes by analysis, comparison and synthesis.
2—The definitions of terms in clear, concise, arithmetical language.
3—The fundamental operations made simple and easy by short processes.
4—Models in script, with illustrative figures, giving specific, concrete form to the principles discussed.
5—A full treatment of common and decimal fractions, including a new and safe method of decimal division.
6—The logical development of measurements, copiously illustrated by cuts and figures, and by problems from practical life.
7—A short and novel presentation of the operations of percentage, trade discount, commission, interest, and the metric system.
8—A full and practical treatment of stocks and bonds, exchange, partnership and partnership settlements, building and loan associations, averaging and equating accounts, mensuration, etc.
9—The classification and gradation of problems—first, simple mental problems to develop an insight into the principles; second, plain practical questions in calculation to develop facility and rapidity; third, more complicated problems requiring reasoning as well as careful computation.
10—The introduction of many interesting and helpful notes.

No effort has been spared to write a simple, logical, comprehensive book on arithmetic that meets the needs of the student and teacher in the class-room, as well as the demands of present-day industrial and commercial life.

Appreciation and thanks are extended to E. J. Conner, J. H. Bryant and R. F. Hayes for subject matter and assistance; to R. S. Collins for script work; and to Thomas May Peirce, Jr., for illustrative figures; to Frederick G. Story of Brown Bros., Philadelphia, George G. Thomas of E. W. Clark & Co., Philadelphia, M. Albert Linton, Actuary of Provident Life & Trust Co., Philadelphia, and Michael J. Brown, Secretary of Building Association League of Pennsylvania, for helpful suggestions and criticism.

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# ARITHMETIC

## DEFINITIONS AND PRINCIPLES

**1. Quantity** is a limited portion of any natural object; as of time, space, weight, etc., or of any solid or fluid substance.

**2.** All mathematical operations deal with *measures of quantity*, and quantity can be measured only by comparing it with some known quantity of the same kind taken as a standard.

**3.** Quantity can be changed, mathematically, only by *increase* or *diminution*.

**4.** A quantity may be *increased* in either of two ways: (a) By combining with it one or more quantities of the *same* kind, greater or less than itself; this is called *Addition*. (b) By combining with it any given number of quantities each exactly equal to itself; this is called *Multiplication*.

**5.** A quantity may be diminished in either of two ways: (a) By taking from it a quantity not greater than itself; this is called *Subtraction*. (b) By continuing to take from it, as many times as possible, a given quantity less than itself; this is called *Division* (because the quantity is separated into a number of equal parts, with or without a *remainder*).

**6.** The two mathematical principles, *increase* and *diminution*, thus give rise to the four fundamental operations of arithmetic—*Addition*, *Subtraction*, *Multiplication* and *Division*.

**7. Arithmetic,** as a science, is the *science of numbers*. As an art, it embraces all known methods of computation by means of figures, all of which are but processes or combinations of adding, subtracting, multiplying and dividing.

**8.** The **Signs** of Addition, Subtraction, Multiplication and Division are respectively: $+$ *plus;* $-$ *minus;* $\times$ *multiplied by;* $\div$ *divided by.*

**9.** The **Result** in addition is called the *sum;* in subtraction, the *difference* or *remainder;* in multiplication, the *product;* and in division, the *quotient.*

**10.** The **Sign of Equality** is $=$, read *equals* or *equal to.*

**11.** A **Unit,** represented by the figure 1, signifies *one thing of any kind;* as, 1 dollar, 1 yard, 1 day. When not applied to any thing, it represents the *abstract* idea of unity.

**12.** A **Number** is a *unit* or a *group of units*, expressed by one or more figures, and considered as *one quantity*. Numbers may be *abstract*, as 1, 5, 12; or *concrete*, as 6 feet, 25 barrels.

# NOTATION AND NUMERATION

**13. Notation** is the art of writing numbers. For purposes of calculation the *Arabic notation*, expressing numbers in figures, is used exclusively; but the *Roman notation*, a system of representing numbers by combinations of letters, is still used for numbering chapters, dates of imprint and inscriptions, dials, etc.

**14. Numeration** is the art of reading or naming numbers.

**15. Arabic Notation** expresses numbers by the following ten characters: **1**, one; **2**, two; **3**, three; **4**, four; **5**, five; **6**, six; **7**, seven; **8**, eight; **9**, nine; **0**, naught, cipher or zero. They are written as follows:

*1 2 3 4 5 6 7 8 9 0*

Any one of these ten characters or figures is called a *digit*, and a whole number represented by any one of them, an *integer*.

The number of units expressed by any one of these figures when standing alone is shown by its name; but when used in combination with other figures the number of units expressed is indicated by its place, as shown in the table which follows. A figure occupying any one of these *places* expresses as many times the number of units it names as is indicated by the name of the place it occupies, being in each case *ten times as many* as it would represent in the next place to the right. Thus, the figure 8, with one figure at the right of it—as in 80, 82, or 86—represents 8 *tens*, because it occupies the second, or tens place; with two figures following to the right—as in 800, 805, or 837—it represents 8 *hundreds*, because it occupies the third, or hundreds place, etc. The cipher, by itself, represents *nothing*, and is used merely to fill vacant *places*.

**16.** **NUMERATION TABLE**

| NAMES OF PLACES | Ten-decillions | Decillions | Hundred-nonillions | Ten-nonillions | Nonillions | Hundred-octillions | Ten-octillions | Octillions | Hundred-septillions | Ten-septillions | Septillions | Hundred-sextillions | Ten-sextillions | Sextillions | Hundred-quintillions | Ten-quintillions | Quintillions | Hundred-quadrillions | Ten-quadrillions | Quadrillions | Hundred-trillions | Ten-trillions | Trillions | Hundred-billions | Ten-billions | Billions | Hundred-millions | Ten-millions | Millions | Hundred-thousands | Ten-thousands | Thousands | Hundreds | Tens | Units |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **10,** | | **999,** | | | **888,** | | | **777,** | | | **666,** | | | **555,** | | | **444,** | | | **333,** | | | **222,** | | | **111,** | | | **654,** | | | **321** | | |
| PLACES OR ORDERS | 35th | 34th | 33d | 32d | 31st | 30th | 29th | 28th | 27th | 26th | 25th | 24th | 23d | 22d | 21st | 20th | 19th | 18th | 17th | 16th | 15th | 14th | 13th | 12th | 11th | 10th | 9th | 8th | 7th | 6th | 5th | 4th | 3d | 2d | 1st |
| PERIODS | 12th | | 11th | | | 10th | | | 9th | | | 8th | | | 7th | | | 6th | | | 5th | | | 4th | | | 3d | | | 2d | | | 1st | | |
| NAMES OF PERIODS | Decillions | | Nonillions | | | Octillions | | | Septillions | | | Sextillions | | | Quintillions | | | Quadrillions | | | Trillions | | | Billions | | | Millions | | | Thousands | | | Units | | |

**17.** Dictionaries of the English language do not contain the names for the periods above *Decillions*, because they are so seldom used. It will be observed that, commencing with Millions (in lieu of *Unillions*) as *one*, Billions *two*, Trillions *three*, etc., these names consist of the Latin numerals in order, with the termination *-illions*.

NOTE.—The names of the higher periods, sometimes used by mathematicians, are formed in the same way; but that such numbers are far beyond human comprehension may be readily shown by the following: If a hollow sphere of the size of the Earth (say, 8000 miles in diameter) were filled with wheat the number of grains in the entire mass would be between thirteen and fourteen *octillions*, (allowing 200 grains to the cubic inch).

**18.** It will also be observed that there is a difference of *two* between the number expressed by the root of the period-name and the number of the period itself—Octillions (*octo*, eight) being the *tenth* period, for instance—just as there is a difference of *two* between the numbers indicated by the roots of September, October, November, December, and the numbers of these months in our year. The reason for this discrepancy in the case of the period-names is that they start with Millions, while with regard to the month-names mentioned it originates in the fact that the Roman year began with March.

**19. English Numeration.** The system of numeration shown in the Numeration Table (Art. 16) is called the French and American system; it is used in nearly all countries, England being the most important exception. The English method is the same as far as Millions; beyond Millions the names are as follows:

| NUMERALS | AMERICAN AND FRENCH NAME | ENGLISH NAME |
|---|---|---|
| 1,000,000,000 | Billions | Thousand millions |
| $10^{12}$ | Trillions | Billions |
| $10^{15}$ | Quadrillions | Thousand billions |
| $10^{18}$ | Quintillions | Trillions |
| $10^{21}$ | Sextillions | Thousand trillions |
| $10^{24}$ | Septillions | Quadrillions |
| $10^{27}$ | Octillions | Thousand quadrillions |
| $10^{30}$ | Nonillions | Quintillions |
| $10^{33}$ | Decillions | Thousand quintillions |

**20.** In the French and American system, each succeeding period of three figures has a new name, so that each period-name indicates a number *one thousand times* as great as the name preceding. In the English system, the figures are grouped in periods of *six* figures each—Units, Tens, Hundreds, Thousands, Ten-thousands, Hundred-thousands—Millions, Ten-millions, Hundred-millions, Thousand-millions, Ten-thousand-millions, Hundred-thousand-millions—Billions, etc., so that each period-name indicates a number *one million times* as great as the name preceding.

**21. Roman Notation.** The Roman notation expresses numbers by means of seven capital letters, as follows: I, one; V, five; X, ten; L, fifty; C, one hundred; D, five hundred; M, one thousand.

**ROMAN NOTATION TABLE**

| ARABIC | ROMAN | ARABIC | ROMAN | ARABIC | ROMAN |
|---|---|---|---|---|---|
| 1 | I | 16 | XVI | 400 | CD |
| 2 | II | 17 | XVII | 500 | D |
| 3 | III | 18 | XVIII | 600 | DC |
| 4 | IV | 19 | XIX | 700 | DCC |
| 5 | V | 20 | XX | 800 | DCCC |
| 6 | VI | 30 | XXX | 900 | CM |
| 7 | VII | 40 | XL | 1,000 | M |
| 8 | VIII | 50 | L | 1,300 | MCCC |
| 9 | IX | 60 | LX | 1,400 | MCD |
| 10 | X | 70 | LXX | 1,500 | MD |
| 11 | XI | 80 | LXXX | 1,600 | MDC |
| 12 | XII | 90 | XC | 1,800 | MDCCC |
| 13 | XIII | 100 | C | 1,900 | MCM |
| 14 | XIV | 200 | CC | 2,000 | MM |
| 15 | XV | 300 | CCC | 1,000,000 | $\overline{\text{M}}$ |

The principles by which the letters are combined to express numbers, as illustrated in the foregoing table, are as follows:

*1.* The letters I, X, C and M are written twice or three times in succession to indicate double or triple their values; as II, two, III, three; XX, twenty, XXX, thirty; CC, two hundred, CCC, three-hundred; MM, two thousand, MMM, three thousand; but they are not repeated more than three times, because there is a shorter way of representation by principle 2 The other letters are not repeated, because VV = X, LL = C, and DD = M.

*2.* When a letter of *less* value is placed *before* one of *greater* value, the combination represents the *difference* of their values, *i. e.*, the greater *minus* the less; as, IV, four; IX, nine; XL, forty; XC, ninety; CD, four hundred; CM, nine hundred. A letter of less value placed between two of greater value is thus subtracted from their sum; as, XIV, fourteen; XIX, nineteen; CXL, one hundred forty; CXC, one hundred ninety; MCM, nineteen hundred.

*3.* When a letter of *less* value is placed *after* one of *greater* value, the combination represents the sum of their values; as, VI, six; XI, eleven; LV, fifty-five; DL, five hundred fifty.

*4.* A line (dash, bar, or vinculum) placed over a letter or letters increases the value *one thousand times;* as, $\overline{\text{IV}}$, four thousand; $\overline{\text{X}}$, ten thousand; $\overline{\text{LXV}}$, sixty-five thousand; $\overline{\text{MM}}$, two million.

NOTE.—On the dials of watches and clocks IIII is invariably to be seen in place of IV, the correct form. Custom has perpetuated the error. Also, the date of imprint, 1900, may be seen on some books represented by MDCCCC; but the correct form is MCM, by principle 2.

# ADDITION

**22. Addition** is the process of finding the *sum* of two or more numbers.

**23.** The **Sum** or **Amount** of several numbers is one number which contains as many units as there are in all the numbers added.

**24.** The **Sign of Addition** is +, read *plus*, and denotes that the numbers between which it is placed are to be added.

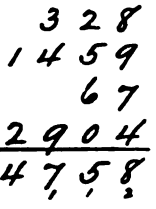
**25.** The **Sign of Equality** is =, read *equals* or *equal to*, and denotes that the numbers or quantities between which it is placed are equal.

**26.** The **Principles of Addition** are:

1. *Only like numbers can be added.*
2. *Units of only the same order can be directly combined.*
3. *The sum is a number of the same kind as the numbers added.*
4. *The sum is the same in whatever order the numbers may be added.*

**27. To find the sum of two or more numbers.**

EXAMPLE.—Add 328, 1459, 67 and 2904




Write the first number, then place the others beneath it in such a manner that the units figures shall be directly under units, tens under tens, hundreds under hundreds, etc.—being careful to keep the columns vertical and evenly spaced. Beginning with the units (first column at the right) add 4+7+9+8, naming result only at each step thus: "Eleven, twenty—eight" (28), which gives 2 tens and 8 units. (Do not say, "Four and 7 are 11, and 9 are 20, and 8 are 28.") Write 8 under the units column and carry 2 to the tens column. Add the tens column by combining the 2 (carried) with the 6, and the 5 with the 2, saying, "Eight—fifteen," which gives 5 tens, and 1 hundred to carry. Write 5 under the tens column, and carry 1 to the hundreds column. Add the hundreds column by combining the 1 (carried) with the 9, and the 4 with the 3, saying, "Ten, seventeen"—1 thousand and 7 hundreds. Write 7 under the hundreds column and carry 1 to the thousands column, which is now read as "Four," and the 4 written under it. Result, 4758—the sum.

PROOF OF ADDITION.—Add the columns again, reversing the order of the operation; that is to say, if the first adding has been from bottom of column to top, let the second be from top of column to bottom, or *vice versa*.

**28.** RULE.—*Write the numbers so that units of the same order stand in the same column.*

*Beginning with the units column, add each column separately, and write the sum beneath it, if less than ten.*

*If the sum of any column is ten, or more than ten, set down only the right-hand figure under the column added, and add the remaining figure or figures to the next column.*

*Write the entire sum of the last column.*

**29.** In addition there are but 45 possible combinations of two figures. The first step toward proficiency is the thorough mastery of these combinations, so that the sum of any two figures can be named instantly.

**30.** The combinations of two figures, one above another, as they appear to the eye in adding columns, present 81 different forms, from $\frac{1}{1}$ to $\frac{9}{9}$, as follows:

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 3 | 1 | 4 | 1 | 5 | 1 | 6 | 1 | 7 | 1 | 8 | 1 | 9 | 2 | 2 | 3 | 2 |
| 2 | 1 | 3 | 1 | 4 | 1 | 5 | 1 | 6 | 1 | 7 | 1 | 8 | 1 | 9 | 1 | 2 | 3 | 2 | 4 |
| 4 | 2 | 5 | 2 | 6 | 2 | 7 | 2 | 8 | 2 | 9 | 3 | 3 | 4 | 3 | 5 | 3 | 6 | 3 | 7 |
| 2 | 5 | 2 | 6 | 2 | 7 | 2 | 8 | 2 | 9 | 2 | 3 | 4 | 3 | 5 | 3 | 6 | 3 | 7 | 3 |
| 3 | 8 | 3 | 9 | 4 | 4 | 5 | 4 | 6 | 4 | 7 | 4 | 8 | 4 | 9 | 5 | 5 | 6 | 5 | 7 |
| 8 | 3 | 9 | 3 | 4 | 5 | 4 | 6 | 4 | 7 | 4 | 8 | 4 | 9 | 4 | 5 | 6 | 5 | 7 | 5 |
| 5 | 8 | 5 | 9 | 6 | 6 | 7 | 6 | 8 | 6 | 9 | 7 | 7 | 8 | 7 | 9 | 8 | 8 | 9 | 9 |
| 8 | 5 | 9 | 5 | 6 | 7 | 6 | 8 | 6 | 9 | 6 | 7 | 8 | 7 | 9 | 7 | 8 | 9 | 8 | 9 |

**31.** After practising until he is able to name the sum of any of these combinations at a glance, the student's next step is to learn to read at sight the sum of such combinations as the following:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 38 | 47 | 56 | 69 | 78 | 87 | 28 | 37 | 49 |
| 3 | 4 | 5 | 3 | 4 | 5 | 5 | 6 | 5 |
| 58 | 69 | 78 | 89 | 98 | 43 | 65 | 54 | 75 |
| 6 | 6 | 7 | 7 | 9 | 8 | 6 | 8 | 7 |

**32.** After a thorough drill on the two-figure combinations the student should begin practise upon three-figure groups. The following are a few of the most practical of the 728 possible combinations of three figures. It will be found profitable to prepare pages of similar combinations, and to practise upon them frequently until they are as familiar to the eye as the three-letter words one sees in books and papers. Two or more figures should not be regarded as separate characters, but as forming a word.

## THREE-FIGURE COMBINATIONS FOR "SIGHT READING"

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 1 | 3 | 2 | 5 | 4 | 7 | 5 | 2 | 4 | 4 | 7 | 6 | 4 | 2 | 6 | 5 | 3 | 1 | 3 |
| 4 | 3 | 4 | 7 | 4 | 2 | 1 | 1 | 6 | 5 | 3 | 2 | 1 | 1 | 3 | 2 | 2 | 5 | 7 | 1 |
| 4 | 6 | 3 | 1 | 1 | 4 | 2 | 4 | 2 | 1 | 3 | 1 | 3 | 5 | 5 | 2 | 3 | 2 | 2 | 6 |
| 3 | 4 | 2 | 2 | 2 | 8 | 3 | 3 | 7 | 1 | 5 | 2 | 6 | 4 | 1 | 2 | 3 | 6 | 3 | 4 |
| 2 | 2 | 7 | 6 | 5 | 1 | 7 | 6 | 2 | 3 | 4 | 8 | 2 | 5 | 2 | 8 | 2 | 2 | 4 | 5 |
| 6 | 5 | 2 | 3 | 4 | 2 | 1 | 2 | 2 | 7 | 2 | 1 | 3 | 2 | 8 | 2 | 7 | 4 | 5 | 3 |
| 5 | 3 | 2 | 7 | 6 | 8 | 2 | 4 | 3 | 2 | 6 | 8 | 3 | 4 | 9 | 4 | 3 | 6 | 7 | 8 |
| 2 | 2 | 6 | 2 | 1 | 1 | 9 | 6 | 6 | 7 | 5 | 2 | 7 | 3 | 2 | 2 | 2 | 4 | 2 | 2 |
| 5 | 7 | 4 | 3 | 5 | 3 | 1 | 2 | 3 | 3 | 1 | 2 | 2 | 5 | 1 | 7 | 8 | 3 | 4 | 3 |
| 2 | 6 | 5 | 2 | 4 | 2 | 6 | 4 | 2 | 7 | 9 | 1 | 3 | 5 | 4 | 3 | 5 | 8 | 6 | 7 |
| 7 | 3 | 2 | 8 | 8 | 9 | 2 | 3 | 5 | 4 | 1 | 8 | 4 | 6 | 5 | 7 | 4 | 4 | 1 | 5 |
| 4 | 4 | 6 | 3 | 1 | 2 | 5 | 6 | 6 | 2 | 3 | 4 | 6 | 2 | 4 | 3 | 4 | 1 | 6 | 1 |
| 4 | 2 | 4 | 7 | 3 | 6 | 9 | 2 | 5 | 7 | 3 | 5 | 4 | 9 | 2 | 8 | 5 | 4 | 5 | 3 |
| 1 | 3 | 2 | 2 | 4 | 3 | 2 | 4 | 2 | 3 | 8 | 3 | 5 | 4 | 9 | 2 | 7 | 3 | 6 | 3 |
| 9 | 9 | 8 | 5 | 7 | 5 | 3 | 8 | 7 | 4 | 3 | 6 | 5 | 1 | 3 | 4 | 2 | 7 | 3 | 8 |
| 3 | 7 | 5 | 8 | 9 | 4 | 6 | 5 | 7 | 9 | 8 | 3 | 7 | 9 | 5 | 8 | 4 | 6 | 7 | 6 |
| 8 | 9 | 6 | 7 | 3 | 7 | 9 | 7 | 8 | 4 | 3 | 9 | 4 | 5 | 8 | 9 | 9 | 5 | 5 | 7 |
| 9 | 4 | 9 | 5 | 8 | 9 | 5 | 8 | 5 | 7 | 9 | 8 | 9 | 6 | 7 | 3 | 7 | 9 | 8 | 7 |

**33.** Addition being the most important of all commercial calculations, thorough drill in its rapid performance should be given daily. The student will get much practise in writing up accounts, but unless this is supplemented by daily practise for speed, he will not become rapid and accurate. It is noticeable that, as a rule, the most rapid adders are the most accurate. All work for speed should be dictated, and the time required for the operation should be limited or noted. In adding columns insist upon the student's naming only the results of combinations, omitting the unnecessary words "and are" or "and is."

The examples on pages 14 and 15 are simply suggestive. While the time limit and the nature of the example should be governed by the ability of students, yet grammar-school students can, by diligent practise, in six months, add sums known as "six-by-tens" in from fifteen to thirty seconds.

**34.** A convenient method for class-drill is to give tables, as shown below, of a prescribed number of columns and rows, suited to the ability of the students. Dictate the sums given below in pairs of two figures each, in the following manner, (taking number 1 as an example): (1) Forty-five, sixty-one, sixty-two; (2) twenty-one, forty-nine and forty-five, etc.

**1**

| | | | | | |
|---|---|---|---|---|---|
| 4 | 5 | 6 | 1 | 6 | 2 |
| 2 | 1 | 4 | 9 | 4 | 5 |
| 8 | 4 | 3 | 4 | 2 | 3 |
| 3 | 2 | 7 | 6 | 4 | 1 |
| 7 | 8 | 2 | 5 | 2 | 9 |
| 2 | 1 | 8 | 2 | 1 | 2 |
| 3 | 9 | 3 | 8 | 3 | 6 |
| 5 | 3 | 2 | 3 | 1 | 2 |
| 1 | 7 | 5 | 4 | 6 | 7 |
| 9 | 2 | 6 | 3 | 4 | 3 |

**2**

| | | | | | |
|---|---|---|---|---|---|
| 7 | 8 | 6 | 5 | 3 | 4 |
| 3 | 2 | 4 | 1 | 7 | 2 |
| 4 | 7 | 3 | 4 | 2 | 4 |
| 2 | 1 | 2 | 8 | 8 | 3 |
| 4 | 2 | 5 | 2 | 5 | 7 |
| 5 | 6 | 4 | 6 | 4 | 2 |
| 1 | 4 | 2 | 4 | 1 | 8 |
| 4 | 5 | 4 | 3 | 8 | 1 |
| 2 | 3 | 8 | 7 | 2 | 9 |
| 8 | 5 | 2 | 5 | 6 | 4 |

**3**

| | | | | | |
|---|---|---|---|---|---|
| 8 | 5 | 1 | 3 | 8 | 3 |
| 3 | 1 | 9 | 2 | 3 | 2 |
| 7 | 4 | 3 | 5 | 7 | 5 |
| 4 | 3 | 7 | 4 | 1 | 4 |
| 6 | 2 | 5 | 2 | 8 | 6 |
| 2 | 5 | 3 | 4 | 1 | 5 |
| 8 | 2 | 2 | 3 | 4 | 3 |
| 3 | 4 | 4 | 7 | 6 | 2 |
| 2 | 3 | 6 | 9 | 2 | 7 |
| 5 | 4 | 7 | 1 | 8 | 3 |

**4**

| | | | | | |
|---|---|---|---|---|---|
| 6 | 8 | 5 | 6 | 3 | 3 |
| 5 | 7 | 4 | 2 | 7 | 4 |
| 8 | 2 | 3 | 8 | 9 | 7 |
| 4 | 8 | 9 | 9 | 3 | 2 |
| 7 | 2 | 4 | 6 | 5 | 1 |
| 2 | 8 | 9 | 2 | 6 | 7 |
| 5 | 4 | 7 | 9 | 9 | 3 |
| 9 | 8 | 5 | 2 | 3 | 1 |
| 5 | 5 | 8 | 5 | 8 | 4 |
| 9 | 6 | 4 | 3 | 9 | 5 |

**5**

| | | | | | |
|---|---|---|---|---|---|
| 7 | 7 | 6 | 4 | 8 | 3 |
| 9 | 1 | 7 | 6 | 4 | 4 |
| 8 | 7 | 9 | 5 | 6 | 9 |
| 5 | 8 | 4 | 6 | 7 | 5 |
| 4 | 1 | 5 | 9 | 3 | 4 |
| 2 | 9 | 7 | 1 | 4 | 7 |
| 6 | 5 | 6 | 3 | 3 | 9 |
| 8 | 9 | 8 | 1 | 9 | 6 |
| 5 | 6 | 5 | 3 | 6 | 7 |
| 6 | 9 | 9 | 7 | 3 | 3 |

**6**

| | | | | | |
|---|---|---|---|---|---|
| 4 | 5 | 5 | 4 | 3 | 4 |
| 2 | 2 | 4 | 1 | 4 | 7 |
| 6 | 4 | 6 | 2 | 7 | 8 |
| 2 | 7 | 9 | 1 | 8 | 3 |
| 8 | 9 | 3 | 9 | 5 | 1 |
| 7 | 2 | 6 | 3 | 3 | 9 |
| 4 | 5 | 9 | 6 | 6 | 5 |
| 5 | 4 | 4 | 7 | 9 | 3 |
| 7 | 1 | 5 | 2 | 2 | 5 |
| 6 | 7 | 8 | 4 | 8 | 7 |

NOTE.—Addition may also be proved by "casting out the 9's" in each line, writing the remainder, or "excess figure," to the right. By adding these several remainders and casting out the 9's, or by carrying the remainder from each line into the next line and continuing back and forth to the last line, an excess figure is obtained which should agree with that obtained by casting the 9's out of the sum of the addition. If they do not agree, there is an error in the work. See example 5, page 15.

The excess figure by 11's may be obtained by subtracting the sum of the figures of the even places (2d, 4th, 6th, etc.) beginning at units, from the sum of the figures of the odd places (1st, 3d, 5th, etc.), increasing the latter by 11 or a multiple of 11, if necessary.

**35. Tables of "eight by fifteen,"** as shown below, may be given to more advanced students in the same manner as the "six by ten," having the time limit of ninety seconds or less, depending on the accuracy and skill of students in handling figures. Teachers should prepare many similar examples for daily drill.

| 1 | 2 | 3 | 4 | 5 | |
|---|---|---|---|---|---|
| 57836489 | 91445827 | 68563394 | 26497764 | 37195183 | 1 |
| 25377836 | 79213152 | 57427435 | 88239176 | 98446712 | 5 |
| 77492682 | 38768699 | 82389724 | 53668795 | 26582358 | 3 |
| 59344193 | 47547362 | 48993286 | 62275846 | 71323494 | 6 |
| 94563324 | 53974829 | 72465137 | 25784159 | 84951873 | 0 |
| 28959271 | 29438196 | 28926784 | 43852971 | 65267962 | 7 |
| 35425723 | 74884675 | 54799321 | 36616389 | 64269762 | 6 |
| 58517261 | 89366339 | 98523173 | 22458165 | 71951831 | 8 |
| 83649177 | 44277686 | 55858487 | 68665653 | 84467125 | 1 |
| 62785921 | 79939831 | 96439565 | 29926997 | 65823589 | 1 |
| 49269482 | 38243299 | 24944828 | 71814554 | 13234947 | 6 |
| 34412847 | 54714523 | 32382649 | 82992241 | 49518736 | 7 |
| 56339768 | 53666987 | 73935986 | 76647862 | 52679624 | 5 |
| 92678429 | 85439665 | 44793198 | 21252791 | 63742615 | 7 |
| 83764791 | 44762786 | 57885837 | 36528939 | 97643284 | 7 |
| | | | | 947099095 | 7 |

**36. Addition of more than one column in one operation.**

```
 3 2
 1 3
 2 5
 5 1
 2 4
-----
 1 4 5
```

PROCESS

24+50= 74+1= 75
75+20= 95+5=100
100+10=110+3=113
113+30=143+2=145

This method proceeds by adding the tens and then the units.

```
 3 4 2
 1 2 1
 2 3 4
 3 7 2
-------
 1 0 6 9
```

PROCESS

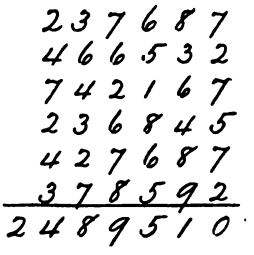
$372+\overbrace{200+30+4}+\overbrace{100+20+1}+\overbrace{300+40+2}=1069$

The student should be encouraged to use above methods. They give variation of method and train to quickness in mental extension.

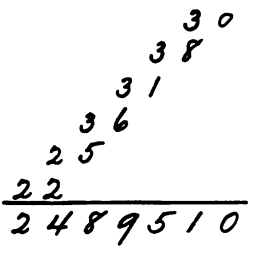
**37.** A common practise among bookkeepers and bankers, especially when frequent interruptions occur, is to indicate at the right the sum (including carrying

figure) of each column, the result being the addition of the last column and the last figure or cipher of each preceding sum in regular order. By the civil service method, the sums of the columns are set to the right in manner indicated and re-added.

BOOKKEEPER'S METHOD



CIVIL SERVICE METHOD



## CROSS ADDITION AND TABULAR WORK

**38.** The ability to add numbers horizontally, quickly and accurately, is almost as valuable as the ability to add vertically. Frequent drills should be given, and care should be taken to add only figures of the same order.

### WRITTEN PROBLEMS

**39.** *1.* Find the total number of yards in ten pieces containing the following numbers of yards: 25, 42, 37, 29, 35, 51, 47, 39, 31, 43.

*2.* Cash Sales

| Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Total |
|---|---|---|---|---|---|---|
| $123.45 | $272.68 | $89.07 | $431.22 | $345.16 | $650.05 | |

*3.* Coal Shipments—Pounds

| | Egg | Stove | Pea | Buckwheat | Totals |
|---|---|---|---|---|---|
| Monday | 327050 | 472150 | 72140 | 27040 | |
| Tuesday | 438160 | 384060 | 48120 | 38160 | |
| Wednesday | 358100 | 538010 | 38100 | 35100 | |
| Thursday | 376080 | 368070 | 68170 | 36080 | |
| Friday | 420120 | 240180 | 40080 | 18040 | |
| Saturday | 110060 | 125100 | 25010 | 10020 | |
| Totals | | | | | |

Total pounds of each kind shipped during week? Stove? Pea? Buckwheat? Total coal shipped each day? Total of all kinds for the week?

*4.* Sums disbursed from Peabody Fund, 1876-1880, inclusive.

| | 1876 | 1877 | 1878 | 1879 | 1880 | Totals |
|---|---|---|---|---|---|---|
| Virginia | $17800 | $18250 | $15350 | $ 9850 | $ 6800 | |
| North Carolina | 8050 | 4900 | 4500 | 6700 | 3050 | |
| South Carolina | 4150 | 4300 | 3600 | 4250 | 2700 | |
| Georgia | 3700 | 4000 | 6000 | 6500 | 5800 | |
| Tennessee | 10100 | 15850 | 14600 | 12000 | 10900 | |
| Totals | | | | | | |

*5.* Five salesmen made the following returns of sales during a week: On Monday —No. 1, $307.50; No. 2, $416.70; No. 3, $178.25; No. 4, $281.05; No. 5, $198.67. On Tuesday—No. 1, $209.10; No. 2, $258.40; No. 3, $227.72; No. 4, $203.50; No. 5, $268.80. On Wednesday—No. 1, $201.48; No. 2, $165.10; No. 3, $245; No. 4, $176.50; No. 5, $198.79. On Thursday—No. 1, $146.60; No. 2, $160.36; No. 3, $236.45; No. 4, $245.15; No. 5, $351.70. On Friday—No. 1, $185.50; No. 2, $240.30; No. 3, $149.30; No. 4, $194.82; No. 5, $282.70. On Saturday—No. 1, $301.70; No. 2, $337.40; No. 3, $341.80; No. 4, $380.78; No. 5, $417.80. Arrange in tabular form and answer the following questions: Upon what day was the greatest amount of sales made? Which salesman sold the greatest amount of goods? What was the total amount of sales for the week?

*6.* The following are the daily receipts of the treasurer of a borough for fourteen consecutive weeks:

| Weeks | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday | Totals |
|---|---|---|---|---|---|---|---|
| First | 486 24 | 360 50 | 48 21 | 516 18 | 233 40 | 108 96 | |
| Second | 132 25 | 897 56 | 437 52 | 213 78 | 478 09 | 980 43 | |
| Third | 731 64 | 930 48 | 376 83 | 481 63 | 704 45 | 173 26 | |
| Fourth | 897 26 | 270 45 | 103 48 | 862 24 | 238 27 | 527 28 | |
| Fifth | 188 25 | 390 31 | 976 82 | 932 21 | 752 34 | 438 21 | |
| Sixth | 297 11 | 434 56 | 542 26 | 276 43 | 177 30 | 222 43 | |
| Seventh | 729 55 | 721 14 | 297 31 | 274 90 | 249 32 | 576 87 | |
| Eighth | 253 54 | 490 78 | 864 44 | 831 82 | 757 42 | 543 85 | |
| Ninth | 238 80 | 809 66 | 198 23 | 582 64 | 523 45 | 350 67 | |
| Tenth | 487 56 | 789 40 | 169 45 | 346 24 | 434 56 | 456 78 | |
| Eleventh | 913 27 | 841 54 | 786 32 | 123 45 | 876 43 | 784 27 | |
| Twelfth | 1963 29 | 1187 95 | 864 21 | 487 91 | 798 63 | 1428 95 | |
| Thirteenth | 1048 63 | 968 42 | 1298 74 | 689 40 | 1698 75 | 1078 64 | |
| Fourteenth | 857 93 | 1236 97 | 1187 42 | 1693 75 | 1574 83 | 1870 74 | |
| Totals | | | | | | | |

Find the total receipts for each day; total receipts for each week; total receipts for the fourteen weeks.

# SUBTRACTION

**40. Subtraction** is the process of finding the *difference* between two numbers.

**41.** The **Difference** between two numbers is a number which, added to the less, will make it equal to the greater.

**42.** The **Subtrahend** is the number to be subtracted.

**43.** The **Minuend** is the number from which the subtrahend is taken.

**44.** The **Sign of Subtraction** is $-$, read *minus*, and denotes that the number following it is to be taken from the number preceding it.

**45.** The **Principles of Subtraction** are:

1. *Only like numbers can be subtracted.*
2. *Units of the same order only can be directly subtracted.*
3. *The difference is a number of the same kind as the minuend and subtrahend.*
4. *If the minuend and subtrahend be equally increased or diminished, the difference remains the same.*

**46. To find the difference between two numbers.**

EXAMPLES.—(1) Subtract 476 from 847; (2) from 10000 take 1346

$$\begin{array}{r} 847 \\ 476 \\ \hline 371 \end{array} \qquad \begin{array}{r} 10000 \\ 1346 \\ \hline 8654 \end{array}$$

After writing the subtrahend under the minuend so that units are under units, tens under tens, etc., we begin with units and say, "Six from seven leaves one," and write 1 under the units column. In the tens column we cannot take 7 from 4, so we borrow 1 (hundred) from the 8, which equals 10 tens, and add it to the 4 and say, "Seven from fourteen leaves seven," and write 7 under the tens. Having decreased 8 by borrowing from it, we now say, "Four from seven leaves three," and write 3 under the hundreds, which gives for a result, 371—the difference.

NOTE.—Another method of subtracting which has its advantages in such cases as Example 2 is as follows: Instead of borrowing 1 from the highest order and regarding it as a succession of 9's in each of the other orders excepting the units, where it must be considered as 10, the operation is performed by saying, "Six from ten leaves four; five from ten leaves five; four from ten leaves six; two from ten leaves eight"—increasing the subtrahend figure by 1 at each step after the first, and regarding the minuend each time as 10.

NOTE.—PROOF OF SUBTRACTION: The difference added to the subtrahend will equal the minuend, or by casting out 9's, the difference between the excess figure of minuend and excess figure of subtrahend (adding 9 to excess figure of minuend, if the smaller) will equal excess figure of remainder.

**47.** RULE.—*Write the subtrahend under the minuend so that units of the same order stand in the same column.*

*Beginning with the units, subtract each figure from the one above it, writing the difference under the figure subtracted.*

*If any figure in the minuend is less than the figure to be subtracted, increase it by* TEN, *by taking 1 from the next order above, then subtract.*

**48.** The following exercises are merely suggestive. When a figure in the minuend is the smaller, borrow a unit from the next higher order which is equal to ten of the lower order. Add ten and then subtract.

In oral drills, avoid such expressions as "from" and "leaves." Name results only.

### MENTAL EXERCISE

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 7 | 9 | 8 | 6 | 5 | 7 | 8 | 6 | 9 | 7 | 7 | 8 | 6 | 8 | 9 |
| 3 | 4 | 5 | 2 | 3 | 4 | 2 | 4 | 3 | 5 | 2 | 3 | 3 | 6 | 6 |
| 27 | 36 | 45 | 18 | 19 | 37 | 39 | 35 | 28 | 38 | 48 | | | | |
| 12 | 14 | 13 | 13 | 12 | 11 | 15 | 14 | 17 | 18 | 23 | | | | |
| 49 | 56 | 45 | 48 | 54 | 47 | 65 | 44 | 36 | 37 | 67 | | | | |
| 33 | 32 | 21 | 27 | 30 | 35 | 43 | 23 | 23 | 22 | 25 | | | | |
| 57 | 76 | 84 | 96 | 79 | 84 | 59 | 46 | 86 | 75 | 66 | | | | |
| 23 | 32 | 31 | 43 | 67 | 52 | 24 | 21 | 53 | 21 | 22 | | | | |
| 43 | 32 | 34 | 37 | 51 | 34 | 41 | 36 | 50 | 42 | 35 | | | | |
| 19 | 25 | 18 | 19 | 16 | 15 | 19 | 17 | 21 | 27 | 19 | | | | |
| 46 | 53 | 49 | 40 | 52 | 34 | 47 | 64 | 45 | 52 | 32 | | | | |
| 18 | 26 | 23 | 22 | 23 | 25 | 19 | 27 | 28 | 23 | 19 | | | | |

### ORAL EXERCISE

**49.** *1.* From 28 take 17. From 63 take 36. From 52 take 25. From 35 take 19.

*2.* 5 from 9 leaves what? 3 from 8? 15 from 35? 37 from 73? 18 from 56? 34 from 43?

*3.* $28-19=?$ $34-23=?$ $37-28=?$ $75-59=?$ $37-18=?$

*4.* What is the difference between 34 and 57? 48 and 29? 85 and 57? 91 and 19? 57 and 75? 49 and 94?

*5.* What is the difference between \$17 and \$65? \$83 and \$29? \$67 and \$83? \$.53 and \$.27? \$.67 and \$1? \$.75 and \$2?

*6.* Two boys had respectively \$1.10 and \$.85; if each spends \$.35, how much money will they have left?

*7.* A boy had 73 cents and his father gave him a half dollar; if he spends 48 cents, how much has he remaining?

*8.* Two newsboys together bought newspapers to the value of 75 cents; one sold 55 cents worth, the other 65 cents worth. What did they gain?

*9.* A man has two fields, one of 32 acres, the other of 57 acres; he puts 48 acres in wheat and the rest in corn; how many acres in corn?

*10.* A man sold two horses for $135 and $110, respectively; what did he gain or lose, if he had paid $250 for the pair?

*11.* What is the value of 13+15−9? Of 23+33−17? Of 83−50+23? Of 24+48−13? Of 55−16−7+6?

*12.* Add 46 to 33 and take away 27. Take away 29 from 77 and add 15.

*13.* Add 7, 9, 8, 6 and 5 and from the result take the sum of 6, 4 and 7.

*14.* I sell a piano for $225, and receive cash $165 and a note to balance account. What is the face of the note?

*15.* A merchant buys a bill of merchandise for $383, giving a note for $175 and cash for balance. How much cash does he pay?

## CASHIERS' CALCULATIONS

**50.** *1.* A salesman handed in a sales check for $3.73 and a $5 bill. Hand him the change. Ans.—2 pennies, a quarter and a dollar.

*2.* Required the change, in the fewest denominations, for a sale of $4.19 to be taken out of a $5 bill. Ans.—1 penny, 1 nickel, 1 quarter, 1 half.

| *3.* Amount of sale | Money tendered |
|---|---|
| $ 3.19 | $10 bill |
| .75 | 5 bill |
| 3.79 | 20 bill |
| .18 | 2 bill |
| 1.17 | 1 bill and $\$\frac{1}{2}$ silver |
| 15.68 | Two 10 bills |
| 4.49 | 10 bill |
| 3.33 | 5 bill |
| 8.90 | 5 bill and two $2 bills |
| .65 | 2 bill |
| 14.91 | 20 bill |
| 1.93 | 5 bill |
| 1.67 | 1 bill and $\$\frac{1}{2}$ and $\$\frac{1}{4}$ silver |
| 13.23 | 10 bill and $5 bill |
| 3.09 | 2 bill and $1 and $\$\frac{1}{4}$ silver |
| 11.21 | 50 bill |
| 83.37 | 100 bill |
| 68.12 | Four 20 bills |
| 16.71 | 10 bill and $5 bill and $2 bill |

*4.* A salesman's check shows items 35 cents, 70 cents and 18 cents; a $20 bill is tendered. What change is handed back?

| *5.* Salesman's items | Money tendered |
|---|---|
| $.50, .20, .18 | $2 bill |
| .39, .15, .25, .10 | 5 bill |
| .75, .20, .30 | 10 bill |
| .36, .13, 1.20 | 2 bill |
| .87, .33, .37 | 20 bill |

## WRITTEN PROBLEMS

**51. To balance accounts.**

EXAMPLE.—Find the balance of

| DR. | CR. |
|---|---|
| $237.75 | $400.00 |
| 81.209 | 600.00 |
| 43.67 | 80.00 |
| 82.96 | |
| 133.45 | Balance, 229.92 |
| $1309.92 | $1309.92 |

**52.** RULE.—*Find the sum of the larger side, set the total beneath: add the columns of the other side, setting down in the place for the balance the figure necessary to make the required figure in the total: verify by adding and totaling all the items of the smaller side.*

**53.** *1.* Find the balance of

| | | | | |
|---|---|---|---|---|
| Debits | $243.75 | $18.94 | $309.90 | $475.60 |
| Credits | $385.20 | 10.50 | 930.27 | 50.40 |

NOTE.—Arrange amounts in columns and find balance as in model.

*2.* The debit items of a trial balance are $644.50, $454.70, $27, $44.80, $104; and the credit items, excepting capital account, are $212.50, $62.50. What is the amount of capital?

*3.* Cash items received during a day were $1000, $28.50, $63, $75; and the cash items paid out were $300, $50, $100, $72. What is the balance on hand?

*4.* The amount of bills receivable received during a month were $400, $100, $200, $150, and the amount paid on them was $150. How much remains unpaid?

*5.* Merchandise items bought in a month were respectively: $2440, $74.50, and $901.25. Sales made: $286.25, $400, $116.25, $314, $441.50, $572, $240, $309, $227.50, and $755.25. If all the merchandise was sold, what amount of gain or loss does the balance show?

*6.* The debit items of a loss and gain account are as follows: $250, $225, $153.60, $150, $254.76, $398.83, $12, $8.17, $71.27, $17.50. The credit items are $14.53, $150. $310. What is the balance of the account, and does it show a gain or a loss?

*7.* During the month of March a merchant allowed discounts to his customers as follows: $2.20, $5, $1.25, $1.53, $1.83, $14.58. He was allowed discounts by others $1.75, $25.39, $4.20, $9.58. Does his discount account show a gain or loss, and how much?

*8.* A merchant deposited in bank during a month $8692.60, $11833.93, $7933.93, and checked out $1600, $2800, $2670.60, $5870.60, $5792.60. What is his balance in bank?

*9.* A farmer made an inventory of his effects and found that he had land worth $12600, buildings worth $3450, live stock worth $2365, wheat $218.75, farming implements $680, household furniture $1475, J. Wright's note $300. He owed a mortgage of $6000, notes held by others $1575, and outstanding accounts $2367.80. What was his net worth?

*10.* A business has cash on hand $11780, notes on hand $650. It owes $3475. What is the net worth of the business?

*11.* A business man makes the following bank deposits: $1578.25, $614, $342.50, $595.69, $734, $110.10; and draws the following checks: $224.73, $219.20, $163.57, $163.75, $591.32, $176.67. What is his balance in bank?

*12.* Cash balance on hand, $9462.50; receipts for the day, $14165.75; cash paid out, $18328.62. Find cash balance at close of day.

*13.* To the merchandise credits, $17342.33, add the value of inventory, $4555.70, and take away the merchandise debits, $21876.66. What was gained?

*14.* Jones's account shows sundry debits as follows: $345.50, $427.15, $87.93, $1503.34, and the following credits: $250, $300.50, $475, $290.90, $75, $100, $175, $120.25. What is the balance of his account?

*15.* The gross weight of a car and its contents is 82525 pounds; if the car weighs 38500 pounds, what is the weight of the contents?

*16.* The resources of a business are: Cash $2351.23, Merchandise $5781.16, Real Estate $8200, Bills Receivable $1250, Personal Accounts due $1462.50. The liabilities are: Bills Payable $2550, Personal Accounts owing $714.69. The capital is the excess of the resources above the liabilities. What is its amount?

*17.* An erroneous trial balance is as follows:

| | *Dr.* | *Cr.* |
|---|---|---|
| T. W. Baylor | $11600.00 | $15000.00 |
| Cash | 60000.00 | 55000.00 |
| Real Estate | 4000.00 | 5000.00 |
| Merchandise | 21000.00 | 27500.00 |
| Expense | 4500.50 | |
| Bank Stock | 800.00 | 1050.50 |
| R. R. Stock | 1000.00 | |
| Interest | 350.75 | 100.75 |
| Commission | | 95.00 |
| Personal Accounts | 13910.00 | |
| Personal Accounts | | 2499.00 |

How much is it out?

*18.* Check Stub. Add deposits. Subtract checks drawn.

| | |
|---|---|
| Balance | $1500.00 |
| Deposits | 465.50 |
| Drawings | 332.25 |
| Balance | |
| Deposits | 825.10 |
| Drawings | 342.90 |
| Balance | |
| Deposits | 119.62 |
| Drawings | 485.00 |
| Balance | |
| Deposits | 526.30 |
| Drawings | 310.80 |
| Balance | |
| Deposits | 48.60 |
| Drawings | 1200.00 |
| Balance | |

*19.* Arrange items and perform operation as in problem No. 18.

William Bray had, on Monday, a bank balance of $685.61; he deposited $412.80 and drew checks for $122.59, $145.80, and $157.10. On Tuesday he drew checks for $158.99, $100.67, $149.27, and $120. What is the balance to his credit on Tuesday night?

*20.* John Cake had, on Monday, a bank balance of $22.60 and deposited $768.25. On Tuesday he drew checks for $156.28, $191.75, $156.28, and $201. What is the balance to his credit on Tuesday night?

# MULTIPLICATION

**54. Multiplication** is the process of finding the *product* of two numbers.

**55.** The **Product** of two numbers is the sum obtained by adding one of the numbers to itself as many times as there are units in the other number.

**56.** The **Multiplicand** is the number to be multiplied.

**57.** The **Multiplier** is the number which shows how many times the multiplicand is to be repeated.

**58.** The **Sign of Multiplication** is $\times$, read *multiplied by*, and denotes that the number preceding it is to be multiplied by the number following it.

**59.** The **Principles of Multiplication** are:

*1. The product is a number of the same kind as the multiplicand.*

*2. The multiplier is an abstract number, because it denotes the number of times the multiplicand is repeated to produce the product.*

*3. The product of two abstract numbers is the same, whichever is made the multiplier.*

*4. If the multiplier be separated into parts, and the multiplicand be multiplied by each part separately, the* SUM *of the partial products will be the entire product; as, fifteen times a number is equal to ten times the number plus five times the number.*

**60. To find the product of two numbers.**

EXAMPLE.—Multiply 436 by 287

```
  436
  287
 3052
3488
 872
125132
```

Write the multiplier beneath the multiplicand, and draw a line. Seven times 6 units are 42 units—write 2 directly under the 7 (so that it will be in units column) and carry 4 tens. Seven times three tens are 21 tens (4 tens carried) are 25 tens—write 5 in the tens column and carry 2 hundreds. Seven times 4 hundreds (2 hundreds carried) are 30 hundreds—write 30.

Eight times 6 are 48—write 8 under the tens figure (5) of the first *partial product*, and carry 4. Eight times 3 (4 carried) are 28 hundreds—write 8 under hundreds column, and carry 2. Eight times 4 (2 carried) are 34—write 34.

Two times 6 are 12—write 2 under hundreds column, and carry 1. Two times 3 (1 carried) are 7—write 7 under thousands column. Two times 4 are 8—write 8 under ten-thousands column.

Draw a line and add the three *partial products*, which gives 125132, or 287 times 436.

**61.** RULE.—*Write the multiplier under the multiplicand, so that units of the same order shall stand in the same column.*

*Beginning with the units, multiply the multiplicand by each figure of the multiplier, separately, writing the first figure of each partial product under that figure of the multiplier which produced it.*

*Add the partial products.*

**62. To prove correctness of result.**

EXAMPLES.—(1) 327×632; (2) 526×437

```
  327 — 3          526 — 4
  632 — 2          437 — 5
  654   6         3682  20 — 2
  981             1578
 1962            2104
206664 — 6       229562 — 2
```

NOTE.—Multiply the multiplier by the multiplicand; if the same result is obtained, it is almost sure to be correct.

A shorter method is to cast out the 9's of the multiplicand and multiplier and multiply the excess figures together. If correct, the excess figures of the two products will agree. (Also see Example 1, page 79.)

**63.** The multiplication of any two numbers, however large, is but a succession of multiplications of a single figure by a single figure. Any multiplication whatever may be performed by the student who has mastered the following table, which contains all the products of any two of the figures 2, 3, 4, 5, 6, 7, 8, 9:

**MULTIPLICATION TABLE**

| | | | |
|---|---|---|---|
| 2 × 2 = 4 | 3 × 3 = 9 | 4 × 4 = 16 | 5 × 5 = 25 |
| 2 × 3 = 6 | 3 × 4 = 12 | 4 × 5 = 20 | 5 × 6 = 30 |
| 2 × 4 = 8 | 3 × 5 = 15 | 4 × 6 = 24 | 5 × 7 = 35 |
| 2 × 5 = 10 | 3 × 6 = 18 | 4 × 7 = 28 | 5 × 8 = 40 |
| 2 × 6 = 12 | 3 × 7 = 21 | 4 × 8 = 32 | 5 × 9 = 45 |
| 2 × 7 = 14 | 3 × 8 = 24 | 4 × 9 = 36 | 6 × 6 = 36 |
| 2 × 8 = 16 | 3 × 9 = 27 | 7 × 7 = 49 | 6 × 7 = 42 |
| 2 × 9 = 18 | 8 × 8 = 64 | 7 × 8 = 56 | 6 × 8 = 48 |
| 9 × 9 = 81 | 8 × 9 = 72 | 7 × 9 = 63 | 6 × 9 = 54 |

**64.** While the foregoing table is all that is absolutely necessary, the student, in order to become a rapid calculator, should, from time to time, increase his store of memorized products by careful study and drill upon the following extended tables, until he can name instantly the product of any two numbers up to 25 times 25. This will enable him to perform with great rapidity many commercial calculations, especially in making extensions on invoices, etc.

**Multiplication Table Extended to 12 × 12**

**65.**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | **4** | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | **9** | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | **16** | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | **25** | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | **36** | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | **49** | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | **64** | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | **81** | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | **100** | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | **121** | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | **144** |

NOTE.—The numbers in BOLD-FACED TYPE—products of two equal factors, or a number multiplied by itself—are called *squares;* as, 16 is the square of 4.

## ORAL MULTIPLICATION DRILL

**66.** A very useful drill to enable the student to master readily the extended tables is as follows: Name all the products of one factor in regular order from the lowest to the highest, by mentally adding at each step, as in addition, and then repeat them again from highest to lowest, mentally subtracting at each step, as in subtraction.

It is, of course, necessary to name the factors as well as the products—otherwise it would be merely an addition and subtraction drill. "Twice 13 are 26, three times 13 are 39, four times 13 are 52, five times 13 are 65, six times 13 are 78," etc., to "twenty-five times 13 are 325." Then reverse—"Twenty-four times 13 are 312, twenty-three times 13 are 299," etc.

**Multiplication Table Extended to 20×20**

**67.**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 13 | 26 | 39 | 52 | 65 | 78 | 91 | 104 | 117 | 130 | 143 | 156 | **169** | 182 | 195 | 208 | 221 | 234 | 247 | 260 |
| 14 | 28 | 42 | 56 | 70 | 84 | 98 | 112 | 126 | 140 | 154 | 168 | 182 | **196** | 210 | 224 | 238 | 252 | 266 | 280 |
| 15 | 30 | 45 | 60 | 75 | 90 | 105 | 120 | 135 | 150 | 165 | 180 | 195 | 210 | **225** | 240 | 255 | 270 | 285 | 300 |
| 16 | 32 | 48 | 64 | 80 | 96 | 112 | 128 | 144 | 160 | 176 | 192 | 208 | 224 | 240 | **256** | 272 | 288 | 304 | 320 |
| 17 | 34 | 51 | 68 | 85 | 102 | 119 | 136 | 153 | 170 | 187 | 204 | 221 | 238 | 255 | 272 | **289** | 306 | 323 | 340 |
| 18 | 36 | 54 | 72 | 90 | 108 | 126 | 144 | 162 | 180 | 198 | 216 | 234 | 252 | 270 | 288 | 306 | **324** | 342 | 360 |
| 19 | 38 | 57 | 76 | 95 | 114 | 133 | 152 | 171 | 190 | 209 | 228 | 247 | 266 | 285 | 304 | 323 | 342 | **361** | 380 |
| 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 | 260 | 280 | 300 | 320 | 340 | 360 | 380 | **400** |

**Multiplication Table Extended to 25×25**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 21 | 42 | 63 | 84 | 105 | 126 | 147 | 168 | 189 | 210 | 231 | 252 | 273 | 294 | 315 | 336 | 357 | 378 | 399 | 420 | **441** | 462 | 483 | 504 | 525 |
| 22 | 44 | 66 | 88 | 110 | 132 | 154 | 176 | 198 | 220 | 242 | 264 | 286 | 308 | 330 | 352 | 374 | 396 | 418 | 440 | 462 | **484** | 506 | 528 | 550 |
| 23 | 46 | 69 | 92 | 115 | 138 | 161 | 184 | 207 | 230 | 253 | 276 | 299 | 322 | 345 | 368 | 391 | 414 | 437 | 460 | 483 | 506 | **529** | 552 | 575 |
| 24 | 48 | 72 | 96 | 120 | 144 | 168 | 192 | 216 | 240 | 264 | 288 | 312 | 336 | 360 | 384 | 408 | 432 | 456 | 480 | 504 | 528 | 552 | **576** | 600 |
| 25 | 50 | 75 | 100 | 125 | 150 | 175 | 200 | 225 | 250 | 275 | 300 | 325 | 350 | 375 | 400 | 425 | 450 | 475 | 500 | 525 | 550 | 575 | 600 | **625** |

**ORAL EXERCISE**

**68.** Ex.—If a train runs 15 miles an hour, how far will it run in 8 hours?

Ans.—It will run 8 times 15 miles, which is 120 miles.

*1.* What will 15 roses cost at 5 cents each?

*2.* At 9 cents a yard, what will 18 yards of ribbon cost?

*3.* A clerk earns $14 a week; how much will he earn in 9 weeks?

*4.* A steamer goes 14 miles an hour; how many miles will it go in 11 hours?

*5.* What will be the cost of 6 oranges at 5 cents each, and 12 lemons at 3 cents each?

*6.* What is the value of 7 cows at $25 each?

*7.* What will be the cost of 7 yards muslin at 12 cents a yard, 3 spools thread at 2 cents each, and a paper of needles 5 cents?

*8.* If three men can do a piece of work in 14 days, how long would it requir one man to do it?

*9.* B having 35 sheep sold 18 of them and then bought 3 times as many as h had left; how many did he then have?

*10.* An engine travels 30 miles an hour and a trolley car 22 miles an hour; hov much farther will the engine travel in 8 hours than the trolley car?

## SALESMEN'S AND CASHIERS' CALCULATIONS

**69.** Required the amount of change in the fewest denominations for each of th following sales:

| | Items of Sale | | | Money Tendered |
|---|---|---|---|---|
| *1.* | 6 lbs. Coffee | @ | $.28 | $2 bill |
| *2.* | 3 " Tea | " | .60 | Two silver dollars |
| *3.* | 25 " Sugar | " | .05 | $1 bill and $\frac{1}{2}$ silver |
| *4.* | 24 cans Corn | " | .06 | 5 bill |
| *5.* | 6 doz. Oranges | " | .30 | 2 bill |
| *6.* | 12 cans Tomatoes | " | .08 | 2 bill |
| *7.* | 5 lbs. Tea | " | .75 | 10 bill |
| *8.* | 11 " Beef | " | .12 | 5 bill |
| *9.* | 6 " Butter | " | .27 | 2 bill |
| *10.* | 8 " Prunes | " | .14 | 5 bill |
| *11.* | 10 " Soap | " | .07 | 2 bill |
| *12.* | 5 " Starch | " | .08 | 1 bill |
| *13.* | 6 " Rice | " | .07 | $\frac{1}{2}$ silver |
| *14.* | 3 yds. Silk | " | .65 | 5 bill |
| *15.* | 12 " Ribbon | " | .15 | 5 bill |
| *16.* | 12 spools Cotton | " | .03 | 2 bill |
| *17.* | 12 yds. Chintz | " | .18 | 5 bill |
| *18.* | 25 " Muslin | " | .08 | 10 bill |
| *19.* | 6 pairs Hose | " | .25 | 5 bill |
| *20.* | 12 yds. Flannel | " | .18 | 2 bill and $1 silver |

## WRITTEN EXERCISE

**70.** *1.* Multiply $475.25 by 25
*2.* Multiply 28375 by 153
*3.* Multiply 62137 by 328
*4.* Multiply 1756 by 488
*5.* Multiply 3991 by 998
*6.* Multiply $486.65 by 117
*7.* Multiply 3937 by 4256
*8.* Multiply 5236 by 973
*9.* Multiply 215042 by 9364
*10.* Multiply 231 by 9997
*11.* Multiply 28317 by 472
*12.* Multiply 6452 by 324
*13.* Multiply 28375 by 756
*14.* Multiply 25293 by 4938
*15.* Multiply 311035 by 426
*16.* Multiply 16387 by 729
*17.* Multiply $442.37 by 674
*18.* Multiply 5029 by 943
*19.* Multiply $316.75 by 749
*20.* Multiply 91442 by 3664

## SPECIAL RULES AND SHORT METHODS

**71. To multiply by 10, 100, 1000, etc.**

**72.** RULE.—*At the right of the multiplicand place as many ciphers as are found in the multiplier.*

**73. To find the product of two numbers when either or both end in ciphers.**

EXAMPLE.—Multiply 2590 by 38000

**74.** RULE.—*Find the product of the significant figures of the two factors, and to the right annex as many ciphers as are found at the right of both.*

$$\begin{array}{r} 2590 \\ \times\ 38000 \\ \hline 20720 \\ 7770 \\ \hline 98420000 \end{array}$$

NOTE.—The judgment of the teacher must be exercised as to the introduction of short rules. Some may be used to advantage with beginning grades; others with more advanced students.

**75. To multiply by 25.**

$$4)\overline{3141600}$$
$$785400$$

EXAMPLE.—Multiply 31416 by 25

By annexing two ciphers the number is multiplied by 100, or 4 times the value of 25.

**76.** RULE.—*Multiply by 100 (by annexing 2 ciphers) and divide by 4.*

### EXERCISE

**77.** 1. Multiply 3048 by 25
2. Multiply 9144 by 25
3. Multiply 5029 by 25
4. Multiply 16093 by 25
5. Multiply 6452 by 25
6. Multiply 92903 by 25
7. Multiply 8361 by 25
8. Multiply 25293 by 25
9. Multiply 4047 by 25
10. Multiply 16387 by 25

**78. To multiply by 250, 125,** etc., is but an extension of the above rule.

*Multiply by 1000 (by annexing three ciphers) and divide by 4 and 8 respectively, 1000 being 4 times the value of 250, or 8 times the value of 125. Use Exercise 77 for drill.*

**79. To multiply by a number a little less or more than 100, 1000,** etc.

EXAMPLE.—Multiply 3937 by 998.

$$\begin{array}{r} 3937000 \\ \quad 7874 \\ \hline 3929126 \end{array}$$

(3937×2) Subtract

**80.** RULE.—*Annex to the multiplicand 2 ciphers if the multiplier is 99 or 101; 3 ciphers if 999 or 1001, etc. To this result add the multiplicand when the multiplier is 1 more, or subtract when the multiplier is 1 less; if 2 more, add twice the multiplicand; if 2 less, subtract twice the multiplicand, etc.*

### WRITTEN EXERCISE

**81.** *1.* Multiply \$1875.24 by 97
*2.* Multiply \$234.70 by 99
*3.* Multiply \$558.96 by 998
*4.* Multiply \$437.10 by 95
*5.* Multiply \$442.37 by 997
*6.* Multiply \$120.92 by 9997
*7.* Multiply \$316.75 by 96
*8.* Multiply \$242.71 by 999
*9.* Multiply \$716.32 by 94
*10.* Multiply \$176.19 by 9999

**82. To multiply when one part of the multiplier is a factor of the other part.**

EXAMPLES.—(*1*) Multiply \$325.50 by 357; (*2*) 7854 by 428

\$325.50
357
2278 50 Product by 7
1139250 " " 5×7
\$116203.50

7854
428
31416 Product by 4
219912 " " 7×4
3361512

**83.** RULE.—*First multiply by one factor; then multiply the partial product by the other factor.*

NOTE.—The factors of a number are the numbers which when multiplied will produce it; as 7 and 4 are the factors of 28.

### WRITTEN EXERCISE

**84.** *1.* Multiply 28317 by 284
*2.* Multiply 7645 by 328
*3.* Multiply 3624 by 324
*4.* Multiply 9463 by 637
*5.* Multiply 3785 by 426
*6.* Multiply 3524 by 735
*7.* Multiply 2835 by 636
*8.* Multiply 9072 by 954
*9.* Multiply 648 by 432
*10.* Multiply 311035 by 864

**85. To multiply by 11, 111, etc.**

EXAMPLE.—Multiply 193 by 11, the result by 111, and that result by 1111

193
11
2123
111
235653
1111
26181048 3

**86.** RULE.—*To multiply by 11, set down the right-hand figure of the multiplicand, then the sum of the first and second figures, the sum of the second and third, and so on, carrying as usual, and writing the left-hand figure of the multiplicand (plus any carrying figure) as the last figure or figures of the product.*

*To multiply by 111, set down the right-hand figure of the multiplicand as before, then the sum of the first and second, then the sum of the first, second and third, of the second, third and fourth, and so on, carrying as usual. For 1111, use each figure four times.*

### WRITTEN EXERCISE

**67.** *1.* 3732 × 11, 111
*2.* 3937 × 11, 111
*3.* 10936 × 11, 111
*4.* 26417 × 11, 111
*5.* 28375 × 11, 111
*6.* 3527 × 11, 111
*7.* 22046 × 11, 111
*8.* 15432 × 11, 111
*9.* 3215 × 11, 111
*10.* 2679 × 11, 111
*11.* 2150 × 11, 111
*12.* 1728 × 11, 111

### CROSS MULTIPLICATION

**88. Cross Multiplication** consists in multiplying one number by another without writing partial products, setting down the result only, performing all of the calculations mentally. It is especially useful in bill work. By practise, operations can be performed readily with multipliers of three or more figures.

**89. To multiply two figures by two figures, writing product only.**

EXAMPLE.—Multiply 46 by 32

46
32
1472

"2 times 6 are 12"—write 2, and carry 1. "3 times 6 are 18 and 1 (carried) makes 19; 2 times 4 are 8, plus 19 makes 27"—write 7, and carry 2. "3 times 4 are 12, and 2 (carried) makes 14."

**90.** RULE.—*Multiply units by units and set down right-hand figure. Multiply units of multiplicand by tens of multiplier, add the carried figure, and to the sum add product of tens of multiplicand by units of multiplier, setting down right-hand figure. Multiply tens by tens, add the carried figure, and write both figures of result.*

### WRITTEN EXERCISE

**91.** *1.* 27 × 27 = ?
*2.* 49 × 26 = ?
*3.* 33 × 63 = ?
*4.* 57 × 75 = ?
*5.* 37 × 86 = ?
*6.* 39 × 74 = ?
*7.* 48 × 34 = ?
*8.* 93 × 89 = ?
*9.* 72 × 67 = ?
*10.* 64 × 53 = ?
*11.* 28 × 35 = ?
*12.* 52 × 29 = ?
*13.* 45 × 73 = ?
*14.* 68 × 56 = ?
*15.* 47 × 69 = ?
*16.* 37 × 43 = ?
*17.* 79 × 38 = ?
*18.* 54 × 83 = ?
*19.* 87 × 94 = ?
*20.* 95 × 58 = ?

**92. To multiply three figures by two figures, writing product only.**

EXAMPLE.—Multiply 436 by 78

436×78=34008

8×6=48—write 8.
7×6=42, plus 4, 46 plus (8×3) 24=70—write 0.
7×3=21, plus 7, 28 plus (8×4) 32=60—write 0.
7×4=28, plus 6=34—write 34.

**93.** RULE.—*1. Multiply the* UNITS *(of the multiplicand) by units (of the multiplier), write right-hand figure.*

*2. Multiply* UNITS *by tens, add carried figure, then* TENS *by units, add, write right-hand figure.*

*3. Multiply* TENS *by tens, add carried figure, then* HUNDREDS *by units, add, write right-hand figure.*

*4. Multiply* HUNDREDS *by tens, add carried figure, write result.*

### WRITTEN EXERCISE

**94.** *1.* $234 \times 46 = ?$ *5.* $613 \times 28 = ?$ *9.* $845 \times 64 = ?$ *13.* $282 \times 48 = ?$
*2.* $368 \times 32 = ?$ *6.* $438 \times 47 = ?$ *10.* $529 \times 52 = ?$ *14.* $375 \times 37 = ?$
*3.* $421 \times 26 = ?$ *7.* $347 \times 63 = ?$ *11.* $672 \times 74 = ?$ *15.* $913 \times 82 = ?$
*4.* $732 \times 34 = ?$ *8.* $764 \times 78 = ?$ *12.* $438 \times 65 = ?$ *16.* $742 \times 94 = ?$

**95. To multiply three figures by three figures, writing product only.**

EXAMPLE.—Multiply 521 by 346

$521 \times 346 = 180266$

| | | | |
|---|---|---|---|
| | | $1 \times 6$ | $= 6.$ |
| $(1 \times 4) + 0$ | carried | $+ (2 \times 6)$ | $= 16.$ |
| $(1 \times 3) + 1$ | " | $+ (2 \times 4) + (5 \times 6)$ | $= 42.$ |
| $(2 \times 3) + 4$ | " | $+ (5 \times 4)$ | $= 30.$ |
| $(5 \times 3) + 3$ | " | | $= 18.$ |

**96.** RULE.—*1.* UNITS $\times$ *units.*

*2.* (UNITS $\times$ *tens*) $+$ (TENS $\times$ *units.*)

*3.* (UNITS $\times$ *hundreds*) $+$ (TENS $\times$ *tens*) $+$ (HUNDREDS $\times$ *units.*)

*4.* (TENS $\times$ *hundreds*) $+$ (HUNDREDS $\times$ *tens.*)

*5.* HUNDREDS *by hundreds.*

### WRITTEN EXERCISE

**97.** *1.* $329 \times 642 = ?$ *5.* $918 \times 475 = ?$ *9.* $678 \times 343 = ?$
*2.* $473 \times 567 = ?$ *6.* $237 \times 646 = ?$ *10.* $834 \times 937 = ?$
*3.* $234 \times 319 = ?$ *7.* $786 \times 372 = ?$ *11.* $712 \times 846 = ?$
*4.* $748 \times 325 = ?$ *8.* $589 \times 716 = ?$ *12.* $576 \times 672 = ?$

**98. To square a number of two figures.**

EXAMPLES.—(*1*) Find the square of 42; (*2*) of 64

| | | | |
|---|---|---|---|
| 42 | $(2 \times 2)$ | 64 | $(4 \times 4)$ |
| 42 | $(4 + 4) \times 2$ | 64 | $(6 + 6) \times 4 + 1$ |
| 1764 | $(4 \times 4) + 1$ | 4096 | $(6 \times 6) + 4$ |

**99.** RULE.—(*1*) *Multiply together the right-hand figures;* (*2*) *multiply the sum of the left-hand figures by the right-hand figure of the multiplier;* (*3*) *multiply together the left-hand figures; carry at each step, as in ordinary multiplication.*

**100. To find the product of two numbers whose units add to 10 and whose tens are alike.**

EXAMPLES.—(*1*) Multiply $66 \times 64$; (*2*) $88 \times 82$

| | | | |
|---|---|---|---|
| 66 | $6 \times 4$—write 24 | 88 | $8 \times 2$—write 16 |
| 64 | $6 + 1 \times 6$—write 42 | 82 | $8 + 1 \times 8$—write 72 |
| 4224 | | 7216 | |

**101.** RULE.—*Multiply together the units and write the result; add 1 to the tens of the multiplicand, multiply by the tens of the multiplier, and prefix the result to the product of the units.*

NOTE.—It will be observed that but a few of the many methods of short multiplication are presented, but the few given are easily mastered and very practical.

### WRITTEN EXERCISE

**102.**

1. $45 \times 45$
2. $95 \times 95$
3. $27 \times 27$
4. $44 \times 44$
5. $64 \times 64$
6. $36 \times 36$
7. $56 \times 54$
8. $38 \times 32$
9. $67 \times 63$
10. $85 \times 85$

### WRITTEN PROBLEMS

**103.** *1.* What is the cost of 3456 sheep at \$6 per head?

*2.* A man bought a farm containing 495 acres at \$28 an acre. What was the cost of the farm?

*3.* A farmer sold 96 bushels of wheat at 57 cents a bushel. How much money did he receive?

*4.* At 63 cents a bushel, what will 325 bushels of potatoes cost?

*5.* Find the value of 16 crates of eggs, each crate containing 32 dozen at 22 cents a dozen.

*6.* A cotton factory contains 245 looms; if each loom makes 37 yards of cloth daily, how much cloth will the factory make in 70 days?

*7.* There are four fields, each containing 585 hills of potatoes, and every hill averages 12 potatoes. How many potatoes in the four fields?

*8.* If a sawmill saws 5768 feet of boards in a day, how many feet will it turn out in 47 weeks of 6 days each?

*9.* A house requires 4765 shingles on each of two sides of the roof. How many shingles are necessary for an operation of 80 houses?

*10.* A coal dealer bought 45 cars of coal containing 25 tons each. What was the cost of it all at \$4.50 a ton?

*11.* A farmer harvested 42 bushels of grain per acre from a field of 22 acres. What did he receive for it at 95 cents a bushel?

*12.* Mr. H. Humphreys bought 6 bureaus at \$8.75 each; 3 easy-chairs at \$15.32 each; 12 dining-room chairs at \$4.67; 15 mattresses at \$13.75; 2 extension tables at \$17.85; 4 mirrors at \$9.79. What was the total amount of the bill?

*13.* James White bought of John Black 26 yards of cloth at \$3.25 per yard; 147 yards of sheeting at 26 cents a yard; 28 yards of table linen at \$1.75 per yard; 23 dozen towels at \$3.25 per dozen.

He sold him 27 lambs at \$7.15 each; 15 yearlings at \$15 each; 47 chickens at 57 cents each. Which owes the other and how much?

*14.* A railway 1326 miles in length was constructed at an average cost of \$63,275 per mile. What was the total cost?

*15.* A carpenter works every day in the year but Sundays. He receives \$5 per day. How much does he save in one year (52 weeks) if his expenses amount to \$12 per week?

## DIVISION

**104. Division** is the process of finding how many times one number is contained in another.

**105.** The **Dividend** is the number to be divided.

**106.** The **Divisor** is the number by which the dividend is divided.

**107.** The **Quotient** is the number which shows how many times the divisor is contained in the dividend.

**108.** The **Remainder** is that part of the dividend left after dividing, when the dividend does not contain the divisor an exact number of times.

**109.** The **Sign of Division** is ÷, read *divided by*, and denotes that the number preceding it is to be divided by the number following it.

**110.** The **Principles of Division** are:

*1. The divisor must be a number of the same kind as the dividend;* for division is a *continued subtraction* of the divisor from the dividend (the quotient showing how many times it is taken), and it is evident that nothing could be taken from the dividend but a part of itself.

*2. The quotient is an abstract number, because it shows the number of* TIMES *the dividend contains the divisor.*

*3. If both dividend and divisor be multiplied or divided by the same number, the quotient of the results obtained will be the same as the quotient of the original numbers.*

## SHORT DIVISION

**111.** The term **"short division"** is used when the divisor is small enough that the operation may be performed mentally.

**112. To divide by short division.**

EXAMPLES.—(*1*) Divide 32741 by 6; (*2*) 265085 by 12

$$6\overline{)32741} = 5456\tfrac{5}{6} \qquad 12\overline{)265085} = 22090\tfrac{5}{12}$$

**113.** RULE.—*Write the divisor to the left of the dividend, with a curved line between them, and draw a horizontal line under the dividend.*

*Begin with the first figure at the left of the dividend (or, if that figure is less than the divisor, use the first two figures), divide and write the quotient under the figure divided.*

Thus in Ex. 1, since 6 is not contained in 3, we say, "Six in 32 five times, and 2 over," and write 5 under the 2.

*When the division is not even or exact, prefix the remainder (mentally) to the next figure of the dividend before dividing it.*

Thus in Ex. 1, prefixing the remainder 2 to the 7, we say, "Six in 27, four times, and 3 over," and write 4 under 7; then proceeding, as before, we say, "Six in 34 five times, and 4 over, six in 41 six times and 5 over," writing the 4 and 6 in their proper positions and placing the last remainder 5 over the divisor 6 (with a line between them) at the right of the quotient.

*Whenever a partial dividend (except the first) is less than the divisor, write a cipher (0) in the quotient and proceed with the division, including the next figure in the partial dividend.*

Thus in Ex. 2, the second division leaves a remainder of 1, which prefixed to the cipher in the dividend makes 10, which is less than 12, so we write 0 in the quotient, and, prefixing the 10 to the 8, we say, "12 in 108 nine times"—no remainder. Then, 12 being less than 5, we place another cipher in the quotient and write the remainder 5 after it, with the divisor under it in the form of a fraction, $\frac{5}{12}$.

NOTE.—A "partial dividend" is that part of the dividend used at each of the successive steps in the operation.

## ORAL EXERCISE

**114.** *1.* At 3 cents each, how many oranges can be bought for 36 cents?

*2.* At 8 cents a yard, how many yards of ribbon can be bought for 64 cents?

*3.* Gave $60 for sheep at the rate of $5 a head, how many sheep did I buy?

*4.* If a wheelman rides 12 miles an hour, how long will it take him to travel 96 miles?

*5.* How many kegs of 10 gallons each can be filled from barrels containing 140 gallons?

*6.* How many days will it take a man to earn $96 if he receives $6 a day?

*7.* How many bushels of wheat can a man buy for $84 if he pays $6 for 5 bushels?

*8.* A man gave a group of boys 56 cents; if each boy received 14 cents, how many in the group?

*9.* If coal is $5 a ton, how many tons can be bought for $95?

*10.* If 12 hams weigh 180 lbs., what is the average weight of a ham?

### WRITTEN EXERCISE

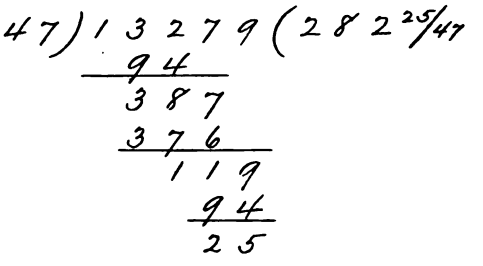
**115.** *1.* Divide 79171923 by 2
*2.* Divide 87682835 by 3
*3.* Divide 96303847 by 4
*4.* Divide 15274652 by 5
*5.* Divide 84205561 by 6
*6.* Divide 65425622 by 7
*7.* Divide 82562828 by 8
*8.* Divide 3453488739 by 4
*9.* Divide 4568392825 by 6
*10.* Divide 7643912848 by 7
*11.* Divide 5324769214 by 8
*12.* Divide 6843179157 by 9
*13.* Divide 7289374673 by 11
*14.* Divide 8924321866 by 12

## LONG DIVISION

**116. Long Division** is applied when the divisor is too large to be handled as a purely mental operation—that is, when the figures of the successive multiplications and subtractions must be written down.

**117. To divide a number by long division.**

EXAMPLE.—Divide 13279 by 47




Result, $282\frac{25}{47}$

**118.** RULE.—*Write the divisor to the left of the dividend with a curved line between them, and another curved line to the right of the dividend to separate the quotient from it.*

*Determine, by inspection, how many times the divisor is contained in the fewest left-hand figures of the dividend, and write the number to the right of the dividend.*

Thus, in the accompanying Example, since 47 is not contained in 1 or in 13, we take 132 as the first partial dividend. By comparing them we see that the left-hand figure of the divisor (4) would be contained in 13 three times; but by inspection, we see that multiplying by 3 would give 2 to carry from the 3 times 7, which added to 3 times 4 would make 14. Hence, we conclude that the first quotient figure should be 2, which we place in the quotient to the right of the dividend.

*Multiply the divisor by the quotient figure just found, and write the product under the partial dividend. Draw a line, subtract and write the remainder beneath.*

Multiplying 47 by 2 gives us 94, which, subtracted from the partial dividend, leaves 38.

*Bring down the next figure of the dividend and place it at the right of the remainder, making the next partial dividend.*

Bringing down the 7 gives us 387 for a partial dividend, which, by inspection as before, we can see will contain 47 eight times; write 8 in the quotient.

*Proceed as with first partial dividend, and continue until all the dividend figures have been brought down.*

NOTE.—PROOF OF DIVISION: Multiply the quotient by the divisor, and add the remainder to the product. The sum should equal the dividend; or, in an even division, after casting out 9's in the dividend and quotient, multiply excess figures of divisor and quotient. The excess figure of product must then equal excess figure of dividend. If there is a remainder, its excess figure is subtracted from the excess figure of the dividend or that figure +9 (if smaller than remainder excess), which gives the excess figure of dividend. See page 80.

### WRITTEN EXERCISE

**119.**

1. Divide 63247 by 53
2. Divide 24876 by 89
3. Divide 31845 by 67
4. Divide 27328 by 73
5. Divide 486329 by 213
6. Divide 675471 by 379
7. Divide 39754 by 437
8. Divide 298546 by 913
9. Divide 3875213 by 634
10. Divide 4783976 by 719
11. Divide 5254361 by 2483
12. Divide 3989768 by 7356
13. Divide 42864374 by 8429
14. Divide 56893573 by 3067
15. Divide 67654826 by 5246
16. Divide 32120487 by 4983
17. Divide 748973254 by 35826
18. Divide 656438602 by 48375
19. Divide 392876348 by 25764
20. Divide 487326454 by 384213

## SPECIAL RULES IN DIVISION

**120. To divide by 10, 100, 1000, etc.**

EXAMPLE.—34837 ÷ 100

$$348\,|\,37 = 348\tfrac{37}{100} = .37$$

**121.** RULE.—*Cut off at the right of the dividend as many figures as there are ciphers in the divisor. The figures remaining to the left will be the quotient, and the figures cut off from the right will be the remainder.*

**122. To divide by any number ending in ciphers.**

EXAMPLE.—Divide 4456328 by 32700

Result, $136\tfrac{2128}{3270}$

$$\begin{array}{r|l} & 136 \\ 3270 \overline{)} & 4456328 \\ & 327 \\ \hline & 1186 \\ & 981 \\ \hline & 2052 \\ & 1962 \\ \hline & 90 \end{array}$$

**123.** RULE.—*Cut off the ciphers at the right of the divisor and an equal number of figures at the right of the dividend.*

*Find the quotient by dividing the remaining figures of the dividend by the significant figures of the divisor, writing the quotient figure at each step directly above the right-hand figure of the partial dividend.*

*To the remainder, after this operation, annex at the right the figures cut off from the dividend, which gives the entire remainder.*

**124. To divide by 25, 250, etc.**

EXAMPLES.—(*1*) Divide 62438 by 25; (*2*) 45659 ÷ 250

$$\begin{array}{r|l} 624 & 38 \\ & 4 \\ \hline 2497 & 52 \end{array}$$

Result, $2497\tfrac{52}{100}$

52 ÷ 4 = 13 Remainder

$$\begin{array}{r|l} 45659 & \\ & 4 \\ \hline 182 & 636 \end{array}$$

**125.** RULE.—*Multiply the dividend by 4, and divide the product by 100.* (*This can be extended, as in multiplication, to 250, 2500, 125, 1250, etc.*)

### WRITTEN EXERCISE

**126.** *1.* Divide 3248 by 25
*2.* Divide 4199 by 25
*3.* Divide 6327 by 25
*4.* Divide 14832 by 25
*5.* Divide 71248 by 25
*6.* Divide 34862 by 25
*7.* Divide 41780 by 25
*8.* Divide 63745 by 25
*9.* Divide 42150 by 25
*10.* Divide 37895 by 25

**127. To divide by a composite number.**

EXAMPLE.—Divide 3479 by 28

7)3479
4)497
$124\frac{1}{4}$

$28 = 7 \times 4$. Dividing by one factor, 7, gives 497, which, divided by the other factor, 4, gives the quotient.

EXAMPLE.—Divide 4265 by 64

8)4265
8)533+1 Rem.
66+5 Rem.
$5 \times 8 = 40$, plus $1 = 41$ True Rem.
Result, $66\frac{41}{64}$

$64 = 8 \times 8$. Divide as above. The second remainder is a part of 533 times 8—a number of *eighths*, and must be multiplied by the first divisor before adding to the first remainder, to get the whole remainder.

**128.** RULE.—*Separate the divisor into two factors, and divide the dividend by one factor; then divide the quotient by the other factor.*

*If the first division leaves no remainder, write the second remainder over the second divisor, and place at the right of quotient.*

*If both divisions leave remainders, multiply the second remainder by the first divisor, add to first remainder, and write sum over the entire divisor, placing at right of quotient.*

### WRITTEN EXERCISE

**129.**

*1.* Divide 34826 by 14 (7 × 2)
*2.* Divide 47563 by 15 (5 × 3)
*3.* Divide 57648 by 16 (4 × 4)
*4.* Divide 37849 by 18 (6 × 3)
*5.* Divide 96876 by 21 (7 × 3)
*6.* Divide 134762 by 22 (11 × 2)
*7.* Divide 312843 by 24 (6 × 4)
*8.* Divide 525301 by 27 (9 × 3)
*9.* Divide 626884 by 28 (7 × 4)
*10.* Divide 317783 by 32 (8 × 4)
*11.* Divide 498632 by 33 (11 × 3)
*12.* Divide 653728 by 35 (7 × 5)
*13.* Divide 329849 by 36 (6 × 6)
*14.* Divide 483764 by 42 (7 × 6)
*15.* Divide 959388 by 45 (9 × 5)
*16.* Divide 1312432 by 48 (8 × 6)
*17.* Divide 3298467 by 49 (7 × 7)
*18.* Divide 1128321 by 54 (9 × 6)
*19.* Divide 2557637 by 56 (8 × 7)
*20.* Divide 4887572 by 63 (9 × 7)
*21.* Divide 5112863 by 64 (8 × 8)
*22.* Divide 6768459 by 72 (8 × 9)
*23.* Divide 8384536 by 77 (11 × 7)
*24.* Divide 9394281 by 81 (9 × 9)
*25.* Divide 3838633 by 84 (12 × 7)
*26.* Divide 5637344 by 88 (11 × 8)
*27.* Divide 8565787 by 108 (12 × 9)
*28.* Divide 3254063 by 121 (11 × 11)
*29.* Divide 4080362 by 132 (12 × 11)
*30.* Divide 6302011 by 144 (12 × 12)

### WRITTEN PROBLEMS

**130.** *1.* At $125 each, how many horses can be bought for $4250?

*2.* A bin contains 128560 lbs. How many bushels of 56 pounds each does it contain?

*3.* If potatoes yield 46 bushels per acre, how many acres will be required to yield 3312 bushels?

*4.* A clerk saves $57 a month. How many months will it require to save $1311?

*5.* A fruit grower received $1755 for 195 barrels of cranberries. What was the price per barrel?

*6.* How many lots at $321 each can be bought for $772326?

*7.* One share of bank stock is worth $98. How many shares can be bought for $22050?

*8.* A public library has a yearly circulation of 5696600 books. How many books are taken daily, if the library is open 313 days in the year?

*9.* A company of 547 men took equal shares in a mine valued at $705083. How much money did each man invest?

*10.* If a saw-mill turns out 1678152 feet of boards in 294 days, how many feet are sawed daily?

*11.* Two cities 294 miles apart are connected by railroad at a cost of $6845630; what is the cost of one mile of railroad?

*12.* A manufacturer produced 8643759 yards of goods at a cost of 129656385 cents. What was the cost of manufacturing one yard?

*13.* The dividend is 72987 and the divisor 45; required the quotient and the remainder.

*14.* If 28 horses cost $3864, what is the cost of one horse?

*15.* A man left $2535 to each of his four children, but, one of them dying, the money was divided among the three living. How much did each receive?

### REVIEW PROBLEMS

#### Addition, Subtraction, Multiplication and Division

**131.** *1.* A merchant's bank balance on Saturday was $564.32. On Monday, he deposited $365.45 and checked out $260.17; on Wednesday, deposited $85.60; on Thursday, deposited $125 and checked out $468.57; on Friday, deposited $93.75. What is his bank balance after these transactions?

*2.* A clerk who earns $65 a month pays $25 a month board, $150 a year for clothes, incidental expenses $138, and invests the remainder in books at $4 a volume. How many volumes can he buy yearly?

*3.* A clerk's present worth is $832. If his salary is $100 a month, and his expenses $14 a week, how many years (52 weeks) must he work to be worth $4000?

*4.* A is worth $525; B is worth 3 times as much, less $80; C is worth 8 times as much as A and B. How many horses valued at $180 each can they buy with their total money?

*5.* Jones owes $864.25, then pays $376.15; he then buys to the amount of $1514.27 and pays $935.63; he then buys for $564.28 and pays $438.20; he now pays $1068.75. How much does he still owe?

*6.* Bought 544 bushels of clover seed at $3.85 a bushel and paid for it with linen worth 17 cents a yard. How many yards did it require?

*7.* Bought a farm of 135 acres. Paid $5130 cash and $145 monthly for 4 years and 6 months. How much did I pay per acre?

*8.* A manufacturer's weekly sheet shows total product as follows: 1st week, 5684 yards; 2d week, 8647 yards; 3d week, 7863 yards; 4th week, 6435 yards; 5th week, 4639 yards; sold 24931 yards at 88 cents a yard, and the remainder at 84 cents a yard. What was the average selling price for each yard produced?

*9.* A lady owns a farm valued at $6300, and 3 stores valued at $15600. She gives her daughter one-fourth of the farm and one-third of the value of the stores; the remainder is shared equally among 5 sons. How much does the daughter receive more than each son?

*10.* Owning 45 acres of land, I sell 15 acres for $2250; the remainder I sell at $135 an acre. I receive $6000 cash and the balance in pigs at $5 each. How many pigs do I receive?

*11.* Bought 35 barrels of pork at $26 per barrel; 3 barrels were damaged and sold for $5 per barrel less than cost; the others were sold at a profit of $3 per barrel. What was the gain?

*12.* A farmer sold a grocer 50 bushels of potatoes at 68 cents a bushel; 12 barrels of vinegar at $6 a barrel; 75 bushels of apples at 72 cents a bushel; 52 pounds of butter at 42 cents a pound, and 84 dozen eggs at 15 cents a dozen. The farmer bought 3 barrels of sugar, 386 pounds each, at 5 cents a pound; 2 barrels of fish at $18 a barrel; 5 boxes raisins, 40 pounds each, at 12 cents a pound; 2 cases prunes, 80 pounds each, at 9 cents a pound; 3 barrels of coal oil, 45 gallons each, at 15 cents a gallon; 18 bushels of salt at 27 cents per bushel. Which owes the other, and how much?

*13.* A farmer buys a farm of 110 acres at $75 an acre; he pays $2200 down and the remainder in 5 yearly instalments. What was paid each year?

*14.* A builder finds the cost of an operation of 65 houses to be as follows: land $325,000, material $185,687, labor $205,947. At what price must he sell the houses to gain $475 on each?

*15.* In constructing a railroad 25 miles long, the grading cost $586,742; 10 bridges were built at an average cost of $45867; a tunnel was dug at a cost of $168,738. What was the average cost per mile?

*16.* In a business exchange 125 horses valued at $31200 were given for 25 pianos. What was gained by selling each piano for $1500?

## PROPERTIES OF NUMBERS

**132.** A **unit** is one; as, one book, one year.

**133.** A **number** is a collection of units; as, three, four boxes, five cents.

**134.** Numbers are divided into *integral* and *fractional*, *concrete* and *abstract*, *prime* and *composite*, *odd* and *even*, *similar* and *dissimilar*.

**135.** An **integral** number expresses whole things; as, six, three years.

**136.** A **fractional** number expresses parts of things; as, one-half, one-fourth of an inch.

**137.** A **concrete** number is a number applied to particular objects; as, two desks, seven apples.

**138.** An **abstract** number is a number considered apart from objects; as, three, four, five.

**139.** A **prime** number has no exact divisor, except itself and one; as, two, three, five, seven.

**140.** A **composite** number is the product of factors other than itself and one, and, consequently, is exactly divisible by them; as, 6, 8, 9, 15.

**141.** An **odd** number is one that leaves a remainder of *one* when divided by two; as, 5, 7, 9.

**142.** An **even** number is one exactly divisible by two; as, 8, 10, 12.

**143.** **Similar** numbers are groups of the same kind of units; as, 2 feet, 6 feet, 8 feet.

**144.** **Dissimilar** numbers are groups of different kinds of units; as, two oranges, five apples.

**145.** The **factors** of a number are the numbers which when multiplied together will produce it. A **prime factor** is a factor that is a prime number. The factors of 12 are 2, 2 and 3; of 15, 3 and 5.

**146.** The result of taking the same number two or more times as a factor is called a **power**; as $5\times5=25$, the second power (called the *square*) of five; $3\times3\times3=27$, the third power (called the *cube*) of three. The power is usually indicated by a small figure, called the exponent, which is written above and at the right of the factor; as, $3^2$ means $3\times3$; $4^3$ means $4\times4\times4$. The small figure indicates the degree of the power; as $2^5$, the fifth power of two; $5^4$, the fourth power of five.

## FACTORING

**147. Factoring** is the process by which a composite number is reduced to its factors. This is done by division, the number which we divide being the *multiple*, and the divisors, the *factors*.

**148. To resolve a number into its prime factors.**

EXAMPLE.—Find the prime factors of 63

$$\begin{array}{r|l} 3 & 63 \\ \hline 3 & 21 \\ \hline 7 & \end{array} \qquad 3\times3\times7, \text{ prime factors}$$

**149.** RULE.—*Divide the number to be factored by any prime factor that will exactly measure it; in like manner divide the quotient thus obtained and the successive quotients until one is obtained that is prime. The several divisors and the prime quotient are the prime factors sought.*

### WRITTEN EXERCISE

**150.** Find the prime factors of

| | | | |
|---|---|---|---|
| *1.* 27 | *6.* 1825 | *11.* 3025 | *16.* 9020 |
| *2.* 81 | *7.* 1890 | *12.* 819 | *17.* 17017 |
| *3.* 132 | *8.* 5346 | *13.* 1785 | *18.* 1009 |
| *4.* 210 | *9.* 3465 | *14.* 8729 | *19.* 971 |
| *5.* 1732 | *10.* 7007 | *15.* 1287 | *20.* 1063 |

## DIVISIBILITY OF NUMBERS

**151.** The following properties of numbers will be found useful in reducing fractions to their lowest terms, in resolving numbers into their prime factors, etc., as they serve to abridge the labor of calculation.

*1.* Even numbers are *divisible by 2.*

*2.* When the sum of the figures of a number is divisible by three, *the number is divisible by 3.*

*3.* When 4 divides the number expressed by the two right-hand figures of a given number, *the given number is divisible by 4.*

*4.* When the right-hand figure of a number is 5 or 0, *the number is divisible by 5.*

*5.* When an even number is divisible by 3, *it is divisible by 6.*

*6.* When 8 divides the number expressed by the three right-hand figures of a given number, *the given number is divisible by 8.*

*7.* When the sum of the figures of a number is divisible by 9, *the number is divisible by 9.*

*8.* 1001 and its multiples, which are usually easily recognized, *are divisible by 7, 11, and 13.*

*9.* When the sum of the odd figures (counting from the right) of a number equals the sum of the even figures, or when the difference between these sums is 11 or a multiple of 11, *the number is divisible by 11.*

## GREATEST COMMON DIVISOR

**152.** A **common divisor** of several numbers is a number that will exactly divide each of them.

**153.** The **greatest common divisor** of two or more numbers is the greatest number that will exactly divide each of them.

**154.** A number is an exact divisor of only such other numbers as contain all its prime factors.

**155. To find the greatest common divisor of two or more numbers by factoring.**

EXAMPLE.—Required the greatest common divisor of 12, 18, and 36

$$\text{The factors of } \begin{cases} 12 = 2\times2\times3 \\ 18 = 2\times3\times3 \\ 36 = 2\times2\times3\times3 \end{cases}$$

The factors common to all are 2 and 3; therefore, their product, $2\times3$, or 6, is the greatest common divisor.

$$\begin{array}{r|l} 2 & 12 - 18 - 36 \\ \hline 3 & 6 - 9 - 18 \\ \hline & 2 - 3 - 6 \end{array}$$

EXPLANATION.—Since 2 is an exact divisor of each number, 2 is a factor of the greatest common divisor. The quotients being divisible by 3, 3 is a factor of the greatest common divisor. The only factors common to all are 2 and 3; their product is 6, the greatest common divisor.

**156.** RULE.—*Divide the numbers by any prime number that exactly divides all of them; proceed in the same manner with the quotients until no number can be found that will divide all of the last quotients. The product of all the divisors used will be the greatest common divisor.*

**157. To find the greatest common divisor when the numbers are large and not easily factored.**

EXAMPLE 1.—Required the greatest common divisor of 437 and 897

$$\begin{array}{r} 437\overline{)897}(2 \\ 874 \\ \hline \text{G. C. D.} - 23\overline{)437}(19 \\ 23 \\ \hline 207 \\ 207 \\ \hline \end{array}$$

EXAMPLE 2.—Required the greatest common divisor of 9388, 16429 and 18776

ABBREVIATED METHOD

| | | | | | | |
|---|---|---|---|---|---|---|
| | 9388 | 16429 | 1 | G. C. D. 2347 | 18776 | 8 |
| | | 9388 | | | 18776 | |
| 1 | 7041 | 7041 | 3 | | | |
| | 2347 | 7041 | | | | |

NOTE.—This arrangement simply avoids the repetition of figures and thus saves time.

**158.** RULE.—*Divide the larger number by the smaller, and each preceding divisor by the remainder until there is no remainder. The last divisor will be the greatest common divisor.*

### WRITTEN EXERCISE

**159.** Find the greatest common divisor of

*1.* 24, 32 and 48
*2.* 53, 63 and 81
*3.* 72, 108 and 120
*4.* 39, 52 and 65
*5.* 81, 105 and 135
*6.* 210, 315 and 420
*7.* 104, 182 and 351
*8.* 180, 216 and 276
*9.* 81, 135 and 216
*10.* 390, 910 and 1365
*11.* 96, 108, 132 and 156
*12.* 36, 84, 108 and 144
*13.* 365, 511 and 803
*14.* 102, 153 and 255
*15.* 140, 250, 360 and 270
*16.* 144, 256, 192 and 204

*17.* A farmer has three strips of timber, 80, 96 and 102 feet, respectively. What is the length of the longest pieces, all of the same length, that may be cut from them?

*18.* Three schools containing 120, 210 and 360 students, respectively, are divided into classes, each containing the same number of students. What is the greatest number of students each class can contain, and how many classes of this size are there in each school?

*19.* A man distributed $240, $336 and $480 among the employees of 3 mills in equal sums, the sums being as large as possible. Required the amount of the equal sums and number of employees.

*20.* A triangular lot, the sides of which are respectively 168, 192 and 180 feet, is to be inclosed with a tight board fence, 7 feet high, the boards to run lengthwise. How many boards will it take if they are 6 inches wide and of the greatest possible length that will require no cutting?

## LEAST COMMON MULTIPLE

**160.** A **multiple** of a number is a number that is exactly divisible by it; as, 10 is a multiple of 5; 12 of 4; 9 of 3.

**161.** A **common multiple** of two or more numbers is a number exactly divisible by each of them. Thus, 18 is a common multiple of 6, 3 and 2.

**162.** The **least common multiple** of two or more numbers is the least number that is exactly divisible by each of them. Thus, 12 is the least common multiple of 3 and 4.

**163.** A common multiple of two or more numbers contains all the prime factors of each of those numbers.

**164. To find the least common multiple of two or more numbers.**

EXAMPLE.—Find the least common multiple of 16, 18, 24 and 60

$$\begin{array}{r|l} 2 & 16-18-24-60 \\ 2 & 8-9-12-30 \\ 2 & 4-9-6-15 \\ 3 & 2-9-3-15 \\ & 2-3-1-5 \end{array}$$

$2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$, or $2^4 \times 3^2 \times 5 = 720$ L.C.M.

**165.** RULE.—*Write the numbers in a row; draw a line underneath, and, selecting for a divisor the smallest prime number that will exactly divide two or more of them, divide as many as are divisible, bringing below the line those numbers which cannot be divided, together with the quotients. Repeat this process with the same divisor until it will no longer divide two of the numbers below the line; then take the next smallest prime factor that will divide two or more, and proceed as before. Continue thus until the numbers below the line are prime to each other. Finally, multiply together the numbers below the last line and all the divisors, and the product will be the least common multiple.*

### WRITTEN EXERCISE

**166.** Find the least common multiple of

1. 20 and 36
2. 21 and 42
3. 12, 18 and 24
4. 15, 30 and 45
5. 16, 24 and 30
6. 42, 84 and 105
7. 45, 72 and 120
8. 54, 90 and 270
9. 40, 80 and 120
10. 63, 84 and 98
11. 45, 60 and 75
12. 18, 36, 72 and 120
13. 8, 10, 40 and 60
14. 27, 36, 45 and 60
15. 28, 35, 49 and 70
16. 96, 120, 160 and 132
17. 36, 60, 120 and 204
18. 150, 180, 320 and 480

19. What is the shortest piece of goods that can be cut into pieces of 12 yds., 18 yds. and 24 yds. and nothing remain?

20. What is the smallest tract of land that can be exactly laid out in fields of either 8, 12, or 16 acres? How many fields of each kind can be made?

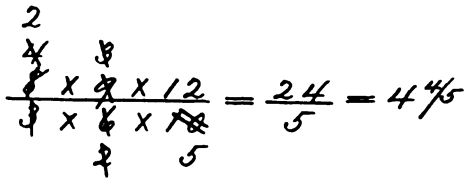
# CANCELATION

**167. Cancelation** is employed to shorten the work of division, and consists in casting out the same factor from both dividend and divisor.

**168.** To cast out a factor of any product is simply to divide that product by the factor eliminated, and to cast out the same or equal factors from both dividend and divisor is equivalent to dividing both dividend and divisor by the same number; it does not change the quotient. Thus, dividing 30 by 10, or three times 30 by three times 10, or one-half of 30 by one-half of 10, produces the same quotient. ($30 \div 10 = 3$; $90 \div 30 = 3$; $15 \div 5 = 3$.)

**169. To find a quotient by cancelation.**

EXAMPLE.—Divide $8 \times 9 \times 12$ by $3 \times 6 \times 10$



The factor 3 is rejected from both dividend and divisor, by canceling the 3 below the line and dividing the 9 above the line by 3, canceling the 9 and writing the quotient 3 above it. The factor 3 is again rejected, canceling the 3 above the 9 and dividing the 6 below the line by 3, canceling the 6 and writing 2 below it. The factor 2 is then rejected, canceling the 2 below the 6 and dividing the 8 above the line by 2, canceling the 8 and writing 4 above it. The factor 2 is again rejected, canceling the 4 above the 8 and writing 2 above it, and canceling the 10 below the line and writing 5 below it. This leaves $2 \times 12$, or 24, above the line, and 5 below it; dividing, we have the result, $4\frac{4}{5}$.

**170.** RULE.—*Write the factors of the dividend above a line, and the factors of the divisor below it. Cancel all of the factors which are found in both dividend and divisor, and divide the product of the remaining factors in the dividend by the product of the remaining factors in the divisor.*

### WRITTEN EXERCISE

**171.** *1.* Divide $12 \times 15 \times 18$ by $4 \times 6 \times 9$

*2.* Divide $15 \times 14 \times 22$ by $3 \times 11 \times 7$

*3.* Divide $18 \times 26 \times 28$ by $14 \times 9 \times 13$

*4.* Divide $20 \times 32 \times 48$ by $12 \times 10 \times 15$

*5.* Divide $72 \times 36 \times 24$ by $18 \times 48 \times 16$

*6.* Divide $56 \times 77 \times 84$ by $63 \times 42 \times 33$

*7.* Divide $180 \times 108 \times 270$ by $120 \times 18 \times 45$

*8.* Divide $121 \times 132 \times 84$ by $144 \times 66 \times 22$

*9.* Divide $75 \times 320 \times 160$ by $20 \times 180 \times 150$

*10.* Divide $240 \times 120 \times 350$ by $180 \times 75 \times 80$

*11.* Divide $360 \times 146 \times 145$ by $365 \times 100 \times 87$

*12.* Divide $1728 \times 195 \times 363$ by $144 \times 65 \times 36$

## WRITTEN PROBLEMS

**172.** *1.* If 10 apples cost 35 cents, what will 14 apples cost?

*2.* If 7 barrels of flour cost $84, what will 10 barrels cost?

*3.* A grocer bought 324 bushels of potatoes at 40 cents per bushel, and paid for them in molasses at 60 cents per gallon. How many gallons did it require?

*4.* How many sheep worth $2.25 per head must be given for 12 tons of hay at $15 per ton?

*5.* What must be paid for shipping 896 pounds of iron at $6 per 2240 lbs.?

*6.* How many bushels of wheat at 90 cents per bushel must be exchanged for 120 bushels of oats at 45 cents per bushel?

*7.* How many times can a cask, holding 3 gallons, be filled from 24 bottles each holding two pints?

*8.* How many cords of wood at $4.50 per cord should be given for 18 barrels of flour at $5 per barrel?

*9.* A sold 15 barrels of pork, each containing 180 pounds, at 18 cents per pound, and received in payment a number of hogsheads of molasses, each containing 90 gallons, at 60 cents per gallon. How many hogsheads did he receive?

*10.* A dairyman sold 36 firkins of butter, each containing 56 pounds, at 25 cents per pound, and with the money bought 30 pieces of calico at 8 cents per yard. How many yards in each piece?

*11.* How many bushels of apples worth 45 cents per bushel must be given in exchange for 60 pieces of muslin, each containing 48 yards at 9 cents per yard?

*12.* A milkman has 20 cows, which give daily 6 quarts each, which he sells at five cents per quart. How many hams, each weighing 15 pounds, at 16 cents per pound, can he buy with 4 days' milk?

*13.* How many barrels of apples, each containing three bushels, at 35 cents per bushel, must be given for 16 barrels of sugar, each containing 240 pounds, at 7 cents a pound?

*14.* If a laborer receives 15 cents per hour, how many days of 10 hours each will it require for him to earn the value of 6 barrels of flour at $4.50 per barrel?

*15.* How many crates of eggs, each containing 30 dozen, at 18 cents per dozen, should be given in exchange for 120 rolls of paper, each containing 15 yards, at 32 cents per yard?

*16.* A dealer bought 625 pounds of cheese at 12 cents per pound and exchanged it for five loads of corn, each load containing 18 bags, and each bag 3 bushels. What was the price per bushel?

*17.* A cotton dealer exchanged 24 bales of cotton, averaging 410 pounds per bale, worth 12 cents per pound, for 48 loads of apples, each load containing 15 bags of 2 bushels each. What was the price of the apples per bushel?

*18.* A coal merchant exchanged 5 car-loads of coal, each containing 15 tons, at $4.80 per ton, for 5 cases of children's shoes at $3 a pair. How many pairs in each case?

# COMMON FRACTIONS

**173.** A **fraction** is one or more of the equal parts into which a unit is divided; as, one half ($\frac{1}{2}$), two thirds ($\frac{2}{3}$).

**174.** A **common fraction** is one which has a written denominator, and is distinguished from a *decimal* fraction, which has its denominator indicated by the decimal point.

**175.** Common fractions are divided into *proper*, *improper*, *simple*, *compound*, and *complex*.

**176.** A **proper** fraction is one whose numerator is less than its denominator; as, three fourths ($\frac{3}{4}$).

**177.** An **improper** fraction is one whose numerator is equal to or greater than its denominator; as, five fifths ($\frac{5}{5}$), four thirds ($\frac{4}{3}$). It is simply a fractional form of a whole or mixed number.

**178.** A **simple** fraction consists of a single fraction; as, five sixths ($\frac{5}{6}$).

**179.** A **compound** fraction is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{1}{4}$ of $\frac{2}{5}$.

**180.** A **complex** fraction is one whose numerator or denominator, or both, contain fractions; as, $\frac{3\frac{1}{2}}{4}$, $\frac{3}{4\frac{2}{3}}$, $\frac{\frac{2}{3}}{\frac{4}{5}}$.

**181.** The **terms** of a fraction are the **denominator** and the **numerator**; the **denominator** is the part written below the line and indicates the number of parts into which the unit is divided; the **numerator** is the part written above the line and indicates how many parts are taken.

**182.** A **mixed number** consists of a whole number and a fraction; as, $2\frac{1}{2}$.

### WRITTEN EXERCISE

**183.** Express in figures

1. One half
2. One third
3. Three fourths
4. Five sixths
5. Seven eighths
6. Eight ninths
7. Nine tenths
8. Ten twelfths
9. Eleven fifteenths
10. Fifteen sixteenths

**184.** What do the following expressions mean?

1. $\frac{2}{3}$
2. $\frac{3}{4}$
3. $\frac{4}{5}$
4. $\frac{5}{6}$
5. $\frac{7}{8}$
6. $\frac{11}{12}$
7. $\frac{12}{13}$
8. $\frac{15}{16}$
9. $\frac{31}{32}$
10. $\frac{48}{51}$

## REDUCTION OF FRACTIONS

**185. Reduction** of fractions is changing their form without changing their value.

**186.** There are five different steps or operations in the reduction of fractions: (1) reducing improper fractions to whole or mixed numbers; (2) reducing mixed numbers to improper fractions; (3) reducing fractions to higher terms; (4) reducing fractions to their lowest term; (5) reducing fractions to their least common denominator.

**187. To reduce improper fractions to whole or mixed numbers.**

EXAMPLE.—Change $\frac{467}{32}$ to a mixed number

$$32\overline{)467}\quad 14\tfrac{19}{32}$$
$$32$$
$$147$$
$$128$$
$$19$$

Result, $14\frac{19}{32}$

**188.** RULE.—*Divide the numerator by the denominator.*

### WRITTEN EXERCISE

**189.** Reduce to whole or mixed numbers

| | | | |
|---|---|---|---|
| 1. $\frac{31}{3}$ | 6. $\frac{88}{5}$ | 11. $\frac{211}{17}$ | 16. $\frac{599}{43}$ |
| 2. $\frac{31}{7}$ | 7. $\frac{80}{7}$ | 12. $\frac{487}{29}$ | 17. $\frac{1305}{61}$ |
| 3. $\frac{59}{8}$ | 8. $\frac{84}{8}$ | 13. $\frac{467}{81}$ | 18. $\frac{3967}{215}$ |
| 4. $\frac{14}{6}$ | 9. $\frac{198}{7}$ | 14. $\frac{578}{37}$ | 19. $\frac{4867}{219}$ |
| 5. $\frac{12}{6}$ | 10. $\frac{271}{12}$ | 15. $\frac{756}{43}$ | 20. $\frac{67891}{889}$ |

**190. To reduce mixed numbers to improper fractions.**

EXAMPLE.—Change $34\frac{5}{23}$ to an improper fraction

$$34\tfrac{5}{23}$$
$$23$$
$$102$$
$$68$$
$$5$$
$$787$$

Result, $\frac{787}{23}$

**191.** RULE.—*Multiply the whole number by the denominator, add the numerator to the product, and write the sum over the denominator.*

### WRITTEN EXERCISE

**192.** Change to improper fractions

| | | | |
|---|---|---|---|
| 1. $8\frac{1}{2}$ | 5. $7\frac{1}{16}$ | 9. $73\frac{3}{88}$ | 13. $227\frac{8}{17}$ |
| 2. $11\frac{7}{8}$ | 6. $9\frac{11}{17}$ | 10. $89\frac{6}{11}$ | 14. $524\frac{3}{19}$ |
| 3. $13\frac{3}{8}$ | 7. $23\frac{7}{16}$ | 11. $108\frac{7}{13}$ | 15. $897\frac{67}{115}$ |
| 4. $16\frac{5}{16}$ | 8. $45\frac{11}{13}$ | 12. $137\frac{11}{11}$ | 16. $967\frac{34}{67}$ |

**193. To reduce to higher or lower terms.**

**194.** Multiplying or dividing both numerator and denominator of a fraction by the same number changes the form but not the value of the fraction.

**195. To reduce fractions to higher terms.**

EXAMPLE.—Change $\frac{7}{9}$ to thirty-sixths $\frac{7}{9} = \frac{28}{36}$

**196.** RULE.—*Multiply both terms of the fraction by such number as will raise its denominator to the required denominator.*

**197.** Change:

1. $\frac{2}{3}$ to 9ths
2. $\frac{3}{4}$ to 16ths
3. $\frac{2}{5}$ to 15ths
4. $\frac{5}{6}$ to 18ths
5. $\frac{4}{7}$ to 28ths
6. $\frac{5}{8}$ to 24ths
7. $\frac{8}{9}$ to 27ths
8. $\frac{9}{10}$ to 30ths
9. $\frac{6}{11}$ to 44ths
10. $\frac{11}{12}$ to 36ths
11. $\frac{2}{3}$ and $\frac{4}{7}$ to 21sts
12. $\frac{5}{6}$ and $\frac{7}{9}$ to 18ths
13. $\frac{3}{4}$ and $\frac{6}{7}$ to 28ths
14. $\frac{3}{4}$ and $\frac{2}{3}$ to 12ths
15. $\frac{4}{5}$ and $\frac{9}{11}$ to 55ths
16. $\frac{8}{9}$, $\frac{5}{8}$ and $\frac{7}{12}$ to 36ths
17. $\frac{2}{5}$, $\frac{5}{7}$ and $\frac{3}{10}$ to 70ths
18. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{5}{12}$ and $\frac{15}{16}$ to 48ths
19. $1\frac{1}{2}$, $\frac{3}{14}$, $\frac{13}{21}$ and $\frac{19}{28}$ to 84ths
20. $\frac{2}{3}$, $\frac{33}{39}$, $\frac{19}{52}$ and $\frac{17}{78}$ to 156ths

**198. To reduce fractions to lowest terms.**

EXAMPLE.—Reduce $\frac{36}{54}$ to lowest terms $\frac{36}{54} = \frac{4}{6} = \frac{2}{3}$

Dividing both numerator and denominator successively by 9 and 2, we obtain the result $\frac{2}{3}$.

**199.** RULE.—*Divide both terms by their greatest common divisor. Or, divide both terms by such numbers as will exactly measure them until the numerator and denominator are prime to each other.*

**ORAL EXERCISE**

**200.** Reduce by inspection to lowest terms

1. $\frac{6}{12}$
2. $\frac{8}{24}$
3. $\frac{9}{36}$
4. $\frac{12}{16}$
5. $\frac{6}{8}$
6. $\frac{15}{24}$
7. $\frac{18}{24}$
8. $\frac{16}{24}$
9. $\frac{14}{35}$
10. $\frac{18}{63}$
11. $\frac{30}{65}$
12. $\frac{25}{75}$
13. $\frac{15}{35}$
14. $\frac{48}{72}$
15. $\frac{21}{63}$
16. $\frac{12}{57}$
17. $\frac{11}{132}$
18. $\frac{132}{144}$
19. $\frac{99}{105}$
20. $\frac{7}{91}$

**WRITTEN EXERCISE**

**201.** Reduce to lowest terms

1. $\frac{66}{105}$
2. $\frac{85}{105}$
3. $\frac{74}{570}$
4. $\frac{118}{188}$
5. $\frac{80}{140}$
6. $\frac{378}{518}$
7. $\frac{197}{316}$
8. $\frac{788}{900}$
9. $\frac{793}{1008}$
10. $\frac{333}{504}$
11. $\frac{433}{616}$
12. $\frac{864}{936}$
13. $\frac{780}{1092}$
14. $\frac{1512}{3528}$
15. $\frac{819}{3015}$
16. $\frac{720}{1155}$
17. $\frac{1008}{2061}$
18. $\frac{1134}{1287}$
19. $\frac{1287}{1716}$
20. $\frac{8024}{17017}$

## ADDITION OF FRACTIONS

**202.** The **denominator** determines the fractional unit, and since only like units can be added or subtracted, there must be a common denominator.

**203.** The **least common denominator** of fractions is the least number into which all the denominators can be evenly divided. This may often be determined by inspection, as 12 is the least common denominator of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

**204.** When the **common denominator** is not seen readily, select the **largest denominator**, observe whether the next largest will divide it evenly; if not, select the **smallest** number into which each of these two will divide evenly; with this number take the third largest denominator and proceed in the same manner until you have a number which all denominators will divide evenly. This number is the least common denominator.

**205.** When the **least common denominator** cannot be determined by inspection, it can be found by getting the least common multiple of the denominators, as indicated in the example. This may be abbreviated by excluding all denominators that will divide evenly into a larger denominator.

**206. To find the least common denominator.**

EXAMPLE.—$\frac{1}{8}$, $\frac{9}{16}$, $\frac{1}{36}$

REGULAR METHOD

$2\overline{)8 - 16 - 36}$
$2\overline{)4 - 8 - 18}$
$2\overline{)2 - 4 - 9}$
$1 - 2 - 9$

$2 \times 2 \times 2 \times 2 \times 9 = 144$

ABBREVIATED METHOD

$4\overline{)16 - 36}$
$4 - 9$

$4 \times 4 \times 9 = 144$

Write denominators as indicated in example and divide in manner shown by any number that will be contained in two or more evenly. Repeat the division of the numbers below the line until you can no longer divide two; then multiply the divisors and numbers below the last line. The result will be the least common denominator.

**207.** Find the least common denominator of

1. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{3}$, $\frac{1}{8}$
2. $\frac{1}{2}$, $\frac{9}{16}$, $1\frac{1}{4}$
3. $\frac{1}{8}$, $\frac{1}{9}$, $\frac{7}{18}$, $1\frac{1}{2}$
4. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{11}$, $\frac{5}{12}$
5. $\frac{3}{5}$, $\frac{7}{15}$, $\frac{5}{16}$, $1\frac{1}{2}$
6. $\frac{3}{4}$, $\frac{3}{7}$, $\frac{5}{8}$, $1\frac{1}{4}$
7. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{5}{14}$, $1\frac{1}{6}$
8. $\frac{3}{5}$, $\frac{3}{4}$, $\frac{5}{8}$, $1\frac{1}{2}$, $\frac{9}{16}$

**208. To add fractions.**

EXAMPLE.—Add $\frac{2}{3}$, $\frac{4}{7}$ and $\frac{3}{5}$

105 *L. C. D.*

$\frac{3}{5} = 63$
$\frac{4}{7} = 60$
$\frac{2}{3} = 70$
193
$\frac{193}{105} = 1\frac{88}{105}$

$5 \times 7 \times 3 = 105$. *least common denominator.*

Since it is impossible to divide two of the denominators evenly by number larger than 1, the least common denominator is obtained by s multiplying the denominators together. Each fraction is reduced to rec parts by dividing the denominator into 105 and multiplying by the nume

Result, $1\frac{88}{105}$

**209.** RULE.—*Find the least common multiple of the denominators. Reduce fraction to this denominator and add the numerators, writing the sum over the con denominator. Reduce the result to its lowest terms.*

**ORAL EXERCISE**

**210.** Reduce to common denominators by inspection and add

1. $\frac{1}{2}$ and $\frac{1}{4}$
2. $\frac{1}{2}$ and $\frac{1}{3}$
3. $\frac{1}{3}$ and $\frac{2}{4}$
4. $\frac{2}{3}$ and $\frac{5}{6}$
5. $\frac{2}{3}$ and $\frac{3}{4}$
6. $\frac{4}{5}$ and $\frac{8}{10}$
7. $\frac{5}{9}$ and $\frac{2}{3}$
8. $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$
9. $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$
10. $\frac{7}{8}$, $\frac{1}{3}$ and $\frac{5}{12}$
11. $\frac{3}{4}$, $\frac{7}{8}$ and $\frac{2}{3}$
12. $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{6}$
13. $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{11}{12}$
14. $\frac{1}{5}$, $\frac{5}{6}$ and $\frac{8}{10}$
15. $\frac{3}{8}$, $\frac{6}{7}$ and $\frac{5}{12}$
16. $\frac{5}{7}$, $\frac{2}{3}$ and $\frac{8}{9}$
17. $\frac{3}{4}$, $\frac{5}{9}$ and $\frac{7}{12}$
18. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and
19. $\frac{3}{7}$, $\frac{5}{9}$ and $\frac{6}{11}$
20. $\frac{4}{11}$, $\frac{5}{12}$ and

**WRITTEN EXERCISE**

**211.** Add

1. $\frac{3}{7}$, $\frac{5}{8}$ and $\frac{7}{12}$
2. $\frac{3}{5}$, $\frac{4}{6}$ and $\frac{7}{18}$
3. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{8}$
4. $\frac{2}{3}$, $\frac{3}{8}$ and $\frac{5}{8}$
5. $\frac{4}{7}$, $\frac{3}{11}$, $\frac{8}{12}$ and $\frac{4}{21}$
6. $\frac{4}{15}$, $\frac{7}{8}$, $\frac{3}{8}$ and $\frac{7}{24}$
7. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{11}{12}$ and $\frac{15}{18}$
8. $\frac{11}{16}$, $\frac{4}{5}$, $\frac{3}{5}$ and $\frac{7}{12}$
9. $\frac{7}{8}$, $\frac{5}{7}$, $\frac{3}{9}$ and $\frac{8}{11}$
10. $\frac{11}{18}$, $\frac{2}{9}$, $\frac{17}{24}$ and $\frac{23}{36}$
11. $\frac{6}{8}\frac{5}{4}$, $\frac{4}{4}\frac{3}{8}$, $\frac{11}{21}$, $\frac{5}{14}$ and $\frac{2}{7}$
12. $\frac{2}{3}$, $\frac{3}{8}$, $\frac{3}{5}$, $\frac{9}{10}$, $\frac{10}{11}$ and $\frac{11}{12}$

**212. To add mixed numbers.**

EXAMPLE.—Add $4\frac{1}{2}$, $3\frac{2}{3}$, $5\frac{3}{4}$ and $4\frac{3}{4}$

12 *L. C. D.*

$4\frac{1}{2} = 6$
$3\frac{2}{3} = 8$
$5\frac{3}{4} = 10$
$4\frac{3}{4} = 9$
$\frac{2}{18\frac{3}{4}}$ $\frac{3}{12} = 2\frac{3}{4}$

Least common denominator of the fractions, 12; fractions equal $2\frac{3}{4}$; adding 2 in with whole numbers gives the result $18\frac{3}{4}$.

Result, $18\frac{3}{4}$

NOTE.—Add the fractions separately, then combine the result with the whole numbers.

## WRITTEN EXERCISE

**213.** Add

*1.* $4\frac{3}{8}$, $3\frac{1}{2}$ and $7\frac{5}{12}$

*2.* $8\frac{3}{4}$, $7\frac{5}{8}$ and $9\frac{1}{2}$

*3.* $3\frac{4}{5}$, $6\frac{2}{3}$ and $8\frac{1}{10}$

*4.* $9\frac{3}{4}$, $5\frac{2}{3}$ and $7\frac{7}{12}$

*5.* $6\frac{1}{2}$, $7\frac{1}{3}$, $4\frac{1}{4}$ and $8\frac{1}{5}$

*6.* $3\frac{3}{4}$, $6\frac{2}{3}$, $8\frac{1}{3}$ and $11\frac{1}{6}$

*7.* $11\frac{1}{2}$, $13\frac{1}{3}$, $6\frac{1}{4}$ and $12\frac{1}{2}$

*8.* $15\frac{1}{5}$, $7\frac{1}{2}$, $3\frac{1}{3}$ and $33\frac{1}{3}$

*9.* $23\frac{1}{2}$, $48\frac{5}{8}$ and $67\frac{3}{8}$

*10.* $87\frac{3}{4}$, $126\frac{7}{8}$ and $168\frac{3}{8}$

*11.* $55\frac{1}{7}$, $67\frac{1}{3}$ and $83\frac{3}{7}$

*12.* $93\frac{5}{8}$, $68\frac{3}{4}$ and $75\frac{5}{8}$

*13.* $37\frac{3}{5}$, $48\frac{3}{4}$, $59\frac{5}{8}$ and $72\frac{1}{8}$

*14.* $124\frac{5}{8}$, $13\frac{4}{7}$, $64\frac{1}{7}$ and $32\frac{5}{11}$

*15.* $83\frac{1}{4}$, $72\frac{1}{8}$, $89\frac{5}{8}$ and $72\frac{3}{8}$

*16.* $16\frac{3}{4}$, $34\frac{5}{6}$, $27\frac{7}{12}$ and $32\frac{5}{8}$

*17.* $32\frac{3}{16}$, $43\frac{3}{8}$, $18\frac{7}{16}$ and $36\frac{3}{16}$

*18.* $127\frac{3}{8}$, $32\frac{5}{6}$, $23\frac{7}{8}$ and $36\frac{1}{14}$

*19.* Find the sum of $357\frac{3}{4}$ bu., $189\frac{5}{8}$ bu., $697\frac{1}{8}$ bu. and $378\frac{5}{12}$ bu.

*20.* Add $897\frac{3}{8}$ ft., $1089\frac{3}{4}$ ft., $878\frac{1}{8}$ ft., $395\frac{1}{2}$ ft. and $689\frac{7}{8}$ ft.

*21.* A merchant had carpets in the respective quantities as follows: $278\frac{1}{2}$ yds., $348\frac{1}{2}$ yds., $195\frac{3}{4}$ yds., $482\frac{1}{4}$ yds. and $378\frac{1}{2}$ yds. How many yards in all?

## WRITTEN PROBLEMS

**214.** *1.* A merchant sold from a piece of muslin $16\frac{3}{4}$ yds., $27\frac{3}{8}$ yds., $47\frac{5}{8}$ yds., and there still remained unsold $25\frac{7}{12}$ yds. How many yards were in the piece?

*2.* A farmer sold ten loads of hay weighing as follows: $37\frac{5}{16}$ cwt., $45\frac{1}{2}$ cwt., $39\frac{1}{16}$ cwt., $38\frac{9}{16}$ cwt., $47\frac{3}{8}$ cwt., $31\frac{1}{2}$ cwt., $34\frac{1}{16}$ cwt., $41\frac{7}{16}$ cwt., $33\frac{1}{16}$ cwt., $50\frac{1}{16}$ cwt. Find total weight.

*3.* A bought $\frac{3}{8}$ of a yard of cloth for $\$\frac{7}{8}$, $\frac{4}{5}$ of a yard for $\$1\frac{1}{2}$, $\frac{2}{3}$ of a yard for $\$\frac{5}{8}$, and $\frac{7}{8}$ of a yard for $\$\frac{7}{16}$. How much money did he spend?

*4.* A family burned in December $3\frac{3}{4}$ tons of pea coal and $1\frac{1}{2}$ tons of nut coal; in January, $2\frac{2}{3}$ tons of pea coal and $1\frac{5}{12}$ tons of nut coal; in February, $3\frac{1}{4}$ tons of pea coal and $2\frac{1}{2}$ tons of nut coal. How much of each kind was burned during the three months?

*5.* A boy worked $5\frac{1}{4}$ days for $\$4\frac{3}{4}$, $7\frac{1}{2}$ days for $\$5\frac{5}{8}$, $3\frac{1}{8}$ days for $\$3\frac{1}{4}$, $6\frac{3}{4}$ days for $\$5\frac{1}{4}$, and $11\frac{3}{8}$ days for $\$12\frac{3}{4}$. How many days did he work, and how much money did he receive?

*6.* I purchased 5 hams, net weight of each being respectively $17\frac{1}{8}$ lbs., $23\frac{5}{8}$ lbs., $31\frac{1}{8}$ lbs., $22\frac{1}{16}$ lbs., $19\frac{3}{4}$ lbs. Find total weight.

*7.* Find the weight of 5 cars of coal weighing respectively $13\frac{3}{8}$ tons, $11\frac{7}{16}$ tons, $12\frac{1}{16}$ tons, $14\frac{5}{16}$ tons and $9\frac{3}{4}$ tons.

*8.* Four cars of iron weighed respectively $28\frac{3}{16}$ tons, $30\frac{3}{16}$ tons, $26\frac{9}{16}$ tons and $29\frac{1}{8}$ tons. What was the total weight?

*9.* A merchant received money in several sums, as follows: £102½, £79¼, £83 15/16, £26 7/16, £5 and £62⅜. What was the sum of these receipts?

*10.* Four wheelmen rode in a day respectively $125\frac{1}{4}$ miles, $107\frac{3}{4}$ miles, $89\frac{3}{4}$ miles and $95\frac{1}{8}$ miles. What was their total mileage for the day?

## SUBTRACTION OF FRACTIONS

**215.** To be subtracted, fractions must have a common denominator.

**216. To find the difference between two fractions.**

EXAMPLE.—Find the value of $\frac{8}{9}-\frac{6}{7}$

63 L.C.D.

$8/9 = 56$

$6/7 = 54$

$2/63$

**217.** RULE.—*Reduce to a common denominator, and write the difference between the numerators over the common denominator.*

### ORAL EXERCISE

**218.** By inspection find the value of

| | | | |
|---|---|---|---|
| 1. $\frac{1}{2}-\frac{1}{3}$ | 5. $\frac{9}{10}-\frac{2}{5}$ | 9. $\frac{2}{3}-\frac{4}{15}$ | 13. $\frac{9}{13}-\frac{2}{3}$ |
| 2. $\frac{3}{4}-\frac{2}{3}$ | 6. $\frac{7}{8}-\frac{5}{12}$ | 10. $\frac{11}{12}-\frac{5}{16}$ | 14. $\frac{16}{19}-\frac{3}{4}$ |
| 3. $\frac{5}{6}-\frac{1}{2}$ | 7. $\frac{7}{8}-\frac{6}{7}$ | 11. $\frac{17}{20}-\frac{3}{4}$ | 15. $\frac{12}{17}-\frac{3}{8}$ |
| 4. $\frac{7}{8}-\frac{3}{4}$ | 8. $\frac{17}{18}-\frac{2}{3}$ | 12. $\frac{8}{9}-\frac{5}{6}$ | 16. $\frac{35}{48}-\frac{5}{9}$ |

**219.** Perform the subtractions

| | | |
|---|---|---|
| 1. From $\frac{29}{34}$ take $\frac{7}{12}$ | 5. From $\frac{87}{100}$ take $\frac{3}{7}$ | 9. From $\frac{78}{87}$ take $\frac{37}{78}$ |
| 2. From $\frac{17}{24}$ take $\frac{3}{7}$ | 6. From $\frac{9}{11}$ take $\frac{14}{87}$ | 10. From $\frac{82}{960}$ take $\frac{57}{456}$ |
| 3. From $\frac{78}{96}$ take $\frac{18}{18}$ | 7. From $\frac{881}{888}$ take $\frac{117}{188}$ | 11. From $\frac{288}{887}$ take $\frac{3}{8}$ |
| 4. From $\frac{22}{26}$ take $\frac{8}{11}$ | 8. From $\frac{88}{99}$ take $\frac{58}{68}$ | 12. From $\frac{59}{61}$ take $\frac{11}{18}$ |

**220. To find the difference between two mixed numbers.**

EXAMPLE.—Find the difference between $14\frac{2}{7}$ and $3\frac{1}{3}$

$14\frac{2}{7} = 14\frac{6}{21} = 13\frac{27}{21}$

$3\frac{1}{3} = 3\frac{7}{21} = 3\frac{7}{21}$

$10\frac{20}{21}$

In practise it is sufficient to write:

$14\frac{2}{7} - \frac{6}{21}$

$3\frac{1}{3} = \frac{7}{21}$

$10\frac{20}{21}$

**221.** RULE.—*Subtract the fractions and the whole numbers separately, then combine the results.*

NOTE.—If the fraction of the minuend is smaller than the fraction of the subtrahend, borrow one unit from the whole number, reduce it to an improper fraction and add it to the fraction of the minuend.

### WRITTEN EXERCISE

**222.** Find the difference between

| | | |
|---|---|---|
| 1. $6\frac{5}{7}$ and $2\frac{4}{11}$ | 9. $13\frac{1}{4}$ and $2\frac{3}{4}$ | 17. $678\frac{15}{16}$ and $399\frac{18}{20}$ |
| 2. $1\frac{1}{2}$ and $\frac{2}{3}$ | 10. $15\frac{3}{8}$ and $7\frac{2}{3}$ | 18. $1037\frac{8}{23}$ and $674\frac{4}{16}$ |
| 3. $3\frac{4}{5}$ and $1\frac{5}{8}$ | 11. $237\frac{7}{10}$ and $159\frac{3}{5}$ | 19. $367\frac{7}{17}$ and $287\frac{11}{17}$ |
| 4. 17 and $1\frac{4}{5}$ | 12. $587\frac{1}{2}$ and $258\frac{3}{8}$ | 20. $973\frac{5}{9}$ and $587\frac{7}{8}$ |
| 5. $2\frac{5}{6}$ and $1\frac{2}{3}$ | 13. 781 and $275\frac{3}{10}$ | 21. $198\frac{5}{8}$ and $132\frac{7}{11}$ |
| 6. $3\frac{1}{4}$ and $2\frac{5}{6}$ | 14. $627\frac{5}{8}$ and $422\frac{5}{12}$ | 22. $622\frac{4}{5}$ and $419\frac{1}{3}$ |
| 7. $5\frac{7}{15}$ and $3\frac{4}{5}$ | 15. $580\frac{2}{3}$ and $316\frac{3}{4}$ | 23. $792\frac{11}{12}$ and $125\frac{1}{8}$ |
| 8. $3\frac{1}{3}$ and $1\frac{7}{8}$ | 16. $642\frac{4}{5}$ and $417\frac{2}{3}$ | 24. $716\frac{3}{7}$ and $242\frac{5}{8}$ |

## WRITTEN PROBLEMS

**223.** *1.* What is the value of $(\frac{5}{6}+\frac{3}{4}-\frac{7}{8})+\frac{2}{3}$?

*2.* Find the value of $3\frac{3}{8}+5\frac{3}{4}+3\frac{1}{8}-2\frac{5}{8}$.

*3.* Find the value of $(16\frac{3}{4}+12\frac{1}{2})-(5\frac{1}{8}+6\frac{3}{10})$.

*4.* What is the value of $56\frac{1}{4}-38\frac{3}{4}+19\frac{1}{2}-8\frac{1}{8}$?

*5.* From the sum of $14\frac{1}{8}$ and $11\frac{3}{8}$ take the difference between $35\frac{1}{3}$ and $26\frac{1}{6}$.

*6.* From $\frac{8}{9}$ take $\frac{3}{101}$ and add $\frac{1}{11}$.

*7.* From a barrel containing $37\frac{3}{4}$ gallons, $29\frac{3}{8}$ gallons leaked out. How much remains in the barrel?

*8.* A farmer sold $560\frac{7}{12}$ bu. from a bin containing $927\frac{3}{8}$ bu. How many bushels still in the bin?

*9.* A lady bought a shawl for $\$13\frac{7}{8}$, a hat for $\$12\frac{3}{4}$, and a dress for $\$25\frac{2}{3}$, and in payment gave six ten-dollar bills. How much change should she receive?

*10.* A man's family expenses for the month of January are: rent, $\$25\frac{3}{4}$; clothes, $\$17\frac{5}{8}$; coal, $\$4\frac{1}{2}$; groceries, $\$15\frac{1}{12}$. If he receives $\$87\frac{1}{2}$ a month, how much can he save?

*11.* I bought $9\frac{1}{2}$ tons of coal and had it delivered in 5 loads, the first containing $1\frac{3}{4}$ tons, the second $1\frac{7}{8}$ tons, the third $1\frac{1}{8}$ tons, and the fourth $1\frac{1}{4}$ tons. What did the fifth load contain?

*12.* A had 375 lbs. of wire and sold $25\frac{3}{4}$ lbs. to B, $75\frac{5}{8}$ lbs. to C, and $37\frac{5}{16}$ lbs. to D. How much had he left?

*13.* From a piece of cloth a merchant sold $37\frac{1}{4}$ yards to A, $23\frac{3}{4}$ to B, $12\frac{2}{3}$ to C, and there remain unsold $27\frac{1}{16}$ yards. Find number of yards in the piece.

*14.* From a piece of muslin of 74 yds., there were sold to different persons $6\frac{1}{4}$ yds., $8\frac{1}{2}$ yds., $12\frac{1}{4}$ yds., $15\frac{3}{4}$ yds. and $27\frac{3}{4}$ yds.; how many yards remain unsold?

*15.* From a tract of land of $472\frac{3}{4}$ acres there were sold three farms, respectively $112\frac{1}{2}$ acres, $220\frac{1}{8}$ acres and $88\frac{3}{8}$ acres. How many acres were retained?

*16.* In locating a park, tracts of land were bought as follows: $33\frac{3}{8}$ acres, $178\frac{1}{4}$ acres, $82\frac{1}{2}$ acres, $13\frac{3}{8}$ acres and $3\frac{3}{4}$ acres. Roads and building sites occupied $18\frac{3}{4}$ acres, and woodlands, $87\frac{5}{8}$ acres; the remainder was in grass. How many acres in grass?

*17.* From 25000 bushels of wheat, there were taken four car-loads of $837\frac{1}{2}$ bushels, $936\frac{1}{4}$ bushels, $1006\frac{3}{4}$ bushels and $883\frac{1}{4}$ bushels. How many bushels were left?

*18.* A trip of 3230 miles was made as follows: $478\frac{1}{4}$ miles the first day, $518\frac{3}{8}$ miles the second day, $490\frac{7}{8}$ miles the third day, $537\frac{3}{4}$ miles the fourth day, $487\frac{7}{8}$ miles the fifth day and $427\frac{1}{8}$ miles the sixth day. How many miles were traveled on the seventh day?

*19.* From a hogshead of molasses containing $72\frac{3}{4}$ gallons there were drawn $9\frac{3}{4}$ gallons, $15\frac{1}{8}$ gallons, $13\frac{3}{4}$ gallons and $27\frac{5}{8}$ gallons. How many gallons remain?

*20.* A wheelman traveled the first day $126\frac{1}{4}$ miles, the second day $137\frac{1}{8}$ miles, the third day $117\frac{3}{4}$ miles, the fourth day $27\frac{5}{8}$ miles, the fifth day $141\frac{3}{4}$ miles. How many miles must he ride to complete $752\frac{1}{8}$ miles?

## MULTIPLICATION OF FRACTIONS

**224. Multiplication** of fractions is the process of finding the product of two factors when one or both of them are fractions.

NOTE.—When any number is multiplied by a number greater than 1, whether a whole or mixed number, the product is greater than the multiplicand; but when a number is multiplied by a fraction less than 1, the product is less than the multiplicand.

**225. To multiply a fraction by a fraction.**

EXAMPLE.—What is the cost of $\frac{7}{12}$ of a yard of cloth at $\$\frac{9}{10}$ per yard?

$\frac{9}{10} \times \frac{7}{12} = \frac{63}{120} = \frac{21}{40}$ or $\frac{\overset{3}{\cancel{9}}}{10} \times \frac{7}{\cancel{12}_{4}} = \frac{21}{40}$

**WRITTEN EXERCISE**

**226.** Find the product of

| | | | |
|---|---|---|---|
| 1. $\frac{2}{3} \times \frac{3}{4}$ | 4. $\frac{2}{10} \times \frac{7}{9}$ | 7. $\frac{8}{17} \times \frac{68}{84}$ | 10. $\frac{2}{3} \times \frac{7}{12} \times \frac{3}{5}$ |
| 2. $\frac{4}{5} \times \frac{9}{16}$ | 5. $\frac{11}{16} \times \frac{48}{88}$ | 8. $\frac{6}{19} \times \frac{1}{24}$ | 11. $\frac{4}{9} \times \frac{3}{7} \times \frac{28}{39}$ |
| 3. $\frac{7}{8} \times 1\frac{2}{5}$ | 6. $\frac{8}{11} \times 1\frac{1}{10}$ | 9. $\frac{3}{4} \times \frac{5}{8} \times \frac{6}{7}$ | 12. $\frac{5}{6} \times \frac{3}{5} \times \frac{9}{15}$ |

NOTE.—Compound fractions are connected by the word *of* and are simplified by the operation of multiplication.

Find the product of

| | |
|---|---|
| 13. $\frac{5}{6}$ of $\frac{3}{8} \times 1\frac{6}{9}$ | 17. $1\frac{4}{15} \times \frac{7}{10} \times 1\frac{33}{55} \times 1\frac{7}{32}$ |
| 14. $\frac{2}{3}$ of $\frac{7}{9} \times \frac{4}{15}$ of $\frac{2}{3}$ | 18. $\frac{2}{5}$ of $\frac{5}{7}$ of $\frac{7}{9}$ of $\frac{9}{15}$ |
| 15. $\frac{3}{8}$ of $1\frac{1}{5}$ of $1\frac{9}{11} \times \frac{4}{9}$ | 19. $1\frac{2}{17} \times 1\frac{3}{15} \times 4\frac{1}{43}$ |
| 16. $\frac{3}{14} \times 1\frac{7}{28} \times \frac{5}{11}$ of $\frac{9}{15}$ | 20. $\frac{4}{5} \times \frac{6}{7} \times \frac{8}{9} \times 1\frac{1}{13} \times 1\frac{5}{16}$ |

**227.** RULE.—*Multiply the numerators for a new numerator and the denominators for a new denominator.*

NOTE.—The work may often be shortened by cancelation.

**228. To multiply a whole number by a fraction.**

EXAMPLE.—Multiply 57 by $\frac{5}{9}$

$57 \times \frac{5}{9} = \frac{285}{9}$ $9\overline{)285}$ $31\frac{6}{9} = 31\frac{2}{3}$

or $\frac{\overset{19}{\cancel{57}}}{} \times \frac{5}{\cancel{9}_{3}} = \frac{95}{3} = 31\frac{2}{3}$

**WRITTEN EXERCISE**

**229.** Multiply

| | | |
|---|---|---|
| 1. $\frac{2}{3}$ by 21 | 5. $1\frac{1}{2}$ by 38 | 9. $\frac{34}{35}$ by 165 |
| 2. $\frac{7}{8}$ by 16 | 6. $1\frac{7}{18}$ by 68 | 10. $\frac{37}{43}$ by 85 |
| 3. $\frac{4}{5}$ by 17 | 7. $\frac{13}{25}$ by 25 | 11. 66 by $\frac{3}{8}$ |
| 4. $\frac{7}{9}$ by 26 | 8. $\frac{33}{75}$ by 75 | 12. 14 by $\frac{5}{14}$ |

**230.** RULE. *Multiply the whole number by the numerator, and divide the product by the denominator.*

**231. To multiply a mixed number by a whole number.**

EXAMPLE.—Multiply $229\frac{2}{3}$ by 7

$$\begin{array}{r} 229\frac{2}{3} \\ \times\quad 7 \\ \hline 1607\frac{2}{3} \end{array}$$

7 times $\frac{2}{3} = \frac{14}{3} = 4\frac{2}{3}$; write $\frac{2}{3}$ and carry 4

**WRITTEN EXERCISE**

**232.** Multiply

| | | |
|---|---|---|
| *1.* $337\frac{3}{4}$ by 9 | *5.* $397\frac{3}{8}$ by 38 | *9.* $1378\frac{15}{16}$ by 423 |
| *2.* $426\frac{3}{4}$ by 15 | *6.* $439\frac{1}{4}$ by 45 | *10.* $3789\frac{3}{8}$ by 629 |
| *3.* $568\frac{7}{8}$ by 21 | *7.* $897\frac{8}{9}$ by 87 | *11.* $4288\frac{7}{8}$ by 325 |
| *4.* $287\frac{1}{3}$ by 35 | *8.* $989\frac{7}{8}$ by 237 | *12.* $5063\frac{6}{8}$ by 724 |

**233.** RULE.—*Multiply the fraction by the whole number, adding the result to the product of the whole numbers.*

**234. To multiply a mixed number by a mixed number.**

EXAMPLE.—Multiply $37\frac{1}{2}$ by $35\frac{1}{2}$

(1) $37 \times 35$
(2) $37 \times \frac{1}{2}$
(3) $\frac{1}{2} \times 35$
(4) $\frac{1}{2} \times \frac{1}{2}$

$$\begin{array}{r} 37\frac{1}{2} \\ \times\quad 35\frac{1}{2} \\ \hline 185 \\ 111 \\ 18\frac{1}{2} \\ 17\frac{1}{2} \\ \frac{1}{4} \\ \hline 1331\frac{1}{4} \end{array}$$

**WRITTEN EXERCISE**

**235.** Multiply

| | | |
|---|---|---|
| *1.* $56\frac{3}{4}$ by $27\frac{1}{2}$ | *5.* $178\frac{3}{4}$ by $15\frac{5}{8}$ | *9.* $468\frac{3}{8}$ by $239\frac{1}{8}$ |
| *2.* $37\frac{1}{4}$ by $25\frac{3}{4}$ | *6.* $267\frac{9}{10}$ by $37\frac{3}{4}$ | *10.* $567\frac{9}{16}$ by $256\frac{3}{4}$ |
| *3.* $87\frac{1}{4}$ by $19\frac{3}{4}$ | *7.* $468\frac{7}{8}$ by $35\frac{5}{8}$ | *11.* $608\frac{4}{5}$ by $197\frac{1}{16}$ |
| *4.* $437\frac{5}{8}$ by $18\frac{3}{8}$ | *8.* $276\frac{1}{16}$ by $127\frac{3}{4}$ | *12.* $763\frac{9}{16}$ by $230\frac{7}{8}$ |

**236.** RULE.—*Multiply, separately, the whole numbers, each fraction by the opposite whole number, and the fraction by the fraction; add these products.*

**WRITTEN PROBLEMS**

**237.** Find the cost of

*1.* $385\frac{1}{2}$ pounds of paper at $2\frac{3}{5}$ cents per pound.
*2.* $378\frac{3}{4}$ pounds of beef at $7\frac{3}{4}$ cents per pound.
*3.* $878\frac{1}{2}$ yards of muslin at $9\frac{1}{4}$ cents per yard.
*4.* $676\frac{3}{4}$ bushels of timothy seed at $\$5.62\frac{1}{2}$ per bushel.
*5.* $173\frac{3}{4}$ dozen eggs at $18\frac{3}{4}$ cents per dozen.
*6.* $375\frac{7}{8}$ gallons of wine at $\$2.37\frac{1}{2}$ per gallon.
*7.* $739\frac{3}{4}$ quarts of berries at $5\frac{1}{2}$ cents per quart.
*8.* $378\frac{8}{11}$ acres of land at $\$127\frac{1}{8}$ per acre.
*9.* $39\frac{1}{4}$ pecks of apples at $37\frac{1}{2}$ cents per peck.
*10.* $837\frac{3}{4}$ pounds of coffee at $33\frac{3}{4}$ cents per pound.

## DIVISION OF FRACTIONS

**238. Division** of fractions is the process of finding the quotient when the dividend or divisor, or both, are fractions.

NOTE.—The quotient is larger than the dividend if the divisor is less than 1, and smaller if the divisor is greater than 1.

**239.** The **reciprocal** of a number is 1 divided by that number; as, the reciprocal of 5 is $\frac{1}{5}$; of 9 is $\frac{1}{9}$.

**240.** The **reciprocal of a fraction** is 1 divided by that fraction, and simply inverts the fraction; as, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$; of $\frac{7}{8}$ is $\frac{8}{7}$.

**241. To divide a whole number by a fraction.**

EXAMPLE.—Divide 2 by $\frac{2}{3}$

$$2 \div \frac{2}{3} = 2 \times \frac{3}{2} = \frac{6}{2} = 3$$

*or*

$$\frac{2}{1} \div \frac{2}{3} = \frac{2}{1} \times \frac{3}{2} = \frac{6}{2} = 3$$

It will be observed by the following steps in reasoning that the divisor naturally inverts itself. To divide by $\frac{2}{3}$ is to divide by $\frac{1}{3}$ of 2. If we divide 2 by 2, we have as a quotient 1. A divisor $\frac{1}{3}$ as great would produce a quotient 3 times as great, that is, 3 times 1 or 3; or $\frac{1}{3}$ is contained 3 times as often as 1, or $3 \times 2$ or 6 times, and $\frac{2}{3}$ is contained $\frac{1}{2}$ as often as $\frac{1}{3}$ or $\frac{1}{2}$ of 6 times or 3. One may be expressed under the whole number, and the division performed similar to a fraction by a fraction.

**242.** RULE.—*Multiply the dividend by the divisor inverted.* (In other words, multiply the dividend by the *reciprocal* of the divisor.)

### WRITTEN EXERCISE

**243.** Divide

| | | | |
|---|---|---|---|
| *1.* 1 by $\frac{3}{4}$ | *4.* 15 by $\frac{10}{11}$ | *7.* 39 by $\frac{18}{19}$ | *10.* 224 by $\frac{32}{33}$ |
| *2.* 9 by $\frac{2}{3}$ | *5.* 20 by $\frac{15}{16}$ | *8.* 45 by $\frac{9}{10}$ | *11.* 375 by $\frac{25}{26}$ |
| *3.* 16 by $\frac{4}{5}$ | *6.* 25 by $\frac{20}{21}$ | *9.* 168 by $\frac{32}{33}$ | *12.* 670 by $\frac{40}{41}$ |

**244. To divide a fraction by a whole number.**

EXAMPLE 1.—Divide $\frac{4}{5}$ by 2

$\frac{4}{5} \div 2 = \frac{2}{5}$ or $\frac{4}{5} \div \frac{2}{1} = \frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$ Dividing the numerator divides the fraction.

EXAMPLE 2.—Divide $\frac{4}{5}$ by 3

$\frac{4}{5} \div 3 = \frac{4}{15}$ or $\frac{4}{5} \div \frac{3}{1} = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$ Multiplying the denominator divides the fraction.

**245.** RULE.—*Divide the numerator by the whole number, or multiply the denominator by the whole number.*

WRITTEN EXERCISE

**246.** Divide

| | | | |
|---|---|---|---|
| *1.* $\frac{1}{2}$ by 2 | *4.* $\frac{7}{8}$ by 4 | *7.* $\frac{10}{12}$ by 5 | *10.* $\frac{1}{3}\frac{5}{6}$ by 10 |
| *2.* $\frac{2}{3}$ by 3 | *5.* $\frac{11}{16}$ by 9 | *8.* $\frac{13}{16}$ by 7 | *11.* $\frac{3}{8}\frac{4}{5}$ by 15 |
| *3.* $\frac{1}{4}$ by 2 | *6.* $\frac{13}{16}$ by 8 | *9.* $\frac{3}{4}\frac{1}{9}$ by 9 | *12.* $\frac{1}{6}\frac{7}{5}$ by 25 |

**247. To divide a fraction by a fraction.**

EXAMPLE.—Divide $\frac{3}{4}$ by $\frac{2}{3}$

$\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$

To divide by a fraction, the principle is the same, whether the dividend be a whole number or a fraction.

**248.** RULE.—*Multiply the dividend by the divisor inverted.*

WRITTEN EXERCISE

**249.** Divide

| | | | |
|---|---|---|---|
| *1.* $\frac{3}{4}$ by $\frac{3}{4}$ | *4.* $\frac{9}{7}$ by $\frac{3}{8}$ | *7.* $1\frac{5}{16}$ by $\frac{3}{4}$ | *10.* $1\frac{1}{2}$ by $\frac{3}{8}$ |
| *2.* $\frac{7}{8}$ by $\frac{1}{2}$ | *5.* $1\frac{3}{8}$ by $\frac{6}{10}$ | *8.* $\frac{15}{32}$ by $\frac{9}{7}$ | *11.* $2\frac{1}{2}$ by $1\frac{1}{2}$ |
| *3.* $\frac{9}{10}$ by $\frac{3}{5}$ | *6.* $\frac{11}{12}$ by $\frac{9}{11}$ | *9.* $\frac{7}{12}$ by $\frac{9}{10}$ | *12.* $3\frac{1}{3}$ by $1\frac{1}{4}$ |

**250. To divide a mixed number by a whole number.**

EXAMPLE.—Divide $37\frac{1}{2}$ by 2

$2\overline{)37\frac{1}{2}}$

$18\frac{3}{4}$

Rem. $1/2 = 3/2 \times 1/2 = 3/4$

37 divided by 2 gives 18, with a remainder of 1. Adding this remainder to the fraction of the dividend, and reducing to an improper fraction, we have $\frac{3}{2}$. Dividing $\frac{3}{2}$ by the divisor we have $\frac{3}{4}$, which completes the quotient.

**251.** RULE.—*First divide the integral part of the mixed number; if there be a remainder, add it to the fractional part; then divide the fractional part.*

WRITTEN EXERCISE

**252.** Divide

| | | |
|---|---|---|
| *1.* $75\frac{1}{2}$ by 4 | *5.* $3971\frac{3}{8}$ by 27 | *9.* $87378\frac{5}{8}$ by 129 |
| *2.* $137\frac{2}{3}$ by 9 | *6.* $4781\frac{9}{7}$ by 33 | *10.* $98375\frac{9}{10}$ by 137 |
| *3.* $148\frac{5}{8}$ by 19 | *7.* $5678\frac{1}{8}$ by 35 | *11.* $87634\frac{1}{2}$ by 147 |
| *4.* $267\frac{3}{4}$ by 23 | *8.* $6438\frac{7}{8}$ by 39 | *12.* $93675\frac{1}{11}$ by 165 |

**253. To divide a whole number by a mixed number.**

EXAMPLE.—(1) Divide 243 by $4\frac{1}{3}$; (2) 7854 by $30\frac{1}{4}$

$4\frac{1}{3})24.3$

$\frac{.3}{13})\frac{3}{729}(56\frac{1}{13}$

$\frac{65}{79}$

$\frac{78}{1}$

$\frac{7854}{1} \div 30\frac{1}{4} =$

$\frac{7854}{1} \times \frac{4}{121} = \frac{2856}{11} = 259\frac{7}{11}$

**254.** RULE.—*Reduce both dividend and divisor to improper fractions having a common denominator (by multiplying both by the denominator of the fraction in the divisor), then divide the numerator of the dividend by the numerator of the divisor.*

*Or, give the whole number a denominator of 1, invert the divisor and proceed as in multiplication.*

### WRITTEN EXERCISE

**255.** Divide

| | | |
|---|---|---|
| *1.* 46 by $6\frac{1}{2}$ | *5.* 364 by $11\frac{5}{7}$ | *9.* 1219 by $56\frac{3}{16}$ |
| *2.* 32 by $3\frac{2}{3}$ | *6.* 892 by $13\frac{3}{8}$ | *10.* 3924 by $42\frac{9}{11}$ |
| *3.* 79 by $7\frac{1}{8}$ | *7.* 478 by $24\frac{1}{12}$ | *11.* 4685 by $74\frac{3}{47}$ |
| *4.* 122 by $9\frac{3}{4}$ | *8.* 572 by $31\frac{4}{13}$ | *12.* 62830 by $126\frac{7}{16}$ |

**256. To divide a mixed number by a mixed number.**

EXAMPLE.—Divide $358\frac{1}{3}$ by $18\frac{3}{4}$

$18\frac{3}{4})358\frac{1}{3}$

$12 \quad 12$

$225)4300(19\frac{1}{9}$

$225$

$2050$

$2025$

$25 = \frac{1}{9}$

$225$

Reducing both dividend and divisor to twelfths, we divide the numerator of the dividend by the numerator of the divisor; or reducing to improper fractions, we invert the divisor and multiply, canceling if possible.

$358\frac{1}{3} \div 18\frac{3}{4} =$

$\frac{4300}{3} \times \frac{4}{75} = \frac{172}{9} = 19\frac{1}{9}$

**257.** RULE.—*Reduce both dividend and divisor to improper fractions having a common denominator (by multiplying both by the least common multiple of the denominators of the fractions), then divide the numerator of the dividend by the numerator of the divisor. Or, reduce to improper fractions, invert the divisor and proceed as in multiplication.*

### WRITTEN EXERCISE

**258.** Divide

| | | |
|---|---|---|
| *1.* $67\frac{1}{2}$ by $12\frac{1}{4}$ | *5.* $637\frac{9}{10}$ by $25\frac{3}{5}$ | *9.* $6789\frac{5}{16}$ by $97\frac{1}{12}$ |
| *2.* $87\frac{3}{4}$ by $15\frac{2}{3}$ | *6.* $938\frac{7}{12}$ by $32\frac{5}{6}$ | *10.* $9878\frac{5}{24}$ by $104\frac{5}{16}$ |
| *3.* $327\frac{3}{5}$ by $24\frac{1}{2}$ | *7.* $1267\frac{7}{8}$ by $87\frac{3}{4}$ | *11.* $9783\frac{5}{7}$ by $129\frac{3}{11}$ |
| *4.* $837\frac{5}{6}$ by $19\frac{3}{4}$ | *8.* $2376\frac{4}{5}$ by $39\frac{2}{3}$ | *12.* $24389\frac{3}{5}$ by $214\frac{1}{7}$ |

### WRITTEN PROBLEMS

**259.** *1.* If $\frac{1}{8}$ of a pound of coffee costs 24 cents, what will a pound cost?

*2.* At $\$3\frac{1}{3}$ per bushel, how many bushels of apples can be bought for \$8?

*3.* If $\frac{1}{3}$ of a pound of butter costs $12\frac{1}{2}$ cents, what will $\frac{1}{4}$ of a pound cost?

*4.* At $\$\frac{3}{4}$ per yard, how many yards of silk can be bought for $\$7\frac{1}{2}$?

*5.* If a man saves $\$\frac{7}{8}$ each day, in how many days can he save $\$6\frac{1}{8}$?

*6.* If a horse can eat $\frac{9}{7}$ of a bushel of oats in one day, in what time can he eat $8\frac{3}{8}$ bushels?

*7.* If a train runs $155\frac{3}{4}\frac{9}{0}$ miles in $3\frac{3}{5}$ hours, what is its average rate per hour?

*8.* How many barrels of flour can be purchased for \$21, if $\frac{7}{8}$ of a barrel costs \$4?

*9.* What will $3\frac{3}{4}$ tons of coal cost, if $7\frac{1}{2}$ tons cost \$35?

*10.* If 7 men can dig a cellar in $3\frac{1}{2}$ days, in what time can 3 men dig it?

*11.* 18 men work 6 days for \$300, how much should 5 men receive for the same time?

*12.* If a pound of butter costs $18\frac{3}{4}$ cents, how much can be bought for $12\frac{1}{2}$ cents? For $90\frac{1}{2}$ cents?

*13.* If $7\frac{1}{2}$ barrels of oil contain $319\frac{3}{4}$ gallons, how many gallons will $3\frac{1}{3}$ barrels contain?

*14.* If $\frac{7}{8}$ of the value of a house is \$13706, what is the value of $\frac{4}{5}$ of it?

*15.* If $4\frac{1}{2}$ tons of coal cost $\$19\frac{1}{8}$, what will $37\frac{3}{4}$ tons cost? How many tons could be bought for $\$337\frac{1}{4}$?

*16.* B bought a house and lot; the lot cost \$3700, which was $\frac{2}{7}$ of the cost of the house. What did he pay for both?

*17.* If $23\frac{4}{5}$ bags of coffee cost \$410, what is the cost of $12\frac{3}{5}$ bags?

*18.* If $17\frac{1}{2}$ tons of hay cost $\$258\frac{3}{4}$, what will be the value of $6\frac{4}{5}$ tons?

*19.* If $69\frac{5}{8}$ acres of land cost $\$7660\frac{4}{11}$, what is the value of $311\frac{5}{8}$ acres?

*20.* If $86\frac{3}{4}$ cords of wood cost $\$806\frac{5}{20}$, what will be the price of $91\frac{1}{4}$ cords?

*21.* If $28\frac{1}{2}$ tons of bran cost $\$718\frac{3}{4}$, what will $43\frac{7}{8}$ tons cost?

*22.* How many bushels of potatoes at $\$\frac{7}{8}$ a bushel will pay for $16\frac{3}{4}$ bushels of wheat at $\$1\frac{1}{4}$ a bushel?

*23.* If $1\frac{5}{8}$ tons of coal cost $\$7\frac{7}{8}$, how many tons can be bought for $\$145\frac{3}{4}$?

*24.* If $39\frac{3}{8}$ pounds of sugar cost $\$2\frac{1}{8}$, what will $72\frac{3}{4}$ pounds cost?

*25.* A man exchanged $46\frac{1}{4}\frac{7}{0}$ acres of land worth $\$120\frac{5}{8}$ an acre for land worth $\$166\frac{2}{3}$ an acre. How many acres did he receive?

*26.* If $6\frac{2}{3}$ tons of hay cost $\$93\frac{1}{3}$, how much hay will be worth $\$157\frac{1}{16}$?

*27.* A piece of muslin containing $42\frac{5}{8}$ yards costs $\$5\frac{7}{8}$; what will a piece containing $63\frac{1}{4}$ yards cost?

*28.* If $47\frac{1}{2}$ gallons of vinegar are worth $\$9\frac{3}{16}$, what will be the cost of 5 barrels of $43\frac{1}{2}$ gallons each?

*29.* How many yards of sheeting worth $19\frac{3}{4}$ cents a yard should be given for 7 hams weighing $10\frac{5}{16}$ pounds each, at $9\frac{11}{16}$ cents a pound?

*30.* How many pounds of coffee at $25\frac{1}{2}$ cents a pound should be exchanged for 3 rolls of butter weighing $4\frac{1}{16}$ pounds each, worth $30\frac{3}{4}$ cents a pound?

## COMPLEX FRACTIONS

**260.** A **complex fraction** is simply an indicated division, and to simplify it is to perform the operation indicated.

**261. To simplify a complex fraction.**

EXAMPLE.—Simplify $\frac{\frac{3}{4}}{\frac{7}{8}}$ $\qquad \frac{3/4}{7/8} = \frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{6}{7}$

**262.** RULE.—*Divide the numerator by the denominator.*

### WRITTEN EXERCISE

**263.** Change to simple fractions

1. $\frac{\frac{2}{3}}{\frac{3}{4}}$
2. $\frac{\frac{7}{8}}{\frac{9}{10}}$
3. $\frac{2\frac{1}{2}}{\frac{1}{5}}$
4. $\frac{5\frac{1}{3}}{7\frac{1}{2}}$
5. $\frac{21}{3\frac{5}{9}}$
6. $\frac{4\frac{1}{3}}{39}$
7. $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{4}-\frac{1}{2}}$
8. $\frac{\frac{3}{8} \text{ of } 1\frac{2}{15}}{\frac{5}{6} \text{ of } \frac{7}{8}}$
9. $\frac{2\frac{2}{3}+3\frac{1}{2}-\frac{5}{6}}{6\frac{7}{8}-2\frac{1}{3}+5\frac{1}{4}}$
10. $\frac{\frac{9}{10}}{1\frac{5}{16}} + \frac{\frac{7}{8}}{\frac{3}{4} \text{ of } \frac{8}{7}} - \frac{\frac{1}{2} \text{ of } \frac{3}{5}}{\frac{2}{3}+\frac{3}{5}}$
11. $\frac{\frac{2}{3}+\frac{3}{4}}{\frac{7}{8}-\frac{1}{2}} \div \frac{\frac{3}{5} \text{ of } 1\frac{1}{2}0}{\frac{4}{5} \div \frac{2}{3}} + \frac{2\frac{1}{2}}{3\frac{5}{16}}$
12. $\frac{\frac{1}{2}}{\frac{2}{3}} - \frac{\frac{4}{5}}{\frac{7}{8}}$

### SHORT METHODS IN THE OPERATIONS OF FRACTIONS

#### ADDITION AND SUBTRACTION

**264. To add or subtract two simple fractions.**

EXAMPLE.—Add $\frac{3}{4}$ and $\frac{5}{7}$

$\frac{3}{4}+\frac{5}{7}=\frac{21+20}{28}=\frac{41}{28}=1\frac{13}{28}$

**265.** RULE.—*Multiply each numerator by the opposite denominator and write the sum or the difference over the product of the denominators.*

#### MULTIPLICATION AND DIVISION

**266. To multiply a whole number by a mixed number that is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., of 10, 100, 1000, etc.**

EXAMPLE.—(1) Multiply 256 by $12\frac{1}{2}$; (2) 1896 by $33\frac{1}{3}$

| | | |
|---|---|---|
| $8\overline{)2560}$<br>3200 | EXPLANATION<br>(1) $12\frac{1}{2}=\frac{1}{8}$ of 100<br>(2) $33\frac{1}{3}=\frac{1}{3}$ of 100 | $3\overline{)189600}$<br>63200 |

**267.** RULE.—*Annex to the multiplicand one cipher if the multiplier is a part of 10, two, if a part of 100, etc., and divide by the denominator of the fractional part.*

**WRITTEN EXERCISE**

**268.** Multiply

| | | |
|---|---|---|
| *1.* 432 by $12\frac{1}{2}$ | *5.* 960 by $16\frac{2}{3}$ | *9.* 325 by $6\frac{2}{3}$ |
| *2.* 276 by $8\frac{1}{3}$ | *6.* 593 by $14\frac{2}{7}$ | *10.* 750 by $9\frac{1}{11}$ |
| *3.* 348 by $14\frac{2}{7}$ | *7.* 878 by $33\frac{1}{3}$ | *11.* 832 by $3\frac{1}{3}$ |
| *4.* 96 by $2\frac{1}{2}$ | *8.* 498 by $11\frac{1}{9}$ | *12.* 1248 by $6\frac{1}{4}$ |

**269.** If the part is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc., greater or less than 10, 100, etc., add the result to or subtract from the increased multiplicand as indicated in examples.

EXAMPLES.—(1) Multiply 187 by $116\frac{2}{3}$; (2) 436 by $91\frac{2}{3}$

$6\overline{)18700}$ $3116\frac{4}{6} = \frac{2}{3}$ $2181\frac{6}{3}$

EXPLANATION

(3) $16\frac{2}{3} = \frac{1}{6}$ of 100

(4) $8\frac{1}{3} = \frac{1}{12}$ of 100

$12\overline{)43600}$ $3633\frac{4}{12} = \frac{1}{3}$ $39966\frac{3}{3}$

**270.** RULE.—*Add or subtract the fractional part greater or less than 10, 100, etc.*

**WRITTEN EXERCISE**

**271.** Multiply

| | | |
|---|---|---|
| *1.* 574 by $112\frac{1}{2}$ | *5.* 472 by $83\frac{1}{3}$ | *9.* 384 by $133\frac{1}{3}$ |
| *2.* 872 by $87\frac{1}{2}$ | *6.* 784 by 75 | *10.* 784 by $66\frac{2}{3}$ |
| *3.* 196 by $116\frac{2}{3}$ | *7.* 847 by $114\frac{2}{7}$ | *11.* 1210 by $109\frac{1}{11}$ |
| *4.* 572 by $111\frac{1}{9}$ | *8.* 686 by $85\frac{5}{7}$ | *12.* 750 by 125 |

**272. To divide a whole number by a mixed number that is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, of 10, 100, etc.**

EXAMPLES.—(1) Divide 960 by $12\frac{1}{2}$; (2) 385 by $33\frac{1}{3}$

$9\overline{)960}$ $8$ $768$ Rem. $\frac{40}{100} = \frac{4}{5}$

$3\overline{)385}$ $3$ $115$ Rem. $\frac{55}{100} = \frac{11}{20}$

**273.** RULE.—*Multiply the dividend by the denominator of the equivalent fraction. Mark off one right hand figure if the divisor is a fractional part of 10; two, if a fractional part of 100, etc. The remainder will be tenths if a fractional part of 10, and hundredths if a fractional part of 100, etc.*

**WRITTEN EXERCISE**

**274.**

| | |
|---|---|
| *1.* Divide 432 by $12\frac{1}{2}$ | *5.* Divide 384 by $33\frac{1}{3}$ |
| *2.* Divide 276 by $8\frac{1}{3}$ | *6.* Divide 189 by $12\frac{1}{2}$ |
| *3.* Divide 595 by $16\frac{2}{3}$ | *7.* Divide 297 by $9\frac{1}{11}$ |
| *4.* Divide 348 by $14\frac{2}{7}$ | *8.* Divide 1462 by $133\frac{1}{3}$ |

## REVIEW PROBLEMS

**275.** *1.* Simplify $\frac{\frac{3}{4} \text{ of } \frac{7}{8}}{\frac{2}{3} \text{ of } \frac{5}{6}}$

*2.* Add $27\frac{3}{4}$, $67\frac{1}{2}$, $38\frac{5}{6}$ and $\frac{12\frac{3}{4}}{\frac{7}{8}}$

*3.* What is the value of $1\frac{4}{3}\frac{8}{3}$ of $\frac{35}{64}+7\frac{1}{2}\times3\frac{1}{3}$ of $1\frac{6}{25}$?

*4.* The product of two numbers is 405 and one of them is $27\frac{3}{5}$. What is the other?

*5.* There are two numbers whose difference is $49\frac{1}{2}$ and one number is $\frac{3}{8}$ of the other; what are the numbers?

*6.* If a man travels 4 miles in $\frac{5}{7}$ of an hour, how far would he travel in $2\frac{1}{4}$ hours at the same rate?

*7.* At $\$\frac{5}{8}$ a yard, how many yards of silk can be bought for $\$10\frac{3}{8}$?

*8.* If 48 is $\frac{3}{7}$ of some number, what is $\frac{2}{3}$ of the same number?

*9.* A gentleman owning $\frac{7}{16}$ of a vessel sold $\frac{1}{3}$ of his share; how much of the vessel does he now own?

*10.* If a person contracts to finish a piece of work in 27 days, what part of it should he have done in 12 days?

*11.* A merchant lost $\frac{1}{8}$ of his capital, after which he gained $700 and then had $2500; what amount did he lose?

*12.* A and B own a steamboat; A owns $1\frac{7}{5}$ and B the remainder of the interest. If $7000 has been gained during a season, how much should each receive?

*13.* Divide $4500 between two persons so that one shall have $\frac{4}{7}$ as much as the other.

*14.* Suppose the cargo of a vessel to be worth $2\frac{3}{8}$ times the vessel; what is the value of the vessel if the cargo is worth $17500?

*15.* If $8\frac{1}{2}$ tons of hay cost $100, how many tons can be bought for $125 at the same rate?

*16.* If $\frac{1}{3}$ of 10 bushels of wheat cost $7\frac{1}{2}$, how much will $\frac{2}{3}$ of 8 bushels cost at the same rate?

*17.* A cistern full of water sprung a leak, and before the leak was stopped $\frac{2}{5}$ of the water had run out, but $\frac{3}{7}$ as much as had run out ran in. What part of the cistern had been emptied when the leak was stopped?

*18.* How many suits of clothes requiring $5\frac{5}{8}$ yards each can be cut from $121\frac{1}{4}$ yards of cloth?

*19.* A tank has two outlet pipes; one can empty it in 5 hours, the other in 8 hours. In what time will the tank be emptied if both be opened at one time?

*20.* A can do a piece of work in 6 hours and B can do it in 5 hours; in what time can both working together do it?

*21.* A and B together have $1640. If B's money is equal to $\frac{3}{5}$ of A's, how much has each?

*22.* A farmer sold 6 bales of hay, weighing respectively $112\frac{5}{8}$ lbs., $120\frac{1}{4}$ lbs., $98\frac{1}{3}$ lbs., $137\frac{3}{4}$ lbs., $118\frac{3}{8}$ lbs., $108\frac{7}{8}$ lbs.; find selling price at $1\frac{3}{4}$ cents per pound.

*23.* If 17½ bushels of potatoes are equally divided among 7 families of 3 persons each, how many should be given to each family and how many to each person?

*24.* A man paid $35 for a colt, $\frac{3}{5}$ of which cost was $\frac{3}{11}$ of the cost of a horse. What did the horse cost?

*25.* A grocer bought 107½ bushels of potatoes and sold 36⅜ bushels. He afterward purchased 54¼ bushels of one man, 37⅜ bushels of another and then sold 47⅛ bushels. How many bushels had he then remaining?

*26.* What is the total value of $307^1$ yards of prints at $6^2$ cents per yard; $407^3$ yards cloth at $\$2.06^1$; $325^2$ yards cassimere at $\$4.37^2$?

NOTE.—Small figures indicate quarters.

*27.* Bought a watch and chain for $140, the chain costing $\frac{2}{3}$ as much as the watch. Find cost of each.

*28.* A speculator has ¼ of his money in city lots, ⅓ in farm land, and the remainder, $17230, in cash. Find amount invested in city lots and farm land.

*29.* How many bushels of apples worth 62½ cents per bushel must a farmer exchange for 125¾ yards of carpet at $1.12 per yard and 33⅓ yards of cloth at $2¼ per yard?

*30.* From a barrel containing 37½ gallons of syrup, a grocer sold 25¾ gallons. What part of the barrel remains unsold?

*31.* A man's income is $10¾ per week, and his expenses are $8⅞. How many weeks will it require to save $137½?

*32.* An importer bought a number of bales of silk, each containing 157 yards, at $1⅞ per yard, and sold it at $2¾, gaining $623. How many bales did he buy?

*33.* A, B and C have a certain sum of money. A has $\frac{3}{8}$ of the sum, B ¼, and C the remainder. Find the sum each has, if A has $42.

*34.* A jeweler engaging in business lost $\frac{2}{7}$ of the sum invested, after which he sold the remaining stock, gaining $632, and received $3162. Find loss.

*35.* A can do a piece of work in 5 days and B in 6 days. In what time can they do it working together?

*36.* A and B can do a piece of work in 12 days; B can do it alone in 20 days. In what time can A do it?

*37.* A Liverpool merchant bought 56 bales of cotton, each containing 367½ lbs., at $16\frac{1}{2}$ cents per lb., and sold it for $19\frac{1}{2}$ cents per lb. Find his gain if the freight amounted to $97.50.

*38.* How many yards of cloth can a merchant buy for $24½, if $\$\frac{3}{4}$ is paid for $\frac{3}{4}$ of a yard?

*39.* A bushel of wheat produces 40 pounds of flour. How many bushels must be taken to the mill to produce 300 pounds of flour, if the miller takes ⅛ of the wheat for toll?

*40.* What is the value of 9 pieces of goods containing respectively 47, $49^3$, $45^2$, $37^1$, $56^3$, $48^1$, $43^2$, $41^3$ and $46^1$ yards, at 62½ cents per yard?

*41.* A man bought 321¾ acres of land at $67½ per acre, and sold ⅔ of the land at $76½ per acre, and the remainder at $75¾ per acre. Find his gain.

*42.* How many lemons would have to be sold to gain 20 cents, if bought at the rate of 2 for 1 cent and sold at the rate of 5 for 3 cents?

*43.* A man bequeathed $\frac{1}{3}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to his son, and what was still left to his daughter. If the son's share was \$4675$\frac{1}{3}$ more than the wife's, what was the value of the estate and the share of each?

*44.* A man purchased a horse and buggy for \$1267.50, paying $\frac{1}{4}$ more for the horse than for the buggy. Find the cost of each.

*45.* A and B have \$1265. If A has $\frac{2}{3}$ less money than B, how much has each?

*46.* A father divided \$1260 between his son and daughter, giving his daughter $\frac{4}{5}$ as much as his son. How much did each receive?

*47.* A lady spent $\frac{1}{5}$ of her monthly earnings for board, $\frac{1}{2}$ of the remainder for clothes, and still has left \$17$\frac{1}{2}$. What did she receive per month?

*48.* In how many days will a boy earn \$130 by sawing wood at \$$\frac{3}{5}$ per cord, provided he saws 2$\frac{1}{2}$ cords per day?

*49.* How many bushels of potatoes, bought at \$$\frac{3}{5}$ per bushel and sold at 62$\frac{1}{2}$ cents, will realize a gain of \$25?

*50.* I own $\frac{3}{4}$ of a mill worth \$8043 and sell $\frac{2}{3}$ of my share. What part of the mill do I still own and what is its value?

*51.* If I own $\frac{5}{7}$ of a property, and $\frac{2}{3}$ of my share is worth \$3955, what is the entire property worth?

*52.* A, B and C own a factory. A owns $\frac{5}{16}$; B owns $\frac{1}{4}$ more than A. What part does C own?

*53.* Divide \$3000 into two parts so that one shall be $\frac{4}{7}$ as much as the other.

*54.* A can do a piece of work in 8 days, B in 12 days, and C in 15 days. After A and B work together 3 days, how long will it require B and C to finish the work?

*55.* A grocer buys sugar at the rate of 16 pounds for \$1 and sells it at the rate of 12 pounds for \$1. How many pounds will he have to sell to gain \$1?

*56.* A, B and C engage in trade. A puts in \$4000, B \$6000 and C \$5000. At the end of the year they find they have gained \$4500. How much should each receive?

*57.* A grocer bought $\frac{3}{4}$ of a barrel of molasses and sold $\frac{2}{3}$ of it for the entire cost and the remainder at cost, and gained by the transaction \$2$\frac{1}{2}$. Find price per barrel.

*58.* It requires 8 minutes for the larger of two pipes, and 12 minutes for the smaller, to fill a cistern. Find time required for both together to fill it.

*59.* $\frac{7}{8}$ of the difference between two numbers is 882. If $\frac{5}{6}$ of the larger is equal to the smaller, what are the numbers?

*60.* A young man agreed to work for a year for \$600 and a suit of clothes. His employer discharged him at the end of nine months, giving him \$422.50 and the suit. What was the value of the suit?

*61.* A lady spent \$22$\frac{1}{2}$ for a dress, \$9$\frac{3}{8}$ for a hat and \$5$\frac{3}{5}$ for shoes. She then found that she had left $\frac{2}{5}$ of the sum of money she originally had. How much had she at first?

*62.* If $\frac{5}{11}$ of a property is worth \$7621.30, what is the value of $\frac{3}{8}$ at the same rate?

*63.* B has in bank \$237½. His income is \$6¾ per week and his expenses \$9½. How many weeks will his reserve fund hold out?

*64.* A broker invested \$37056 in land, and sold it for $1\frac{1}{8}$ times its cost, gaining \$4 per acre. How many acres did he buy?

*65.* If $\frac{3}{4}$ of $\frac{3\frac{1}{2}}{\frac{7}{8}}$ of $1\frac{1}{3}$ times a man's money is \$2626, what is $\frac{3}{8}$ of $\frac{\frac{8}{9}}{1\frac{1}{2}}$ of $3\frac{1}{3}$ times his money?

*66.* A can mow a field in 6 days, and A and B together in 2½ days. How many days will it require B alone to mow the field?

*67.* A, B and C can do a piece of work in 10, 12 and 15 hours, respectively. How long will it take all of them, working together, to do the work? If they receive \$45, how should it be divided?

*68.* A father divided \$2151 between his two sons, so that $\frac{2}{3}$ of what the younger received equaled $\frac{3}{5}$ of what the older received. Find amount received by each.

*69.* A man owning $\frac{3}{4}$ of a vessel sold $\frac{4}{5}$ of his share for \$21242. Find value of the vessel and of his remaining share.

*70.* How long will it require to fill a cistern with a capacity of 337½ gallons, if there is a pipe discharging into it 23½ gallons per hour and there is a leak by which it loses 2½ gallons per hour?

*71.* If a boy buys peaches at the rate of 6 for 4 cents and sells them at the rate of 4 for 3 cents, how many must he buy and sell to gain 25 cents?

*72.* I spent $\frac{2}{3}$ of my money for an overcoat and $\frac{3}{5}$ of the remainder for a dress suit. If the overcoat cost \$11½ less than the dress suit, what was the cost of each?

*73.* A boy earned \$562½ in two years, and $\frac{3}{4}$ of what he earned the first year and \$50 equals what he earned the second year. Find sum earned each year.

*74.* B lost $\frac{3}{4}$ of his money, and after earning $\frac{2}{3}$ as much as he had lost he had \$237½. How much had he at first?

*75.* A boy and a man receive \$1276 for a year's work. If the boy can do $\frac{2}{3}$ as much work as the man, how should the money be divided?

*76.* A grocer bought a tub of butter containing 84 pounds at 18¾ cents per pound, and sold $\frac{1}{3}$ of it for \$14 and the remainder at a profit of 4½ cents per pound. Find gain.

*77.* A miller bought 72½ bushels of wheat at 87½ cents per bushel and sold $\frac{1}{4}$ of it at 92½ cents per bushel, and the remainder at \$1.12½ per bushel. Find his gain.

*78.* Bought 768 pounds of coffee, and sold sufficient to amount to \$87.50 at 27½ cents per pound. How much is unsold?

*79.* A lady withdrew $\frac{6}{11}$ of her money from bank and spent $\frac{3}{4}$ of what she withdrew for a dress. If the dress cost \$24, how much money has she still in bank?

*80.* A purchased 325 horses and sold them so that $\frac{3}{4}$ of what he received for them equaled the cost. If his gain was \$9875, what did they cost him per head?

*81.* If $\frac{5}{8}$ of a farm is worth \$450 more than $\frac{3}{5}$ of it, how much is the whole farm worth?

*82.* $\frac{3}{4}$ of the sum paid for a house is the sum paid for a lot. What is the cost of each if together they cost \$14014.21?

*83.* $\frac{8}{9}$ of B's money is equal to $\frac{3}{4}$ of A's and the sum of their money is \$19008. Find share of each.

*84.* A speculator bought a tract of land at \$67½ per acre. He sold $\frac{2}{7}$ of it at \$87½ per acre, $\frac{3}{5}$ of the remainder at \$76½ per acre, and $105\frac{3}{7}$ acres, or what still remained, at \$80¼. Find his profit.

*85.* A dealer expended \$5460 for grain, ½ for wheat at \$1.37½ per bushel, ⅓ for corn at \$.60⅔ per bushel, and the remainder for rye at \$.89 per bushel. How many bushels of each kind did he purchase?

*86.* A and B each had $\frac{8}{9}$ of a ton of hay; B sold A $\frac{2}{3}$ of his hay. What part of A's equals B's after the sale?

*87.* A's farm, consisting of $325\frac{5}{8}$ acres, cost him \$62½ an acre. He sold ⅓ of it at \$67 an acre, ½ of it at \$70 an acre, and the remainder at cost. Find his gain.

*88.* I bought 6 hams, weighing respectively 11½ lbs., 12½ lbs., $7\frac{5}{12}$ lbs., 9¾ lbs., $15\frac{7}{8}$ lbs. and $13\frac{13}{16}$ lbs., at 11¾ cents per pound. I sold the first 3 at $15\frac{5}{8}$ cents per pound and the last 3 at 16¼ cents per pound. Find my gain.

*89.* A dealer bought 5 cars of coal of $13\frac{3}{8}$ tons, $11\frac{7}{10}$ tons, $12\frac{1}{2}\frac{3}{0}$ tons, $14\frac{9}{4}\frac{}{0}$ tons and $9\frac{9}{1}\frac{5}{2}$ tons. Find the total cost at \$6.75 a ton.

*90.* From a chain $546\frac{3}{4}$ yards long traces $\frac{7}{8}$ of a yard long were cut. How many pairs were cut, and what part of a yard remained?

*91.* A farm consisting of $253\frac{3}{8}$ acres cost \$62⅜ an acre; ⅓ of it was sold at \$67⅞ an acre, ½ of the remainder at \$72½ an acre and the remainder at cost. Find the gain.

*92.* How many lots containing $2\frac{5}{16}$ acres can be sold from a field containing $45\frac{3}{4}$ acres, and what will the remainder be worth at \$158 an acre?

*93.* A dealer bought 5 turkeys weighing respectively $17\frac{7}{8}$ lbs., $23\frac{9}{16}$ lbs., $21\frac{3}{8}$ lbs., $22\frac{1}{1}\frac{}{2}$ lbs. and $19\frac{3}{4}$ lbs., at $13\frac{5}{8}$ cents per pound, and sold them at $16\frac{7}{8}$ cents per pound. Find his gain.

*94.* Sold $\frac{5}{8}$ of the difference between $758\frac{5}{8}$ and $948\frac{1}{4}$ tons of coal for \$1328¼. What was the price per ton? per cwt.?

*95.* A farmer sold to a merchant 3 hams weighing $10\frac{5}{8}$ lbs., $11\frac{7}{8}$ lbs. and $12\frac{13}{16}$ lbs., at 12½ cents a pound; 12½ dozen eggs at 25½ cents a dozen. He took ½ the value of these commodities in cash, ½ of the remainder in sugar at 6¼ cents a pound, and the balance in muslin at 12½ cents a yard. How much cash did he receive? How many pounds of sugar? How many yards of muslin?

## RELATION OF NUMBERS

**276.** Finding the relation which one number bears to another is a process of division, and the result is expressed in *times* either integrally or fractionally.

Ex.—What is the relation of 10 to 5?

Ans.—10 is 2 times, or twice 5.

Ex.—What is the relation of 5 to 10?

Ans.—5 is one-half of 10.

NOTE.—Only numbers of the same denomination can be thus compared.

### MENTAL PROBLEMS

**277.** *1.* What is the relation of 8 to 4? Of 12 to 6? Of 36 to 12? Of 12 to 84? Of 72 to 6? Of 96 to 8? Of 11 to 121? Of 132 to 12? Of 132 to 11?

*2.* What part of 8 apples is 6 apples? Of 16 bushels is 10 bushels? Of 35 gallons is 20 gallons?

*3.* What part of 100 is $12\frac{1}{2}$? 25? $16\frac{2}{3}$? $14\frac{2}{7}$? $8\frac{1}{3}$? $9\frac{1}{11}$? $33\frac{1}{3}$? 20? $6\frac{1}{4}$? $11\frac{1}{9}$? $3\frac{1}{3}$? $2\frac{1}{2}$? $37\frac{1}{2}$? $62\frac{1}{2}$?

*4.* If 4 yards of silk cost \$12, what will 8 yards cost?

SOLUTION.—8 yards will cost twice as much as 4 yards, or \$24.

*5.* If 10 pounds of sugar cost 50 cents, what will 100 pounds cost?

*6.* If 7 yards of chintz are worth 91 cents, what is the value of 21 yards?

*7.* If 8 pairs of shoes cost \$15, what are 32 pairs worth?

*8.* If 12 oranges cost 35 cents, what is the value of 36 oranges?

*9.* If 15 horses cost \$800, what are 75 horses worth?

*10.* If 9 men can build 17 rods of wall in a given time, how many rods can 54 men build in the same time?

*11.* If a man bought 11 pigs for \$9, how many could he have bought for \$45?

*12.* 4 men plowed 28 acres in 6 days; how many acres could they plow in 18 days?

*13.* How much will 5 books cost if 25 books cost \$15?

SUGGESTION.—5 books are $\frac{1}{5}$ of 25 books.

*14.* What will 7 tons of hay cost if 28 tons cost \$168?

*15.* If a wheelman rode 510 miles in 6 days, how far could he ride in 2 days?

*16.* If 84 bushels of wheat are worth \$77, what is the value of 12 bushels?

*17.* If 63 gallons of molasses are worth \$27, what will 7 gallons cost?

*18.* A man had \$30 and lost \$25, what part of the original amount has he left?

*19.* A man had \$35 and received \$70 additional; how does his present amount compare with the first amount?

*20.* A man has \$45. He spends \$9. What part of the original amount has he spent? What part remains?

*21.* In a bin of 60 bushels of wheat, 20 bushels are damaged. What part of the wheat is sound?

*22.* A pole 60 feet long is sunk 15 feet in the earth; what part of it is in the air?

*23.* Two men engage in a business venture, the gains or the losses to be shared in proportion to money furnished; A puts in $60 and B $90; how should they share the gain?

*24.* In a mixture of clover and timothy 30 pounds are clover and 10 pounds are timothy; what is the proportion of each in the mixture?

*25.* In a blend of coffee 39 pounds are Java and 13 pounds are Mocha; what is the proportion of each in the blend?

*26.* A farmer made an insecticide of 99 parts water and 1 part London purple; what is the ratio of the London purple to the water? Of the water to the mixture?

*27.* If Java and Mocha are blended in the proportion of 4 to 1, how many pounds of each in a mixture of 60 pounds?

*28.* In a blend of tea, the black is to the green as 5 to 3; how many pounds of each in a mixture of 24 pounds?

*29.* In a certain school, the boys are to the girls as 8 to 5; how many of each, the total number on roll being 52?

*30.* Doe's money is to Roe's as 7 to 5; how much has each if both have $96?

*31.* In a mucilage mixture 3 pounds of water are used and $\frac{1}{2}$ pound of gum-arabic; how much of each must be taken to make 70 pounds?

### WRITTEN PROBLEMS

**278.** *1.* A contributed $4500 to a charitable institution, B $7500, and C $3000. What part of the total did each contribute?

*2.* A grocer bought 560 pounds of Rio coffee and 440 pounds of Java. What part of the whole amount is each?

*3.* A merchant bought 3 rolls of carpet. The first contained 360 yards, the second 560 yards, and the third 840 yards. What part of the total in each roll?

*4.* Jones has a capital of $2250 and gains $750; what part of his present worth is his gain?

*5.* A merchant's business expenses outside of rent for a year amount to $3500 and his rent is $700; what part of his total expenses is his rent?

*6.* A shipper sends goods over one railroad 600 miles and over another 400 miles; if his freight charges are $30, how much should each road receive?

*7.* In a shipment of grain there are 6500 bushels of wheat and 3500 bushels of oats; what part of the shipment is wheat? What part is oats?

*8.* The receipts from certain agricultural products amount to $3000 from hay, $2500 from potatoes, and $2000 from straw; what part of the whole amount was received from each?

*9.* In a partnership A invested $4000, B $3000, and C $2000; what part of the gain should each receive if they share in proportion to investment?

*10.* A grocer mixed 40 pounds of Mocha coffee with 120 pounds of Java coffee; what part of the mixture is Java? What part is Mocha?

*11.* At a partnership closing, D received $2000 as his share of gain, E received $3000, and F $1000; what part of the gain did each receive as his share?

*12.* Two kinds of coffee are blended at the rate of 2 pounds of one to 3 pounds of the other; how many pounds of each in 180 pounds of the mixture?

*13.* The relation of $2500 is to a second sum as 5 to 9; what is the second sum?

*14.* A merchant's assets, $15200, are to his liabilities as 5 to 2; what are his liabilities?

*15.* In a shorthand school of 220, the ladies are to the gentlemen as 7 to 4; how many of each sex in the school?

*16.* A business man owes three creditors $2600 in the ratio of 6, 4 and 3; what is due each creditor?

*17.* A business man withdrew from bank $1250, which was $\frac{2}{3}$ of the amount left in bank. How much had he in bank at first?

*18.* I pay $156 for a horse, which is $\frac{4}{5}$ of the amount paid for a carriage; what did both cost me?

*19.* A cycler wheeled 135 miles, which was $\frac{3}{5}$ of the distance he traveled by steamer; what was the length of his trip?

*20.* If $\frac{3}{4}$ of a vessel cost $15000, what is the value of $\frac{1}{8}$ of it?

*21.* If $\frac{7}{16}$ of an estate is worth $101500, what would be a $\frac{5}{8}$ interest in the estate?

*22.* Bought $\frac{3}{5}$ of a mill for $15250; what is the value of $\frac{2}{7}$ of it at the same price?

*23.* If $1\frac{1}{4}$ of a factory is valued at $14014, what is $\frac{1}{3}$ of it worth?

*24.* If I spend $\frac{2}{3}$ of $15, the remainder is what part of my friend's money, if he has $25?

*25.* A has $45 and B has $\frac{2}{3}$ as much, plus $22; what part of A's money equals B's?

*26.* What is the relation of $12\frac{1}{2}$ to $37\frac{1}{2}$? Of $37\frac{1}{2}$ to $12\frac{1}{2}$? What is the relation of 25 to $87\frac{1}{2}$?

*27.* A man having $\frac{4}{5}$ of a ton of coal, sold $\frac{3}{4}$ of it; what part of a ton remained?

*28.* A merchant owned $\frac{5}{8}$ of the stock of a store and sold $\frac{3}{5}$ of his interest; what interest in the entire stock did he retain?

*29.* A and B each had $1\frac{1}{2}$ of a ton of hay; B sold A $\frac{2}{3}$ of his hay; what part of A's equals B's after the sale?

# DECIMAL FRACTIONS

**279.** A **decimal fraction,** or a decimal, is a fraction whose denominator is 10 or a power of 10; as, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$.

**280.** In **decimal fractions,** a unit is divided into 10 equal parts, named *tenths;* the tenths are divided into 10 equal parts, named *hundredths.* In descending, each order is $\frac{1}{10}$ part of the preceding order.

**281.** In a **decimal** the denominator is indicated by a point (.) called the **decimal point.** Sufficient ciphers must be prefixed when necessary to make the places equal the ciphers in the denominator. Thus, $\frac{3}{10}$ is written decimally .3; $\frac{3}{100}$, .03; $\frac{3}{1000}$, .003.

**282.** The **decimal point** is also called the **separatrix** when it separates whole numbers and decimals. In United States money it separates dollars and cents.

**283.** A **pure decimal** consists of decimal figures only; as, .7, .45, .375.

**284.** A **mixed decimal** consists of a whole number and a decimal; as, 6.9, 24.75, 3.1416.

**285.** A **complex decimal** is a decimal which has at its right a common fraction; as, $.16\frac{2}{3}$, $.18\frac{3}{4}$, $.083\frac{1}{3}$.

**286. Decimal notation** applies to integers and decimals, and the processes are the same whether the scale is ascending or descending.

As we ascend from units, each place in the integral scale is increased tenfold, and in the decimal scale decreased in the same ratio, the first place above units being **tens** and the first place below units being **tenths,** etc.

## DECIMAL NUMERATION

**287.** The similarity and relation of the periods and orders in a **decimal** to those in a **whole number** are shown by the following

**NUMERATION TABLE**

| Billions | Hundreds of millions | Tens of millions | Millions | Hundreds of thousands | Tens of thousands | Thousands | Hundreds | Tens | Units | Decimal Point | Tenths | Hundredths | Thousandths | Ten-thousandths | Hundred-thousandths | Millionths | Ten-millionths | Hundred-millionths | Billionths |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | . | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Integral Periods and Orders | | | | | | | | | | | Decimal Periods and Orders | | | | | | | | |

## WHOLE NUMBERS AND DECIMALS COMPARED

**288. General Principles:**

*1.* Both **whole numbers** and **decimals increase** from right to left and **decrease** from left to right in a tenfold ratio.

*2a.* The farther an **integral figure** is from the units order the **greater** its value.

*2b.* The farther a **decimal figure** is from the decimal point the **less** its value.

*3a.* Placing a cipher after a whole number multiplies it by 10; two ciphers, by 100, etc.

NOTE.—Ciphers are not properly placed before whole numbers.

*3b.* Placing ciphers after a decimal does not change its value, while placing one cipher between the figures and the decimal point divides it by 10; two, by 100, etc.

## NUMERATION OF DECIMALS

**289. To read a pure decimal.**

EXAMPLE.—.1375 is read, "One thousand three hundred seventy-five ten-thousandths," or, "Thirteen hundred seventy-five ten-thousandths."

**290.** RULE.—*First: Begin at the point and numerate the decimal figures to determine the name of the right-hand order. The first place, being one-tenth of units, is called* TENTHS; *the second,* HUNDREDTHS; *the third,* THOUSANDTHS; *the fourth,* TEN-THOUSANDTHS, *etc. Observe carefully the written expression in decimals and integers.*

*Second: Read the decimal as if it were a whole number, adding the name of the right-hand order.*

**291. How to read a mixed decimal.**

EXAMPLES.—(1) 24.75 is read, "Twenty-four *and* seventy-five hundredths"; (2) 354.10865 is read, "Three hundred fifty-four *and* ten thousand eight hundred sixty-five hundred-thousandths."

**292.** RULE.—*After numerating the decimal figures, read the whole number, following it with "and" at the separatrix; then read the decimal after the manner of a pure decimal.*

NOTE.—Do not read "and" except at the decimal point. Thus, in the last example do not say, "Three hundred and fifty-four and ten thousand eight hundred and sixty-five," etc.

**293. How to read a complex decimal.**

EXAMPLES.—(1) Read .83⅓; (2) 83.166⅔

NOTE.—The common fraction at the end of a complex decimal is not to be regarded as occupying a decimal place, but as being a part of 1 in the last place before it. Hence, the above examples should be read, (*1*) "Eighty-three one-third hundredths"; (*2*) "Eighty-three and one hundred sixty-six two-thirds thousandths."

### WRITTEN EXERCISE

**294.** Write or read

| | | | |
|---|---|---|---|
| *1.* .5 | *6.* .025 | *11.* .1003 | *16.* 349.52433 |
| *2.* .05 | *7.* .77 | *12.* .00205 | *17.* 2000.0025 |
| *3.* .005 | *8.* 2.5 | *13.* .000109 | *18.* .2025 |
| *4.* .1 | *9.* 7.055 | *14.* 32.10005 | *19.* 100.1875 |
| *5.* .15 | *10.* .055 | *15.* 15.7006 | *20.* 100.18 |

## NOTATION OF DECIMALS

**295.** A **decimal** should contain as many places as there are ciphers in the denominator of the equivalent common fraction. In the common fraction, it will be observed that the denominator is usually written and may be any number; in the decimal, the denominator is simply indicated and must be 10 or some power of 10.

**296. To write a decimal.**

EXAMPLES.—(1) Write forty-five hundredths; (2) fifty-six thousandths; (3) fourteen hundred twenty-five ten-thousandths; (4) three hundred seventy-five hundred-thousandths; (5) nineteen thousand sixty-four millionths.

(1) .45; (2) .056; (3) .1425; (4) .00375; (5) .019064

NOTE.—It is important to distinguish between the *number* part and the *name* part of a decimal expressed in words; as 45 hundredths.

**297.** RULE.—*First write the figures of the decimal as if they comprised a whole number, then place the point to the left of as many figures (counting from the right hand) as there would be ciphers in the denominator if written as a common fraction.*

*Verify the notation by numerating the decimal from left to right, beginning at tenths.*

### WRITTEN EXERCISE

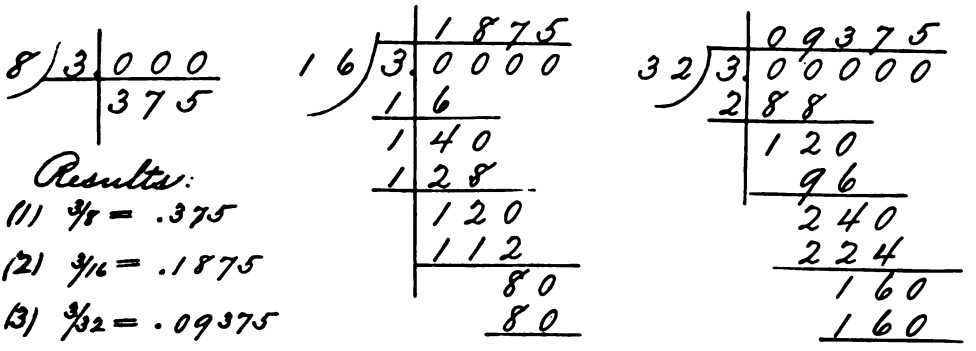
**298.** Write

*1.* 9 tenths
*2.* 27 hundredths
*3.* 83 hundredths
*4.* 125 thousandths
*5.* 65 thousandths
*6.* Ninety-five tenths
*7.* 72 ten-thousandths
*8.* 15 hundred-thousandths
*9.* 95 hundredths
*10.* 4175 thousandths
*11.* One hundred one thousandths
*12.* One hundred and one thousandth
*13.* Fourteen hundredths
*14.* Twenty-five thousandths
*15.* Sixty-six ten-thousandths
*16.* Twelve hundred-thousandths
*17.* Eighty-eight millionths
*18.* Nine and five-tenths
*19.* Ninety-nine ten-millionths
*20.* One hundred-millionths
*21.* Eleven hundred eleven hundredths
*22.* Fifteen hundred and fifteen millionths
*23.* Fifteen hundred fifteen millionths
*24.* Twenty-five thousand and twenty-five thousandths
*25.* Four hundred five and five hundred four hundred-thousandths
*26.* Ninety-nine million nine hundred ninety thousand ninety-nine and nine hundred ninety million ninety-nine thousand nine hundred ninety-nine billionths

## REDUCTION OF DECIMALS

**299. To reduce a common fraction to a decimal.**

EXAMPLES.—(1) Reduce $\frac{3}{8}$ to a decimal; (2) $\frac{3}{16}$; (3) $\frac{3}{32}$




**300.** RULE.—*Place the numerator to the right of the denominator with a curved line between them, as in short division. Draw a vertical line just to the right of the numerator and cross it with a horizontal line either above or below the numerator, as shown in the foregoing examples.*

*Divide by the denominator, annexing ciphers to the numerator as required, and place a quotient figure either above or below each cipher annexed. Continue the division until it results in a pure decimal or in the number of decimal places required.*

**301.** The reduction of common fractions to decimals can often be easily and readily made by inspection and a study of common decimal equivalents. This can be done where it is possible to resolve composite denominators into two or more easy factors; as,

| | |
|---|---|
| $\frac{1}{8}$ = .125 | $\frac{5}{8}$ = .625 |
| $\frac{1}{8}$ ($\frac{1}{4}$) = .25 | $\frac{5}{8}$ ($\frac{3}{4}$) = .75 |
| $\frac{3}{8}$ = .375 | $\frac{7}{8}$ = .875 |
| $\frac{1}{8}$ ($\frac{1}{2}$) = .5 | |

EXAMPLE.—Reduce to a decimal (1) $\frac{7}{16}$; (2) $1\frac{2}{5}$

(1) $\frac{7}{16}$ = $\frac{1}{2}$ of $\frac{7}{8}$ or $\frac{1}{2}$ of .875 = .4375
(2) $1\frac{2}{5}$ = $\frac{1}{5}$ of $\frac{12}{5}$ or $\frac{1}{5}$ of 2.6 = .8666+

**302. Change to decimals by inspection**

| | | | |
|---|---|---|---|
| 1. $\frac{3}{16}$ | 4. $\frac{15}{16}$ | 7. $\frac{7}{16}$ | 10. $1\frac{31}{32}$ |
| 2. $\frac{5}{16}$ | 5. $\frac{11}{16}$ | 8. $\frac{15}{16}$ | 11. $\frac{1}{16}$ |
| 3. $\frac{7}{16}$ | 6. $\frac{13}{16}$ | 9. $\frac{1}{16}$ | 12. $1\frac{17}{32}$ |

### WRITTEN EXERCISE

**303.** Reduce to decimal form

| | | | |
|---|---|---|---|
| *1.* $\frac{1}{5}$ | *12.* $\frac{7}{25}$ | *21.* $\frac{111}{125}$ | *32.* $\frac{18}{83\frac{1}{3}}$ |
| *2.* $\frac{3}{8}$ | *13.* $18\frac{3}{4}$ | *22.* $\frac{249}{250}$ | |
| *3.* $\frac{5}{8}$ | *14.* $8\frac{3}{40}$ | *23.* $\frac{37}{750}$ | *33.* $15\frac{1}{16}$ |
| *4.* $\frac{4}{5}$ | *15.* $\frac{13}{40}$ | *24.* $\frac{5138}{1250}$ | *34.* $\frac{37\frac{1}{2}}{87\frac{1}{2}}$ |
| *5.* $\frac{3}{5}$ | *16.* $37\frac{1}{2}$ | *25.* $60\frac{9}{15}$ | |
| *6.* $\frac{9}{20}$ | *17.* $\frac{16}{33\frac{1}{3}}$ | *26.* $18\frac{13}{20}$ | *35.* $59\frac{3}{64}$ |
| *7.* $\frac{13}{20}$ | | *27.* $11\frac{1}{9}$ | *36.* $17\frac{1}{160}$ |
| *8.* $\frac{19}{20}$ | *18.* $\frac{15}{66\frac{2}{3}}$ | *28.* $9\frac{1}{11}$ | *37.* $25\frac{1}{50}$ |
| *9.* $\frac{13}{25}$ | | *29.* $13\frac{1}{3}$ | *38.* $34\frac{3}{400}$ |
| *10.* $\frac{8}{40}$ | *19.* $\frac{17}{125}$ | *30.* $6\frac{2}{3}$ | *39.* $32\frac{3}{25}$ |
| *11.* $\frac{7}{8}$ | *20.* $\frac{23}{125}$ | *31.* $48\frac{1}{8\frac{1}{2}}$ | *40.* $940\frac{1}{128}$ |

NOTE.—Carrying results to four decimal places is sufficiently accurate for most commercial operations. The operation may often be simplified by reducing the common fraction to its lowest terms before dividing.

**304. To reduce a decimal to a common fraction.**

EXAMPLES.—Reduce to common fractions (1) .25; (2) $.16\frac{2}{3}$; (3) .0625

(1) $.25 = \frac{25}{100} = \frac{1}{4}$

(2) $.16\frac{2}{3} = \frac{16\frac{2}{3}}{100} = \frac{50}{3} \times \frac{1}{\cancel{100}\,{}^{2}} = \frac{1}{6}$

(3) $.0625 = \frac{625}{10000} = \frac{5}{80} = \frac{1}{16}$

**305.** *Write the decimal in the form of a common fraction, omitting the decimal point and any ciphers standing next to it, and using the figures for a numerator; for a denominator, write 1 with as many ciphers annexed to it as there are places in the decimal. Reduce to lowest terms.*

NOTE.—In the case of a complex fraction, simplify according to method in common fractions.

### WRITTEN EXERCISE

**306.** Change to common fractions

| | | | | |
|---|---|---|---|---|
| *1.* .25 | *9.* $.16\frac{2}{3}$ | *17.* 3.75 | *25.* $.98\frac{3}{4}$ | *33.* $2.00\frac{5}{8}$ |
| *2.* .5 | *10.* $.33\frac{1}{3}$ | *18.* 12.5 | *26.* $7.30\frac{1}{3}$ | *34.* $.007\frac{9}{13}$ |
| *3.* .15 | *11.* $.66\frac{2}{3}$ | *19.* $.083\frac{1}{3}$ | *27.* 2.0875 | *35.* $.6266\frac{2}{3}$ |
| *4.* .75 | *12.* $.87\frac{1}{2}$ | *20.* 3.1416 | *28.* $1.62\frac{1}{2}$ | *36.* $3.05\frac{5}{19}$ |
| *5.* .125 | *13.* $.06\frac{1}{4}$ | *21.* $.57\frac{1}{7}$ | *29.* $3.87\frac{1}{2}$ | *37.* 2.0375 |
| *6.* .625 | *14.* $.18\frac{3}{4}$ | *22.* $.066\frac{2}{3}$ | *30.* .5236 | *38.* 6.3178 |
| *7.* .875 | *15.* $.11\frac{1}{9}$ | *23.* $.44\frac{4}{9}$ | *31.* $2.007\frac{3}{8}$ | *39.* 21.675 |
| *8.* .7854 | *16.* $.14\frac{2}{7}$ | *24.* $.124\frac{3}{8}$ | *32.* $3.006\frac{1}{4}$ | *40.* $44.466\frac{2}{3}$ |

## ADDITION OF DECIMALS

**307. To add decimals.**

EXAMPLE.—Add .5, .0075, 80.08, 24.75, 3.06, 800.125

**308.** RULE.—*Write the numbers so that the decimal points are in the same column. Add as in whole numbers, keeping the decimal point in its proper place and carrying as in whole numbers.*

NOTE.—Whenever possible, the teacher should dictate the numbers to be added.

```
   .5
   .0075
 80.08
 24.75
  3.06
800.125
908.52225
```

### WRITTEN EXERCISE

**309.** *1.* Add .79, .473, 61.00057, 457.002073.

*2.* Add 81.004, 73.0102, .08007, 8000.0008, .8008, 368.5.

*3.* Find the sum of 77 ten-thousandths, 751 hundred-thousandths, 319 millionths, 7 tenths, 7 ten-millionths.

*4.* Find the sum of 315 and 75 hundredths, 85 and 85 thousandths, 1523 and 375 ten-thousandths, 415 and 1875 hundred-thousandths, 95 and 9 tenths, 2324 and 17625 millionths.

*5.* Write the decimals from problems 1 to 10 inclusive in the exercise in notation of decimals on page 74, in proper form for addition, and find the sum.

*6.* Write from 11 to 26 inclusive in the same exercise as a problem in addition, and find the sum.

*7.* Add the following items of United States money, placing the dollar sign before the first one only: $175854.75, 954.32, 1324.56, 97.88, 11927.50, 45.12, 245.29, 64381.95, 19.61, 227433.19.

*8.* Write the following in a line and add them horizontally: 4712.25, 820.75, 5162.50, 112.67, 6125.18, 231.45, 15.95, 550.08, 9.84, 1243.36. Prove result.

*9.* Add 31.135, 15.016, 23.25½, 36.47¾, 125.3¾.

*10.* How many tons of coal in 4 car-loads weighing respectively 32.805, 31.675, 29¼, 30.25½ tons?

*11.* A merchant sold 3½ yards of cloth for $5.67½, 2¼ yards for $1.50 and 12.3 yards of another piece for $6.03½. How many yards did he sell in all, and for how much?

*12.* Find the sum of 175 francs 18 centimes, 38 fr. 96 c., 115 fr. 45 c., 24 fr. 65 c., 8 fr. 50 c., 132 fr. 15 c., 56 fr. 80 c., 6 fr. 75 c., 251 fr. 22 c., 48 fr. 25 c.

*13.* Find the sum of 2150 Marks 55 Pfennigs, 459 M. 75 Pf., 91 M. 52 Pf., 1654 M. 25 Pf., 75 M. 50 Pf., 353 M. 85 Pf., 9 M. 95 Pf., 766 M. 20 Pf., 87 M. 35 Pf., 2344 M. 65 Pf.

NOTE.—100 centimes in a franc; 100 pfennigs in a mark.

## SUBTRACTION OF DECIMALS

**310. To subtract decimals.**

EXAMPLES.—(1) From .875 take .75; (2) 18.75 take 1.375; (3) 10 take .0875

| (1) | (2) | (3) |
|---|---|---|
| .875 | 18.75 | 10. |
| .75 | 1.375 | .0875 |
| .125 | 17.375 | 9.9125 |

**311.** RULE.—*Write the numbers so that the decimal points are in the same column and proceed as in simple numbers.*

NOTE.—If there are fewer places in the minuend than in the subtrahend, consider the vacant places in the minuend as being ciphers.

### WRITTEN EXERCISE

**312.** *1.* From twelve and fifty-six thousandths, take three hundred forty-seven ten-thousandths.

*2.* Find the difference between 13.8657 and 9.8769.

*3.* Find the difference between 71.8654 and .587.

*4.* Find the difference between 10.0101 and 9.00999.

*5.* Find the difference between $371.64 and $78.725.

*6.* Find the difference between $2875.625 and $389.15.

*7.* From 46.201 take 17.9624.

*8.* From one take ninety-nine hundredths.

*9.* From .016 take .0016.

*10.* Subtract 101.027 from 206.431.

*11.* Subtract 39.416712 from 100.01.

*12.* What is the difference between 209.41 and 8.73546?

*13.* 102000.102 − 102.00102 = ?

*14.* 19.04039 − 6.52781 = how many?

*15.* From twenty-five hundred and ninety-nine thousandths, take five hundred ninety-nine thousandths.

*16.* Find the difference between 111.38056 and 22.0605942.

*17.* Find the difference between 160 and 101.00101.

*18.* Find the difference between 36.3636 and .304403.

*19.* Find the difference between 100.001 and 10.100101.

*20.* From $19.413\frac{3}{16}$ take $18.45\frac{3}{4}$ and add $17.06\frac{1}{4}$ to result.

*21.* Subtract $4.05\frac{3}{80}$ from $16.812\frac{7}{32}$.

*22.* $89.723\frac{1}{2}\frac{9}{2} - 17.09\frac{3}{4}\frac{3}{8}0 + 250.007\frac{1}{4}$.

*23.* From $72.62\frac{5}{8}$ take $13.614\frac{7}{8}0$ and add $37\frac{1}{8}0$ to the result.

*24.* $3080.015 - 2582.36\frac{5}{16} + 1201\frac{1}{16}$.

*25.* Subtract $92.08\frac{1}{16}$ from $150.3\frac{5}{64}$.

*26.* From $52.008\frac{5}{16}$ take $2.08\frac{3}{32}$.

*27.* From $10.01\frac{5}{16}$ take $(.1001\frac{3}{4} + 8.01\frac{1}{8})$.

## MULTIPLICATION OF DECIMALS

**313. To multiply decimals.**

EXAMPLES.—(1) Multiply .39 by .047; (2) $475.625 by 3.18; (3) $4.8665 by 250

(1)

.39 - 3
.047 - 2/6
273
156
.01833 - 6

(2)

475.625 - 7
3.18 - 10/10 - 1
3805000
475625
1426875
1512.48750 - 4

(3)

414.8665 | 5
1216 | 625
Result 1216.625

**314.** RULE.—*Multiply as in whole numbers and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.*

NOTE.—The small figures to the right in Examples 1 and 2 above are proof-figures, the first being proved by casting out the 9's, and the second by casting out the 11's.

### WRITTEN EXERCISE

**315.** *1.* Multiply 278.05 by 231.

*2.* Multiply 2150.42 by 868.

*3.* Multiply 28.575 by 3.1416.

*4.* Multiply 33.5 by 33.5 by .7854.

*5.* Find the product of 3.5×3.5×.5236.

*6.* Multiply 347.176 meters by 39.37.

*7.* Multiply 875.65 francs by .193.

*8.* Multiply £68.125 by 4.8665.

*9.* Multiply 475.5 liters by 1.0567.

*10.* Multiply 250.05 hectoliters by 2.8375.

*11.* Multiply 478.5 kilometers by .62137.

*12.* Multiply 6.5 meters by 5.4 by 2.5 by 1.375.

*13.* Multiply 175.6 kilos by .18.

*14.* The diameter of a circle is 14.7 feet; to find its circumference multiply the diameter by 3.1416. What is the circumference?

*15.* Multiply 5.5 by 5.5 and that product by .7854.

*16.* Multiply $1575.25 by .06 and that product by 287.17.

*17.* Multiply 5050.65 by .01125.

*18.* 10.89×108.9×1.089 = ?

*19.* 33×33×33×.5236 = ?

*20.* 25.5×25.5×.7854 = ?

*21.* What is the product of 375¾ and 101.29?

*22.* What is the value of 312.2 meters of silk at 5.18 francs a meter?

*23.* Multiply 15.25 liters by 7.55 and that product by .193.

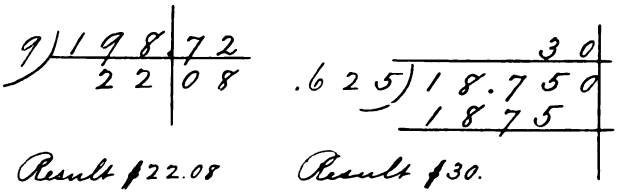
*24.* A ship having a cargo valued at $22500 being disabled at sea, .22½ of its cargo was thrown overboard; what was the loss to a shipper who owned .18 of the cargo?

## DIVISION OF DECIMALS

**316. To divide by a whole number, a pure decimal or a mixed decimal.**

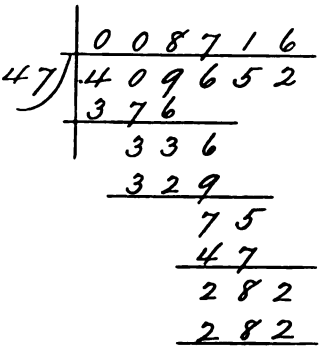
**317.** The following method will be found easy and safe for beginners, as it avoids errors in pointing off the decimal places.

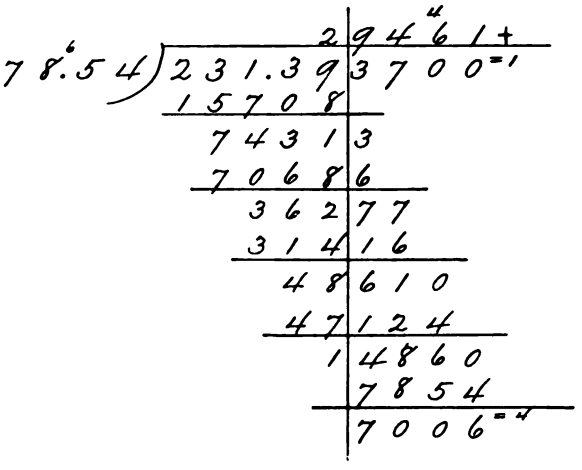
EXAMPLES.—(1) $198.72 ÷ 9 (2) $18.75 ÷ .625
(3) .409652 ÷ 47 (4) 231.3937 ÷ 78.54










Result 2.9461+

Proof

Divisor Excess 6

Quotient Excess 4

$6 \times 4 = 24 = 6$

Dividend Excess 1

Remainder, 4

$(1+9)-4=6$

**318.** RULE.—*Write the divisor to the left of the dividend with a curved line between them, as in the division of whole numbers.*

*Draw a horizontal line below the dividend if the division is to be "short," or above it if the question requires long division.*

*When the divisor contains more decimal places than the dividend, equalize them by annexing ciphers to the dividend.* (*See Ex. 2.*)

*Draw a vertical line through the dividend* (*at right angles to the horizontal line*) *as many places to the right of the decimal point as there are decimal places in the divisor—through the decimal point when the divisor is a whole number.*

*Proceed with the division, being careful to write the first figure of the quotient directly beneath* (*in short division*) *or directly above* (*in long division*) *the right-hand figure of the first partial dividend. All that part of the quotient to the left of the decimal line will be a whole number, and that to the right of it a decimal.*

## WRITTEN EXERCISE 1

**319.** Obtain exact results for

1. Divide .625 by 2.5
2. Divide 15.25 by .05
3. Divide .0156 by .003
4. Divide 23.1 by .07
5. Divide 2.31 by .7
6. Divide 345.15 by .75
7. Divide 7.5 by .015
8. Divide 37.06 by .017
9. Divide 142.74 by .61
10. Divide 110 by .44
11. Divide 24.75 by 2.25
12. Divide 3.65 by 7.3
13. Divide 2425.5 by .32
14. Divide 56.43 by .064
15. Divide 79.64 by 5.5
16. Divide 30.25 by 16
17. Divide 2150.4 by 67.2
18. Divide 5280 by 16.5
19. Divide 43560 by .375
20. Divide .7854 by .0034
21. Divide 9.5 by 19
22. Divide 36.5 by 73
23. Divide 1750 by .875
24. Divide 461.975 by 5.435
25. Divide 3.6 by .18
26. Divide 343.68 by 53.7
27. Divide 44.44 by 1.1
28. Divide 128.625 by .08
29. Divide 169.39 by 13
30. Divide 162.54 by 27
31. Divide 5236 by .008
32. Divide 7276.5 by 31.5
33. Divide .07958 by .16
34. Divide 93.75 by .0625

## WRITTEN EXERCISE 2

NOTE.—Extend to at least four decimal places.

**320.** 1. Divide .175 by 2
2. Divide 35.104 by 5
3. Divide .3562 by 5
4. Divide 360 by .06
5. Divide 14.2672 by .08
6. Divide 360 by .04$\frac{1}{2}$
7. Divide 568.6 by 30.25
8. Divide 765.28 by 2.3$\frac{3}{4}$
9. Divide 360 by .05
10. Divide 4.8665 by .01$\frac{1}{4}$
11. Divide 2150.4 by 8
12. Divide 48.4 by .16
13. Divide 138.48 by 30$\frac{1}{4}$
14. Divide 7500 by 24$\frac{3}{4}$
15. Divide 2747.7 by 57.75
16. Divide 39.37 by 36
17. Divide 5.18 by .193
18. Divide 4.8665 by 240
19. Divide .7854 by 231
20. Divide 329.84 by 5.19
21. Divide 82432 by .37
22. Divide 12.4328 by .61
23. Divide 24.255 by .11
24. Divide 384 by 1.2
25. Divide 3.45 by .005
26. Divide 30.4 by 19
27. Divide 16.9 by .13
28. Divide 37.5 by .00$\frac{1}{4}$
29. Divide 7276.5 by 31.5
30. Divide 864.65 by 272$\frac{1}{4}$
31. Divide 375 by .19$\frac{1}{2}$
32. Divide 56$\frac{3}{4}$ by .13$\frac{1}{2}$
33. Divide 272$\frac{1}{2}$ by .375
34. Divide 34.33$\frac{1}{2}$ by .03$\frac{1}{4}$
35. Divide 17.16$\frac{3}{4}$ by 83$\frac{1}{2}$
36. Divide 3.39924 by 8.716
37. Divide .00540625 by .00865
38. Divide $939.12 by 13416
39. Divide $13878.72 by $48.19
40. Divide $9615.36 by $50.08
41. Divide .409652 by .047
42. Divide 31.833$\frac{1}{3}$ by 18$\frac{3}{4}$
43. Divide 96.8 by .238
44. Divide 175.25 by .000$\frac{3}{4}$

## SHORT METHODS IN DECIMALS

### Multiplication

**321. To multiply by simply moving the decimal point.**

1. Multiply 24.75 by 10
2. Multiply 31.5 by 100
3. Multiply 3.1416 by 1000
4. Multiply .5236 by 100
5. Multiply 2150.4 by 10000
6. Multiply 172.8 by 10

**322.** Rule.—*Moving the decimal point one place to the right multiplies by 10; two places, by 100; three places, by 1000, etc.*

**323. To multiply by a decimal, the equivalent of which is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of 1.**

1. Multiply 43560 by .5
2. Multiply 5280 by .25
3. Multiply 2150.4 by .125
4. Multiply 2747.7 by .33$\frac{1}{3}$
5. Multiply 24.75 by .16$\frac{2}{3}$
6. Multiply 31.5 by .08$\frac{1}{3}$
7. Multiply 4.8665 by .12$\frac{1}{2}$
8. Multiply 231 by .14$\frac{2}{7}$
9. Multiply 160 by .2$\frac{1}{2}$
10. Multiply 78.54 by .333$\frac{1}{3}$

**324.** Rule.—*Divide by the denominator of the equivalent common fraction.*

**325. To multiply by a decimal, the equivalent fraction of which is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., greater or less than 1.**

1. Multiply 2150.4 by .75
2. Multiply 2747.7 by .66$\frac{2}{3}$
3. Multiply 31.5 by .83$\frac{1}{3}$
4. Multiply 5280 by .87$\frac{1}{2}$
5. Multiply 4.86 by .99
6. Multiply 43560 by .91$\frac{2}{3}$
7. Multiply 2150.4 by 1.25
8. Multiply 2747.7 by 1.33$\frac{1}{3}$
9. Multiply 31.5 by 1.16$\frac{2}{3}$
10. Multiply 5280 by 1.125
11. Multiply 4.8665 by 1.10
12. Multiply 43560 by 1.08$\frac{1}{3}$

**326.** Rule.—*From the multiplicand subtract such part of itself as the multiplier is less than 1, or add such part as it is greater than 1.*

**MISCELLANEOUS EXERCISE**

**327.** Multiply

1. 30.25×2.5
2. 3.1416×25
3. 31.5×12.5
4. 24.75×33$\frac{1}{3}$
5. 4.8665×500
6. 30.25×7$\frac{1}{2}$
7. 3.1416×750
8. 24.75×87$\frac{1}{2}$
9. 2150.4×6$\frac{2}{3}$
10. 31.5×66$\frac{2}{3}$
11. 30.25×125
12. 3.1416×1250
13. .238×112.5
14. 2150.41×133$\frac{1}{3}$
15. .193×1500

### Division

**328. To divide by moving the decimal point.**

1. Divide 24.75 by 10
2. Divide 2150.4 by 100
3. Divide 31.5 by 1000
4. Divide 1728 by 100
5. Divide 37.85 by 100
6. Divide 4.8665 by 1000
7. Divide 31.416 by 10
8. Divide 144 by 100

**329.** Rule.—*Moving the decimal point one place to the left divides by 10; two places, by 100; three places, by 1000, etc.*

**330. To divide by a decimal of which the equivalent fraction is ½, ⅓, ¼, etc.**

1. Divide 3.1416 by .5
2. Divide .7854 by .25
3. Divide 31.5 by .33⅓
4. Divide 24.75 by .16⅔
5. Divide 16.5 by .12½
6. Divide 144 by .08⅓
7. Divide 1728 by .14⅜
8. Divide 2150.4 by .125
9. Divide 4.8665 by .2½
10. Divide 30.25 by .333⅓

**331.** RULE.—*Multiply by the denominator of the equivalent fraction.*

**332. To divide by a decimal whose equivalent value is ½, ⅓, ¼, etc., greater or less than 1.**

EXAMPLE.—$36 \div .75 = 36 \div \frac{3}{4} = 36 \times \frac{4}{3} = 48$

SUGGESTION.—$36 \times \frac{4}{3}$ is equivalent to adding ⅓ of the dividend to itself.

1. Divide 24.75 by .75
2. Divide 2150.4 by .87½
3. Divide 2747.7 by .66⅔
4. Divide 144 by .83⅓
5. Divide 1728 by .85⅞
6. Divide 31.5 by .7½; by .6⅔
7. Divide 24.75 by 1.25
8. Divide 2150.4 by 1.33⅓
9. Divide 2747.7 by 1.50
10. Divide 144 by 1.12½
11. Divide 1728 by 1.25
12. Divide 31.5 by 1.16⅔

**333.** RULE.—*Divide by the numerator of the equivalent common fraction and add the result when the divisor is less than 1, or subtract when it is greater than 1.*

**334. To divide by a whole number ending in ciphers.**

1. Divide 435.6 by 300
2. Divide 528.85 by 700
3. Divide 75864.15 by 6000
4. Divide 4576.75 by 4000
5. Divide 2150.4 by 320
6. Divide 9178.39 by 1300
7. Divide 37477.88 by 11000
8. Divide 4.8665 by 1600

NOTE.—See Rule 123, page 37.

## APPLICATION OF DECIMALS TO INVENTORIES

**335.** It is usually sufficient in ordinary commercial operations to carry a decimal division to four places, but in estimating the cost of manufactured products to get a unit of value on millions, it is necessary to carry the result to eight or nine decimal places. This is true also in figuring bankruptcy and partnership settlements by the percentage method, and in constructing compound interest tables.

### WRITTEN PROBLEMS

**336.** *1.* If the cost of producing 7500000 barrels of a certain grade of flour was $31829437.95, what should be the company's inventory value of 2345560 barrels?

*2.* The cost of producing 5000000 tons of pig iron is $47953274.87, what should be the mill inventory value of 2897450 tons?

*3.* If the cost of manufacturing 7586000 rolls of building paper was $4492673.65, what should be the manufacturer's inventory of 3815967 rolls?

*4.* The cost of producing 9847600 rolls of wall paper was $1028922.45, what should be the manufacturer's inventory value of 31472569 rolls?

*5.* If the cost of making 44562975 pounds of a certain commodity was $2943647.50, what should be the manufacturer's inventory value of 29675875 pounds?

*6.* If the cost of refining 24756800 gallons of oil was $1285675.70, what should be the company's inventory value of 6938450 gallons?

*7.* The cost of making 25000000 tons of steel was $487755692, what was the company's inventory value of 9398670 tons?

## REVIEW EXERCISE

**337.** In simplifying expressions connected by the signs of the fundamental operations (+, −, ×, and ÷) observe the following order:

*1.* Multiplication.

*2.* Division.

*3.* Addition and Subtraction as they may occur.

Expressions contained in marks of aggregation (parentheses, brackets, braces) must be simplified according to the above order, and the result used for the whole expression in the marks; then simplify the original expression as directed above.

EXAMPLE.—$4\times\overline{8+14}\div 8+[16-(25\div 5)+7]=29$

| Operations | (Re-statement) $(4\times 22\div 8+18)$ |
|---|---|
| *1.* $8+14=22$ | a. $4\times 22=88$ |
| *2.* $(25\div 5)=5$ | b. $88\div 8=11$ |
| *3.* $(16-5+7)=18$ | c. $11+18=29$ |

**338.** *1.* $12.5\times 12.5\times .7854\div 144=$ ?

*2.* $26.728\times \$1.05=$ ?

*3.* $13.450\times \$18.25=$ ?

*4.* $43.75\times \$11+132.80\times \$7=$ ?

*5.* $184.80\times \$.625=$ ?

*6.* $12.456\times \$8.75=$ ?

*7.* $28165\times \$.08\div \$28.75=$ ?

*8.* £765.5×$4.8665= ?

*9.* £12345.6×$4.8665÷$.193= ?

*10.* $687.5\times 39.37\div 36=$ ?

*11.* $785.125\times 36\div 39.37=$ ?

*12.* $7912\times 25000\times 640=$ ?

*13.* $7912\times 7912\times 7912\times .5236=$ ?

*14.* $\frac{24\times 20\times 8\times 4}{12}\times \$.025=$ ?

*15.* $\frac{60\times 24\times 30}{144}=$ ?

*16.* $60\times 49\times 144\div(14\times 20)=$ ?

*17.* $17\times 22\times 62.5=$ ?

*18.* $(28\times 6600\div 9)\times \$.92=$ ?

*19.* $(\$900+05)\times 1.1675=$ ?

*20.* $(\$9.21\div .03)\times .65=$ ?

*21.* $\frac{75\times 5280\times 48}{43560}+\$720=$ ?

*22.* $60\times 3.1416\times 8\times 1728\div 231=$ ?

*23.* $(1.5\times 5280\times 75\div 272.25)\times \$1.10=$ ?

*24.* $187\times 187\times .7854\div 272.25=$ ?

*25.* $2240\times 7\div 8\times 4\times 168=$ ?

*26.* $14\times 3.1416\times 38\div 9=$ ?

*27.* $59\times 5280\div 60\div 5.5=$ ?

*28.* $\$76000\times .75\times 1.04=$ ?

*29.* $\frac{\$950\times 93\times .06}{360}=$ ?

*30.* $\frac{\$6500\times 61\times .05}{365}=$ ?

*31.* $\frac{\$1896\times(91-8)\times .06}{360}=$ ?

*32.* £124.5025×$4.87+$1.50= ?

*33.* Divide 7.5 by 62.5.

*34.* $\$1890\div .045\times 1.05=$ ?

*35.* $\$1940.50\div 4.875=$ ?

*36.* $640\times \$6.25\times .02=$ ?

*37.* $(\$7200\div .08)\times 1.1625=$ ?

*38.* £37.804×$4.8665= ?

*39.* $4775\times \$.82-(4775\times \$.82\times .04)=$ ?

*40.* $(2\times 40+2\times 60)\times 9\times 3.5\times \$.30=$ ?

## REVIEW PROBLEMS

**339.** *1.* $\$197.87\frac{1}{2}+\$67.37\frac{3}{4}+(\$27.47\frac{1}{4}\times 3\frac{1}{2})-.75\frac{1}{2}=\ ?$

*2.* $\$37\frac{1}{2}+\$1.87\frac{3}{4}+\$_{\frac{5}{8}}-\frac{\$26\frac{3}{8}}{\frac{3}{4}}=\ ?$

*3.* Find the cost of 10 loads of hay at $\$16\frac{1}{2}$ per ton, containing respectively $2\frac{1}{4}$ tons, $1\frac{1}{4}$ tons, 1.375 tons, $2\frac{1}{4}$ tons, 2.8 tons, $1\frac{1}{8}$ tons, $1\frac{3}{4}$ tons, 2.15 tons, $1\frac{7}{8}$ tons and $2\frac{1}{2}$ tons.

*4.* If 9.3 barrels of flour cost \$48.825, what will 15.25 barrels cost at the same rate?

*5.* A farmer sowed three fields in wheat, containing respectively $117\frac{3}{4}$ acres, $125\frac{7}{8}$ acres, and $325.56\frac{1}{4}$ acres. He harvested from the first an average of $27\frac{1}{2}$ bushels per acre, from the second 30.25 bushels per acre, and from the third $29\frac{1}{8}$ bushels per acre. Find the value of his wheat at $\$1.12\frac{1}{2}$ per bushel.

*6.* A speculator bought 325.45 acres of land at $\$15\frac{3}{4}$ per acre and sold it in plots of 1.15 acres each at \$112.50 a plot. Find his gain or loss.

*7.* Two trains are 587.25 miles apart, and are running toward each other. One train has a speed of $45\frac{1}{4}$ miles per hour and the other $53\frac{1}{8}$ miles per hour. After they have been running for 4.6 hours, how far are they apart?

*8.* A grain dealer bought $1275\frac{3}{4}$ bushels of wheat at $\$.87\frac{1}{2}$ per bushel. He sold $\frac{2}{5}$ of it at $\$.90\frac{1}{4}$ per bushel, .25 of the remainder at $\$.94\frac{3}{4}$ per bushel, and what still remained at cost. What did he gain?

*9.* If a bushel contains 2150.4 cubic inches, how many bushels in 37560 cubic inches? What is the value of this quantity at $62\frac{1}{2}$ cents per bushel?

*10.* During a year a man spends $.37\frac{1}{2}$ of his income for clothes, $.27\frac{1}{2}$ for board, .16 for incidentals, and deposits the rest in bank. If his bank deposit is \$570, what is his income?

*11.* A merchant deposited in a bank at one time \$575.25, and at another \$487.30. If he checked out \$387.50, how many yards of cloth can he buy, at \$1.25 per yard, with the money he still has in bank?

*12.* Change $\frac{3\frac{1}{2}}{4\frac{3}{4}}$ to a decimal fraction.

*13.* If .625 of a ton of coal cost \$3.25, what will $7\frac{1}{4}$ tons cost?

*14.* Divide \$9 between two boys so that one shall receive .25 more than the other.

*15.* If a load of hay weighing 1665 pounds costs \$8.75, at that rate how much should be paid for a long ton (2240 pounds)?

*16.* How many barrels of flour at $\$5.12\frac{1}{2}$ must be given for $127.37\frac{1}{2}$ bushels of corn at 30 cents per bushel and $127\frac{1}{2}$ bushels of wheat at $\$.87\frac{1}{2}$ per bushel?

*17.* If the freight on 75.8 barrels of apples is $\$9.47\frac{1}{2}$, what should be paid on $103\frac{1}{4}$ barrels?

*18.* If there are 63360 inches in a mile, and 39.37 inches in a meter, what will it cost to pave a street one mile long at $\$1.12\frac{1}{2}$ per meter?

*19.* The circumference of a body is 3.1416 times the diameter. What is the diameter of the earth if the circumference is 24899.024 miles?

*20.* The value of a franc is $.193. How many francs in $458.37½, and how many yards of silk could be purchased with this sum at 9.5 francs per yard?

*21.* If .37 of a boy's wages for a week is $3.14½, what should he receive for 7¾ weeks?

*22.* If a certain kind of iron ore contains .37½ foreign substance, how much pure iron in 968.9 tons of the ore?

*23.* A tailor bought 32 pieces of cloth containing 23¼ yards each at $3.60½ per yard and sold it so as to gain $372, after paying $14.88 for freight. How much did he receive per yard?

*24.* A man in disposing of his property gave ⅛ to his younger son, ½ of the remainder to his elder son, .125 to a church, and the remainder, $32151.35, to his wife. Find value of his property and amount each son received.

*25.* A clerk spent .37½ of his money and still had remaining $237.25. How much did he spend?

*26.* A house and lot together cost $7805.38. If the house cost 1.75 times as much as the lot, what was the cost of each?

*27.* The product of three numbers is 331.81071, and two of the numbers are 3.79 and 23.1. What is the third number?

*28.* A grocer invested $526.50 in Rio and Java coffee. If .625 of the amount invested in Rio equals the amount invested in Java, what sum was invested in each, and how many pounds of each could he buy at 22½ cents per pound?

*29.* At the end of the first year a merchant's inventory is $37325.62½, which is equal to .375 of his sales for the year. If during the year he gained $12725.37½, what was the amount of his purchases?

*30.* Two men commenced business with equal capital. At the end of the year one had gained $3725.40, the other had lost $2376.80, and together they had $7897.30. How much had each at first?

*31.* A merchant sold 450 yards of cloth from a piece containing 875 yards. What decimal part of the piece remained unsold?

*32.* A, B and C own a mill. A owns .265 of it, B .378 of it, and C the remainder. If A's share is $7234.50, what is the value of B's and C's share?

*33.* If .375 of a ton of coal cost $4.95, what will .0875 of a ton cost?

*34.* A invested .375 of his money in a factory, .24 of the remainder in a city property, and deposited the balance, $17432.50, in a bank. Find amount of his money, and amount invested in the factory.

*35.* The private secretary of the governor receives $2000, of which he spends $525 for rent, $675 for living expenses, $297.50 for clothes. What decimal part of his salary does he save?

*36.* If 25.5 barrels of flour cost $85, how many can you get for $40.25?

*37.* A man traveled 697½ miles by rail and on bicycle. If he traveled .55 as far on bicycle as by rail, how far did he travel by rail and how far on bicycle?

*38.* If the value of an English sovereign is $4.8665, how many sovereigns are worth $100? What is the value of 156 sovereigns?

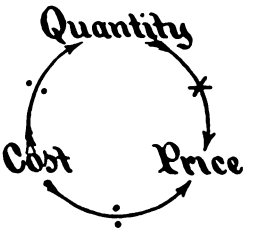
## QUANTITY, PRICE AND COST

**340.** The relation of these three elements is that of **multiplicand, multiplier** and **product.**

**Quantity** is the number of units.

**Price** is the cost of one unit.

**Cost** is the value of a quantity.




Formula:

1. Quantity × Price = Cost
2. Cost ÷ Price = Quantity
3. Cost ÷ Quantity = Price

**341.** An **aliquot** is a number or quantity (either integral or fractional) that is contained in another number or quantity an exact number of times. Thus, 6 is an aliquot of 18; 25 is an aliquot of 100; $\frac{1}{4}$ is an aliquot of $\frac{1}{2}$, and of $\frac{3}{4}$.

**342.** An **aliquot part** of a number or quantity is one of its even parts, which is given a fractional expression by placing it over such number or quantity and reducing it to its lowest terms.

**343.** A **simple aliquot** is one that can be reduced to a fractional expression having a numerator of 1; as $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.

**344.** A **compound aliquot** is comprised of two or more simple aliquots; as, $\frac{5}{8} = \frac{1}{2} +$ ($\frac{1}{2}$ of $\frac{1}{4}$).

**345.** An **aliquot unit** is the number or quantity of which the aliquot is an even part.

**346.** A **commercial unit** is a fixed unit of measure upon which the price of a commodity is based; as, a yard, a bushel, a gallon, a pound, a ton, etc.

**347.** SIMPLE ALIQUOT PARTS OF $1.00

| | | | |
|---|---|---|---|
| 50c. $= \frac{1}{2}$ | $16\frac{2}{3}$c. $= \frac{1}{6}$ | 10c. $= \frac{1}{10}$ | $6\frac{1}{4}$c. $= \frac{1}{16}$ |
| $33\frac{1}{3}$c. $= \frac{1}{3}$ | $14\frac{2}{7}$c. $= \frac{1}{7}$ | $9\frac{1}{11}$c. $= \frac{1}{11}$ | 5c. $= \frac{1}{20}$ |
| 25c. $= \frac{1}{4}$ | $12\frac{1}{2}$c. $= \frac{1}{8}$ | $8\frac{1}{3}$c. $= \frac{1}{12}$ | $3\frac{1}{3}$c. $= \frac{1}{30}$ |
| 20c. $= \frac{1}{5}$ | $11\frac{1}{9}$c. $= \frac{1}{9}$ | $6\frac{2}{3}$c. $= \frac{1}{15}$ | $2\frac{1}{2}$c. $= \frac{1}{40}$ |

**348.** If the price given should be $1.33\frac{1}{3}$, or $\frac{1}{3}$ more than $1.00; or 66$\frac{2}{3}$ cents, $\frac{1}{3}$ less than $1.00, the result can be readily obtained by adding or subtracting the fractional part, as the case may be. This can be applied to every simple aliquot with an easy unit as a base, whether 25c., 50c., $1.00, $100 or 100 per cent., the object being to simplify multiplication and division in business computations.

Require students to make up tables of the aliquot parts of 50c., 25c., 10c., similar to table given above.

## APPLICATION OF ALIQUOT PARTS

**349.** GENERAL RULE.—*Divide the quantity by the number of commercial units that $1 will buy.*

EXPLANATION.—At 12½c., $1 will buy 8 units; at 33⅓c., 3 units; at 50c., 2 units; at 66⅔c., ⅔ units, or 1½; at 75c., ¾ or 1½ units; at $1.25, ⅘ of a unit; at $1.33⅓, ¾ of a unit; at $1.50, ⅔ of a unit.

NOTE.—When the quantity contains a common fraction, it should be reduced to a decimal before dividing. If the fraction will not reduce to a simple decimal, extend it to 4 decimal places.

**350. To find the cost when the price is a simple aliquot of $1.**

EXERCISE 1

Find the cost of

1. 175 yds. @ 50c.
2. 84¾ yds. @ 33⅓c.
3. 127 gals. @ 25c.
4. 75½ gals. @ 20c.
5. 156 lbs. @ 16⅔c.
6. 99⅛ yds. @ 12½c.
7. 350 yds. @ 14⅔c.
8. 89⅘ lbs. @ 10c.
9. 810 qts. @ 11⅑c.
10. 495½ qts. @ $9\frac{1}{11}$c.
11. 480 gals. @ 8⅓c.
12. 230 yds. @ 6¼c.
13. 279⅓ yds. @ 33⅓c.
14. 236⅔ lbs. @ 25c.
15. 789¾ yds. @ 16⅔c.

**351. To find the cost when the price is a simple aliquot less than $1 or more than $1.**

EXERCISE 2

1. 248 gals. @ 87½c.
2. 356½ gals. @ 90c.
3. 189 yds. @ 75c.
4. 456 yds. @ 66⅔c.
5. 360⅛ yds. @ 80c.
6. 356 gals. @ 95c.
7. 750 yds. @ $1.12½
8. 632 yds. @ $1.50
9. 156¾ yds. @ $1.33⅓
10. 256 doz. @ $1.25
11. 169½ doz. @ $1.16⅔
12. 573¾ yds. @ $1.20

**352.** RULE.—*Add to the quantity or subtract from it such aliquot part of itself as the price is greater or less than $1.*

**353. To find the cost when the price is a simple aliquot more or less than $2, $3, $4, $5, etc.**

EXERCISE 3

1. 960 bbls. @ $2.12½
2. 480 bbls. @ $2.16⅔
3. 560 bbls. @ $3.14⅔
4. 728 bbls. @ $3.25
5. 360 bbls. @ $4.33⅓
6. 244 yds. @ $1.87½
7. 342 yds. @ $1.83⅓
8. 572 yds. @ $1.75
9. 456¾ yds. @ $2.33⅓
10. 196 yds. @ $2.75
11. 128 tons @ $5.12½
12. 96¾ tons @ $4.66⅔
13. 240 tons @ $3.87½
14. 85½ tons @ $6.25
15. 375 tons @ $2.66⅔

**354.** RULE.—*Multiply the quantity by the nearest dollar price and add to the result, or subtract from it, such aliquot part of the quantity as the real price is above or below the assumed price.*

**355. To find the cost using 50c., 25c., or 10c. as a base.**

EXERCISE 4

Find the cost of

| | | |
|---|---|---|
| *1.* 164 yds. @ 55c. | *5.* 76½ yds. @ 22½c. | *9.* 48¾ yds. @ 13½c. |
| *2.* 132 gals. @ 45c. | *6.* 120 yds. @ 18¾c. | *10.* 152 lbs. @ 7½c. |
| *3.* 284 gals. @ 37½c. | *7.* 92½ yds. @ 35c. | *11.* 69¾ lbs. @ 6¾c. |
| *4.* 716 gals. @ 56¼c. | *8.* 132 yds. @ 15c. | *12.* 175 lbs. @ 3½c. |

NOTE.—These exercises are to test the student's power of invention.

**356. To find the cost when the price is an aliquot part of $10, $100, $1000, or an aliquot part more or less than $10, etc.**

EXERCISE 5

| | | |
|---|---|---|
| *1.* 318¾ yds. @ $2.50 | *5.* 53⅓ doz. @ $15 | *9.* 160 acres @ $125 |
| *2.* 274½ yds. @ $3.33⅓ | *6.* 28 bbls. @ $7.50 | *10.* 71½ acres @ $250 |
| *3.* 157½ yds. @ $5 | *7.* 67½ tons @ $25 | *11.* 27⅜ acres @ $500 |
| *4.* 894 tons @ $33⅓ | *8.* 39¾ tons @ $33⅓ | *12.* 145¼ acres @ $150 |

**357. To find the cost when the quantity is 10, 100, 1000, etc., or when it may be factored upon some power of 10 taken as an aliquot unit.**

EXERCISE 6

| | | |
|---|---|---|
| *1.* 10 yds. @ $1.35½ | *5.* 250 gals. @ $1.15½ | *9.* 1500 gals. @ 89½c. |
| *2.* 100 yds. @ 73½c. | *6.* 125 gals. @ $1.19¼ | *10.* 2000 lbs. @ 49½c. |
| *3.* 1000 yds. @ 9⅜c. | *7.* 75 lbs. @ 23⅛c. | *11.* 500 tons @ $3.78½ |
| *4.* 500 yds. @ 86⅜c. | *8.* 2500 yds. @ 64⅛c. | *12.* 750 bbls. @ $2.96¾ |

**358. To find the cost when the numerator of the aliquot is greater than 1.**

Illustration: 37½c. = $⅜ = ¼ + (½ of ¼)

EXERCISE 7

| | |
|---|---|
| *1.* 144 yds. @ 37½c. | *6.* 640 bbls. @ $1.72½ |
| *2.* 256 gals. @ 62½c. | *7.* 84 bbls. @ $2.37½ |
| *3.* 196½ gro. @ 85c. | *8.* 56 bbls. @ $3.62½ |
| *4.* 288 yds. @ $1.37½ | *9.* 68 bbls. @ $2.22½ |
| *5.* 512 yds. @ $1.62½ | *10.* 96 tons @ $5.37½ |

**359.** RULE.—*Resolve the aliquot into two or more simple aliquots.*

**360. To find the quantity when the cost is given and the unit price is a simple aliquot of $1.**

EXERCISE 8

| | Cost | Unit Price | | Cost | Unit Price | | Cost | Unit Price |
|---|---|---|---|---|---|---|---|---|
| *1.* | $52.50 | 25c. | *6.* | $14.85 | $3\frac{1}{3}$c. | *11.* | $19.45 | 20c. |
| *2.* | 368.25 | $12\frac{1}{2}$c. | *7.* | 8.75 | $2\frac{1}{2}$c. | *12.* | 73.25 | $8\frac{1}{3}$c. |
| *3.* | 97.75 | $16\frac{2}{3}$c. | *8.* | $4.72\frac{1}{2}$ | 5c. | *13.* | 16.50 | $7\frac{1}{7}$c. |
| *4.* | 142.50 | $33\frac{1}{3}$c. | *9.* | 184.50 | $14\frac{2}{7}$c. | *14.* | 24.75 | $6\frac{1}{4}$c. |
| *5.* | 295.75 | 50c. | *10.* | 59.10 | $11\frac{1}{9}$c. | *15.* | 36.20 | $66\frac{2}{3}$c. |

*16.* $78.75 @ 25c. a yard
*17.* $156.25 @ $33\frac{1}{3}$c. a yard
*18.* $24.75 @ $16\frac{2}{3}$c. a pound
*19.* $147.25 @ $6\frac{1}{4}$c. a gallon
*20.* $385 @ $87\frac{1}{2}$c. yard
*21.* $175.50 @ $1.25 a yard
*22.* $264.80 @ $1.33\frac{1}{3}$ a yard
*23.* $363.75 @ $1.50 per barrel
*24.* $1975 @ $12.50 a ton
*25.* $26.58 @ 75c. a yard

**361.** RULE.—*Multiply the cost by the number of commercial units that $1 will buy.*

**362. To find the price of a commercial unit.**

EXAMPLE.—What is the unit price when 100 yds. cost $43.625?

SOLUTION.—Moving the point in the cost two places to the left gives $.43625, or $43\frac{5}{8}$ cents, as the price of a yard.

EXERCISE 9

**363.** Find the unit price if

*1.* 100 yds. cost $15.87\frac{1}{2}$
*2.* 1000 lbs. cost $66.25
*3.* 250 yds. cost $188.75
*4.* 125 yds. cost $177.50
*5.* 3000 tons cost $116.25
*6.* $333\frac{1}{3}$ yds. cost $575
*7.* 2000 lbs. cost $127.50
*8.* 150 yds. cost $78.75
*9.* 1500 gals. cost $97.12\frac{1}{2}$
*10.* 750 yds. cost $2986.50

EXERCISE 10—REVIEW

**364.** Find the cost of

*1.* 389 yds. @ 25c.
*2.* $246\frac{3}{4}$ yds. @ $33\frac{1}{3}$c.
*3.* 369 lbs. @ $16\frac{2}{3}$c.
*4.* $166\frac{2}{3}$ yds. @ $12\frac{1}{2}$c.
*5.* 750 gals. @ $8\frac{1}{3}$c.
*6.* 156 lbs. @ $1.33\frac{1}{3}$
*7.* $562\frac{1}{2}$ yds. @ $2 50
*8.* 116 gals. @ $1.12\frac{1}{2}$
*9.* 3126 lbs. @ $2\frac{1}{2}$c.
*10.* 964 ft. @ 15c.
*11.* 1190 bbls. @ $4.12\frac{1}{2}$
*12.* $418\frac{3}{4}$ yds. @ $3.16\frac{2}{3}$
*13.* $273\frac{1}{4}$ yds. @ $3 87\frac{1}{2}$
*14.* 168 tons @ $6.25
*15.* $178\frac{1}{2}$ yds. @ $62\frac{1}{2}$c.

## SPECIAL RULES

**365. To find the cost when merchandise is sold by the hundred or the thousand.**

Many kinds of merchandise, such as lumber, cigars, meal, grain, etc., are sold by the hundred or thousand.

NOTE.—C is the abbreviation for 100; M, for 1000. A cental of grain is 100 pounds. A cwt. is 100 pounds avoirdupois. A net ton is 2000 pounds; a long ton, 2240 pounds.

EXAMPLE.—Find the value of 2604 lbs. of wheat at $1.12 per cental

**366.** RULE.—*Point off two places in the quantity if the price is per hundred, or three places if per thousand; multiply by the price and point off as in the multiplication of decimals.*

NOTE.—For net tons, point off three places and divide by 2.

$$\begin{array}{r} 26.04 \\ \times\ 1.12 \\ \hline 5208 \\ 26044 \\ \hline \$29.1648 \end{array}$$

### WRITTEN PROBLEMS

**367.** *1.* Find the cost of 4500 feet of pine boards at $22.50 per M.

*2.* Find the cost of 75 boxes cigars, each containing 50, at $15.50 per M.

*3.* What will 1112 pounds of hay cost at $1.40 per cwt.?

*4.* Find the cost of a car-load of bran, weighing 27350 lbs., at 65 cents a cwt.

*5.* Find the cost of insuring a property valued at $7250 at 87½ cents per $100.

*6.* Find the cost of 40 boxes envelopes, each containing 500, at $2.25 per M.

*7.* What are 17610 feet of oak lumber worth at $27.25 per M?

*8.* Find the cost of

| | | | | |
|---|---|---|---|---|
| 2175 lbs. | Middlings | at $.65 | a | cwt. |
| 875 " | Cracked Corn | " .50 | " | |
| 200 " | Oat Meal | " 1.25 | " | |
| 3125 " | Bran | " .38 | " | |

*9.* Find the cost of

| | | | | |
|---|---|---|---|---|
| 26281 ft. | Hemlock | at $12.25 | per | M. |
| 4278 " | Scantling | " 12.50 | " | |
| 7500 " | Shingles | " 17.50 | " | |

*10.* What is the cost of a car-load of hay, 18750 lbs., at $13.50 per ton?

*11.* Find the freight on hay in preceding problem at 8 cents a cwt.

*12.* 13560 pounds of wheat, purchased at $1.37 a cental, were sold at $1 a bushel of 60 lbs. What amount was gained or lost by the transaction?

*13.* Find the cost of 1812 pounds of hay at $14.50 a ton.

*14.* What is the value of 65 pounds of wheat at $1.25 a cental?

*15.* I sold a bill of hemlock lumber, 585622 feet, and gained $1.30 a thousand. What was my entire gain?

**368. To find cost when merchandise is sold by the ton.**

EXAMPLE.—What is the value of 5 tons 6 cwt. of coal at $5.75 per ton?

5 tons 6 cwt. = 5.30 tons.

**369.** RULE.—*Multiply the hundredweights by .05 and add to the tons; multiply by the price per ton and point off required places.*

NOTE.—The above rule applies to both the "short" or standard ton of 2000 pounds, and the "long" ton of 2240 pounds, the latter having the long hundred of 112 pounds, which equals $\frac{1}{20}$, or .05 of a ton.

$5.75
5.3
1725
2875
$30.475

**WRITTEN PROBLEMS**

**370.** *1.* Find the cost of 11 tons 18 cwt. of coal at $3.25.

*2.* Find the cost of 9 tons 2 cwt. of coal at $5.50.

*3.* What is the value of 2 cars of coal, one containing 11 tons 3 cwt., the other 13 tons 15 cwt., at $2.85 per ton?

*4.* Make the extensions in

| | | | | |
|---|---|---|---|---|
| 1 car | Pea Coal, | 11 tons | 6 cwt. at | $3.25 |
| 1 " | Egg Coal, | 12 tons | 19 cwt. " | 4.75 |
| 1 " | Stove Coal, | 10 tons | 13 cwt. " | 4.50 |
| 1 " | " " | 13 tons | " | 4.50 |
| | | | Total | |

*5.* What is the value of 18 tons 17 cwt. of iron ore at $18.75 per ton?

*6.* What is the freight on 43 tons 7 cwt. of iron at $1.35 a ton?

*7.* A man bought his year's supply of coal, 12 tons 6 cwt., at $5.75; what was the amount of the bill?

*8.* Three cars of iron weighed respectively 25 tons 7 cwt., 23 tons 18 cwt., and 27 tons 2 cwt.; what was the value of the iron at $14.80 a ton?

*9.* What would be the freight on the iron in preceding problem at 35 cents a ton?

*10.* How many tons of 2000 pounds in 25 tons 9 cwt. long tons?

**371. To find the cost of commodities sold by the standard or "short" ton of 2000 pounds, when the weight is given in pounds.**

EXAMPLES.—(1) Find the cost of 2840 pounds of hay at $18 a ton; (2) of 3500 pounds of bran at $25 a ton; (3) of 1960 pounds of fertilizer at $32.50 a ton.

2840 — $20. a ton.
284 — 2. • (1/10)
$25.56 — $18. •

3500 — $20. a ton.
875 — 5. • (1/4)
$43.75 — $25. •

1960 — $20. a ton.
980 — 10. • (1/2)
245 — 2.50 • (1/4 of 1/2)
$31.85 — $32.50 •

Since 1 cent a pound gives $20 a ton, and since most ton prices can be easily figured upon that basis, the above method can be generally applied.

**372.** RULE.—*Point off two decimal places in the number of pounds, which gives $20 a ton; then add to or subtract from this result such part of it as the price is more or less than $20.*

### ORAL EXERCISE

**373.** Analyze the following prices upon a basis of $20 a ton and apply them to such quantities as may be given by the teacher:

| | | |
|---|---|---|
| *1.* $10 a ton | *7.* $12.50 a ton | *13.* $45 a ton |
| *2.* $30 " | *8.* $17.50 " | *14.* $50 " |
| *3.* $16 " | *9.* $22.50 " | *15.* $47.50 " |
| *4.* $24 " | *10.* $27.50 " | *16.* $37.50 " |
| *5.* $22 " | *11.* $40 " | *17.* $21.50 " |
| *6.* $15 " | *12.* $35 " | *18.* $65 " |

**374. To find the cost of commodities sold by the bushel of so many pounds.**

NOTE.—It is evident that 1 cent a pound for any commodity sold by the bushel, is as many cents a bushel as there are pounds in a bushel of the commodity in question. Thus, 1 cent a pound equals 60 cents a bushel for wheat, clover seed, potatoes, beans, etc. (these commodities each weighing 60 pounds to the bushel); it also gives 32 cents a bushel for oats, 45 cents for timothy seed, 56 cents for shelled corn.

EXAMPLES.—(1) Find the cost of 14680 lbs. wheat at 90c. a bu.; (2) of 4368 lbs. oats at 40c. a bu.

$$\begin{array}{r|l} 14680 & = 60\text{¢ a bu. for wheat} \\ 7340 & = 30\text{¢} \\ \$2202.0 & = 90\text{¢} \end{array}$$

$$\begin{array}{r|l} 4368 & = 32\text{¢ a bu. for oats} \\ 1092 & = 8\text{¢} \\ \$1456.0 & = 40\text{¢} \end{array}$$

**375.** RULE.—*Point off two decimal places in the quantity, and increase or diminish this value (as the price may require) by the use of aliquot parts.*

**376. To find cost without changing to bushels.**

### WRITTEN EXERCISE

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *1.* 24370 | lbs. | wheat | at | $0.75 | a bu. | *11.* 29640 | lbs. | oats | at | $0.40 | a bu. |
| *2.* 54940 | " | " | " | .90 | " | *12.* 13664 | " | " | " | .45 | " |
| *3.* 94580 | " | " | " | .97½ | " | *13.* 9900 | " | " | " | .50 | " |
| *4.* 73690 | " | " | " | 1.05 | " | *14.* 97980 | " | shelled corn | " | .70 | " |
| *5.* 66460 | " | " | " | 1.03 | " | *15.* 57428 | " | " " | " | .60 | " |
| *6.* 9720 | " | beans | " | 1.50 | " | *16.* 7245 | " | timothy seed | " | 1.05 | " |
| *7.* 17230 | " | " | " | 2.70 | " | *17.* 6440 | " | barley | " | .78 | " |
| *8.* 8850 | " | peas | " | 1.95 | " | *18.* 5280 | " | millet | " | 1.35 | " |
| *9.* 3190 | " | clover seed | " | 6.90 | " | *19.* 12240 | " | rye | " | .84 | " |
| *10.* 2775 | " | " " | " | 7.50 | " | *20.* 99750 | " | corn in ear | " | .42 | " |

## REVIEW PROBLEMS

**377.** *1.* A farmer hauled to market 25 loads of wheat averaging 2796 lbs. to the load. One-half the wheat was graded as "No. 1," for which he received \$1.05 a bushel; $\frac{1}{4}$ as "No. 2," for which he was paid \$1 a bushel; and the remainder "No. 3," worth 90 cents a bushel. How much did he get for his wheat?

*2.* Five car-loads of wheat, weighing respectively 52475, 54980, 46325, 49590 and 51650 pounds, were shipped from Kansas to Philadelphia. If the dealer who shipped the wheat paid 90 cents a bushel for it, $27\frac{1}{2}$ cents per cwt. for freight, and \$75 for other expenses, and then sold it for \$1.20 a bushel, what was his profit?

*3.* Ten loads of hay, weighing respectively 2450, 2380, 2590, 2725, 2675, 2295, 2885, 2565, 3020 and 2745 pounds, were sold at \$22.50 a ton. How much was received for the hay?

*4.* A stock dealer bought 125 head of hogs, weighing them in five lots: the first lot weighed 10500 lbs.; the second, 9975 lbs.; the third, 11150 lbs.; the fourth, 8995 lbs.; the fifth, 9780 lbs. What was the average weight of the hogs? If he paid $6\frac{1}{4}$ cents a pound for them, and sold them at $7\frac{1}{2}$ cents a pound after paying out \$105 for freight and other expenses, what was his profit?

*5.* A Pittsburgh grain merchant bought 245760 lbs. of shelled corn at 42c. a bu., 385600 lbs. of corn in the ear at 40c. a bu., 296440 lbs. of oats at 36c. a bu., and 582380 lbs. of wheat at 93c. a bu. What was the entire cost?

*6.* If the merchant mentioned in the preceding problem had the corn in the ear shelled, losing thereby $\frac{1}{5}$ of its weight in the cobs, but disposing of the cobs at \$5 a ton, then selling all the corn at 49c. a bu., the oats at 45c. a bu., and the wheat at \$1.03 a bu., what was his net gain on the whole, his expenses amounting to \$593.45?

*7.* How much better would the merchant in the preceding problem have done by having the wheat converted into flour, at an expense of $33\frac{1}{3}$c. per barrel for the milling and 20c. apiece for barrels, and then selling the flour at \$5.25 a bbl. (196 lbs.), the bran and middlings at \$22 a ton, estimating a bushel of wheat to make 42 lbs. of flour and allowing any fraction of a barrel of flour for waste?

*8.* A New Jersey farmer marketed 50 loads of potatoes of 175 baskets each. If the average net weight of the potatoes was $37\frac{1}{2}$ lbs. to the basket and he received 75c. a bushel for them, what did he realize from the crop?

*9.* A grain dealer sold during a year 37856 lbs. corn meal at 8 cents a hundred profit; 139874 lbs. bran at a profit of \$1.15 per ton; a car-load of corn (shelled) 53768 lbs., at $4\frac{1}{2}$ cents per bushel profit; 76328 lbs. oats at $2\frac{1}{2}$ cents per bushel profit; 18376 lbs. clover seed at \$3.75 per bushel profit; 3856 lbs. potatoes at a profit of $15\frac{1}{2}$ cents per bushel; 26375 lbs. timothy seed at $13\frac{1}{3}$ cents per bushel profit; and $328\frac{1}{2}$ bushels rye at $9\frac{1}{2}$ cents per bushel profit. What was his total profit?

# ANALYSIS

**378.** Many arithmetical problems may be solved readily by analysis, proceeding from the known to the unknown, according to the conditions of the problem, using the unit as the basis of comparison.

### MENTAL PROBLEMS

EXAMPLE.—What will be the cost of 5 oranges, if 3 oranges cost 12 cents?

SOLUTION.—1 orange will cost 4 cents and 5 oranges will cost 20 cents.

NOTE.—It is deemed unnecessary that the student go through a long formula of words; let him use sufficient words to show that the mental trend is connected and logical, and require no more.

**379.** *1.* What will 6 lemons cost at the rate of 8 lemons for 16 cents?

*2.* If 7 men can mow 14 acres of grass in a day, how many acres can 12 men mow in the same time?

*3.* If 9 pounds of beef cost 54 cents, what will 7 pounds cost?

*4.* If a man travels 64 miles in 8 days, how far can he travel in 14 days?

*5.* If a drover feeds hay at the rate of 22 tons in 11 weeks, how many tons will he feed in 7 weeks?

*6.* How many apples can be bought for 42 cents at the rate of 14 for 7 cents?

*7.* If 9 men can mow 18 acres of grass in a day, how much can 11 men mow in the same time?

*8.* If 8 men can dig 24 rods of ditch in a day, how much can 9 men dig in the same time?

*9.* If 7 boys can do a piece of work in 15 days, how long will it take 21 boys to do it?

*10.* How many men can mow as much grass in 5 days as 4 men can mow in 40 days?

*11.* If it requires 12 men 6 days to build a wall, how many men will be required to build it in 12 days?

*12.* If a farmer gave 9 bushels of wheat for 2 barrels of flour, what was the wheat worth a bushel if 8 barrels of flour cost $72?

*13.* A farmer exchanged apples worth $2 a barrel for 4 yards of cloth which cost at the rate of $21 for 7 yards; how many barrels of apples did he give?

*14.* If $\frac{3}{4}$ of a yard of muslin cost 9 cents, what will 2 yards cost?

SOLUTION.—$\frac{1}{4}$ of a yard will cost 3 cents, 1 yard will cost 12 cents and 2 yards will cost 24 cents.

*15.* What will 3 boxes of soap cost, if $\frac{3}{4}$ of a box cost $6?

*16.* If A can walk 24 miles in $\frac{6}{7}$ of a day, how far can he walk in 6 days?

*17.* How much will 8 barrels of apples cost, if $\frac{7}{10}$ of a barrel cost 70 cents?

*18.* How much will 7 tons of hay cost, if $20 will buy $\frac{4}{5}$ of a ton?

*19.* A watch cost $42 and $\frac{2}{3}$ of its cost was twice the cost of a chain; what was the cost of the chain?

*20.* A horse cost $100 and $\frac{3}{5}$ of its cost was the cost of a carriage; what did both cost?

*21.* If 11 yards of cloth cost $5\frac{1}{2}$, what will 5 yards cost?

SOLUTION.—$5\frac{1}{2}$ are $\frac{11}{2}$, 1 yard will cost $\frac{1}{2}$ and 5 yards will cost $\frac{5}{2}$ or $2\frac{1}{2}$.

*22.* If 5 oranges are worth $7\frac{1}{2}$ cents, what is one dozen worth?

*23.* What cost 18 bananas at the rate of $7\frac{1}{2}$ cents for 5 bananas?

*24.* If $\frac{1}{6}$ of 42 yards of muslin cost 49 cents, what will $\frac{2}{3}$ of 36 yards cost?

*25.* If 8 pairs of shoes cost $\$\frac{14}{5}$, what will 10 pairs cost?

*26.* How much are 12 lamps worth, if 7 lamps are worth $\$\frac{14}{3}$?

*27.* What is the value of 6 mirrors, if 9 mirrors are worth $\$\frac{18}{2}$?

*28.* A boy is 12 years old, which is $\frac{1}{4}$ of his father's age; what is the father's age?

SOLUTION.—If 12 years is $\frac{1}{4}$ the father's age, the father is 4 times 12 years, or 48 years.

*29.* James has 20 pennies, which is $\frac{2}{3}$ of John's number; how many have both?

*30.* A clerk spent $20, which is $\frac{1}{8}$ of the sum he has left; how much had he at first?

*31.* A man earned $30, which is $\frac{3}{8}$ of what he has in a savings bank; how much has he in bank?

*32.* A watch was bought for $25, which is $\frac{1}{2}$ of 5 times what the chain cost; what was the cost of both?

*33.* The head of a fish is 5 inches long, which is $\frac{1}{8}$ of twice the length of the body; what is the length of the body?

*34.* Wheeler paid $5 for a saddle, which is $\frac{1}{8}$ of $\frac{1}{3}$ of the cost of his wheel; what did his wheel cost?

*35.* The head of a fish is 4 inches long and the tail 5 inches long, which together is $\frac{1}{2}$ of $\frac{1}{3}$ of the length of the body; how long is the fish?

*36.* Thomas, who is 15 years old, is $\frac{5}{6}$ of Harry's age; how old is Harry?

SOLUTION.—$\frac{1}{6}$ of Harry's age is $\frac{1}{5}$ of 15 years or 3 years; Harry is 6 times 3 years, or 18 years old.

*37.* A woman found $12, which was $\frac{4}{5}$ of what money she then had; how much had she at first?

*38.* A drover sold a horse for $120, thereby gaining $\frac{1}{5}$ of its cost; what was its cost?

*39.* Leary sold a book for 80 cents, which was $\frac{4}{5}$ of $\frac{5}{6}$ of its cost when new; what was its cost?

*40.* Harkness sold a horse for $140, which was $\frac{7}{8}$ of $\frac{4}{5}$ of its value; what was the value of the horse?

*41.* 18 feet of a pole is in the water, which is $\frac{3}{5}$ of $\frac{5}{7}$ of the length in the air; what is the length of the pole?

*42.* A horse cost $150, and $\frac{2}{3}$ of the cost of the horse is twice the cost of the harness; what was the cost of both?

*43.* A cow cost $35, which was $\frac{1}{4}$ of twice the cost of a horse; what did both cost?

*44.* A boy's pony cost $44, which was $\frac{4}{5}$ of $\frac{5}{6}$ of the cost of his wagon; what was the cost of his wagon?

*45.* If 5 boys can earn $6\frac{2}{3}$ in a week, how much can 6 boys earn in the same time?

*46.* How far will a man walk in 3 hours, if in 7 hours he walks $19\frac{1}{4}$ miles?

*47.* If a man earns $\$5\frac{1}{4}$ in 3 days, how much can he earn in 4 days?

*48.* If $4\frac{1}{2}$ tons of coal cost \$27, how much will 7 tons cost at the same rate?

*49.* How much will a man earn in a week of 6 days at the rate of $\$2\frac{1}{4}$ a day?

*50.* If 3 loads of hay cost $\$15\frac{1}{2}$, what will 6 loads cost?

*51.* If a vessel can sail 23 miles in $4\frac{3}{4}$ hours, how far can it sail in 9 hours?

*52.* If a man rides 14 miles in $2\frac{1}{3}$ hours, how far will he ride in $4\frac{2}{3}$ hours?

*53.* If 4 cows eat 2 tons of hay in 8 weeks, how long will the same amount last 5 cows?

*54.* If 2 men can do a piece of work in 16 days, how long will it take 8 men to do the same work?

*55.* A watch cost \$50, which is $\frac{5}{8}$ of 3 times what the chain cost. Required the cost of both.

*56.* A man's money increased by its $\frac{2}{3}$ equals \$90; how much money had he?

*57.* The difference between $\frac{2}{3}$ of my money and $\frac{3}{10}$ of my money is \$10; what is the amount of my money?

*58.* A boy having 36 pennies, lost $\frac{2}{3}$ of them and then found $\frac{5}{8}$ as many as he had at first; how many had he then?

*59.* $\frac{2}{3}$ of $\$1\frac{1}{2}$ is $\frac{1}{10}$ of the cost of a chain; how much did the chain cost?

*60.* A hat cost $\frac{1}{3}$ of \$10, which is $\frac{2}{5}$ of the cost of a coat. Required the cost of both.

*61.* If 12 sheep cost \$25, what will 15 sheep cost at the same rate?

*62.* A chain cost \$12 and $\frac{1}{2}$ of its cost is $\frac{3}{4}$ of the cost of a watch; what is the cost of both?

*63.* If 6 men can do a piece of work in $6\frac{2}{3}$ days, how long will it take 5 men to do it?

*64.* A man being asked his age said, "If to my age its $\frac{1}{2}$ and its $\frac{2}{3}$ be added, I will be 52 years old." What is his age?

*65.* A man having lost $\frac{3}{4}$ of his money had \$24 remaining; how much did he lose?

*66.* $\frac{2}{3}$ of the length of a pole is in the air, $\frac{1}{4}$ in the water, and the remainder, 18 feet, in the earth; what is the length of the pole?

*67.* What number is that which being increased by its $\frac{1}{2}$, and that sum diminished by its $\frac{2}{3}$, the remainder is 30?

*68.* What is a man's money if twice his money diminished by \$12 equals \$88?

*69.* A man's money increased by \$80 equals three times his money; how much money has he?

*70.* If a tree's height be increased by $\frac{2}{3}$ its height and 20 feet, the sum will equal twice its height; what is its height?

*71.* A man and a boy earned \$24 in a week; what did each earn if the man earned twice as much as the boy?

*72.* A watch and chain cost \$70; what was the cost of each if $\frac{2}{3}$ of the cost of the watch equals the cost of the chain?

*73.* A pole 40 feet long was broken in two unequal parts so that $\frac{2}{3}$ of the longer equals the shorter; what is the length of each part?

## WRITTEN PROBLEMS

**380.** EXAMPLE.—If 5 men reap ten acres of wheat in a day, how many acres will 7 men reap in the same time?

SOLUTION.—If five men reap ten acres of wheat in a day, one man will reap one-fifth as much or two acres, and seven men will reap fourteen acres.

EXAMPLE.—If 10 men dig a ditch in 4 days of 10 hours each, in how many days will 12 men dig a similar ditch, working 8 hours a day?

SOLUTION.—If 10 men dig it in 4 days of 10 hours each, it would require one man 40 days of 10 hours, or 400 hours; 12 men would do it in $\frac{1}{12}$ of 400 hours, or $33\frac{1}{3}$ hours; and working eight hours a day would require $4\frac{1}{6}$ days.

EXAMPLE.—A can do a piece of work in 10 days, B in 12 days; in what time can both working together do it?

SOLUTION.—A can do $\frac{1}{10}$ of it in one day; B can do $\frac{1}{12}$ in one day; both working together can do $\frac{1}{10}$ and $\frac{1}{12}$, or $\frac{11}{60}$ in one day. They can do $\frac{60}{60}$, the whole piece of work, in $5\frac{5}{11}$ days.

**381.** *1.* If a post 5 feet high casts a shadow 12 feet long, how high is a telegraph pole which at the same time casts a shadow 65 feet long?

*2.* A pipe will fill a cistern in 10 hours, another pipe in 6 hours; if both be opened at the same time, what time will be required to fill the cistern?

*3.* A man can do a piece of work in 12 days, and a boy can do it in 15 days. In what time can they do it working together?

*4.* An inlet pipe will fill a tank in 6 hours, and an outlet pipe will empty it, when full, in 10 hours. If both be left open when the tank is empty, what time will be required to fill the tank?

*5.* Jones can dig a ditch in 12 days, Smith in 15 days, and Brown in 18 days. How long will be required for them to dig a ditch twice as deep, if they all work together?

*6.* B can do $\frac{9}{10}$ as much work in a day as A. How long would it require both working together to do that which B alone can do in 20 days?

*7.* Green can unload a car of corn in 6 hours; Black can [illegible] it in 5 hours. How many hours would both require, working together, to unl[illegible] similar cars?

*8.* A can do a piece of work in 16 days, B in 20 days. [illegible]oth together begin the work, and A leaves when it is $\frac{2}{3}$ done, how long will B [illegible]re to complete it?

*9.* If I buy goods at 4 yards for \$5 and sell at t[illegible] of 5 yards for \$7, how many yards must be sold to gain \$70?

*10.* If 10 men can do a piece of work in 24 da[illegible], and 4 men retire when they have worked 10 days, in what time will the remaining men finish the work?

*11.* If 5 men or 9 boys can do a piece of work in 12 days, in how many days should 9 men and 5 boys do the same work?

*12.* 10 men agreed to do a piece of work, but 2 of them did not report, on account of which the work took 3 days' more time. In what time could the 10 men have done the work?

*13.* A can carry a ton of coal to the fourth story of a building in $\frac{1}{2}$ of a day, B in $\frac{3}{4}$ of a day, and C in $\frac{4}{5}$ of a day. How long will it require A, B and C, together, to carry 1 ton?

*14.* A, B and C can do a piece of work in 8, 10 and 15 days respectively. After the 3 have worked 2 days, how long will it require B and C together to finish it?

*15.* A does a piece of work in $\frac{1}{2}$ day, B in $\frac{2}{3}$ of a day, C in 2 days, and D in $1\frac{1}{4}$ days. What time will it require to perform the work if they all work together?

*16.* If a loaf of bread weighs 8 ounces when flour is \$3.75 a barrel, what should it weigh when flour is \$4.50 a barrel?

*17.* If I gain \$1250 by selling \$20000 worth of goods, how much should I sell to gain \$6250?

*18.* If $9\frac{3}{4}$ pounds of tea cost \$6.75, what would $17\frac{1}{2}$ pounds be worth?

*19.* If \$7.60 will buy 144 pounds of soap, how many pounds can be bought for \$19.75?

*20.* If $11\frac{1}{4}$ acres of land will pasture 33 cows, how many acres will be required for 327 cows?

*21.* If 6 men can do a piece of work in 17 days, how many men would be required to do the work in 11 days?

*22.* If a cycler can ride 225 miles in $2\frac{1}{4}$ days of 8 hours each, how many days of ten hours each would be required to ride 625 miles?

*23.* If 6 men can mow a field in 10 hours, how many men must be added to mow it in 6 hours?

*24.* If a man can do $\frac{1}{3}$ of a piece of work in 6 days of 10 hours each, in what time can he do the remainder, working 8 hours a day?

*25.* A, B and C own a vessel, A owning $\frac{2}{3}$ and each of the others half the remainder; B sells $\frac{1}{2}$ of his share for \$3000; what is the value of A's and C's share at the same rate?

*26.* If 8 horses or 7 cows eat $6\frac{3}{4}$ tons of hay in 23 days, what time will it take 7 horses and 8 cows to eat the same quantity?

*27.* If 15 men can do a piece of work in 10 days, what time will be required to do it if 10 men quit when the work is half done?

*28.* One pump can fill a cistern in 4 hours and another in 5 hours, and an outlet pipe can empty it in 1 hour; if the tank be empty and the pumps both working and outlet pipe closed, in what time can it be filled?

*29.* If the tank in the preceding problem be full with both pumps working and the outlet pipe open, in what time will the tank be emptied?

*30.* A can mow an acre in $5\frac{1}{2}$ hours, and A and B together can mow twice as much in $4\frac{3}{11}$ hours; in what time can B working alone mow $2\frac{1}{2}$ acres?

*31.* Two men start in business with the same amount of capital; the first gains $\frac{1}{4}$ of his capital and the second gains $\frac{1}{5}$ of his capital, when the first has \$100 more than the second. What was the capital of each?

595813 A

32. C can reap a field of grain in 6 days and D in 8 days. In how many days can they do it reaping together?

33. A cistern has two pipes by the first of which it may be filled in 12 hours, and by the second in 8 hours: how long will both be in filling it?

34. D and E working together can make a china closet in 8 days; D working alone could do it in 12 days: in what time could E make it?

35. A owns $\frac{1}{3}$ of a tract of land and B owns $\frac{1}{4}$. A's share is worth $250 more than B's. Find the value of the tract.

36. A and B agree to do a piece of work, A to receive $2 a day and B $3 a day. A works twice as many days as B, and they together receive $70. How many days does each labor?

37. I sold a book and lost $\frac{1}{4}$ of its cost. Had it cost $1 less, I would have gained $\frac{1}{3}$ of its cost. What was its cost?

38. A man sold a book, making $\frac{1}{3}$ of the cost. Had the book cost $1 less and sold for the same profit, he would have gained three times the cost. Find the cost.

39. A and B invest equal sums of money; A gains $\frac{1}{5}$ of his investment and B loses $20; B's money is then equal to $\frac{1}{2}$ of A's. How much does each invest?

40. A can do a piece of work in 5 days; B can do it in 6 days. In how many days can they do it working together?

41. C can reap a field of grain in $6\frac{2}{3}$ days and D in $8\frac{3}{4}$ days. In how many days can they do it reaping together?

42. A tank has two pipes, by the first of which it may be filled in 12 hours, and by the second in 15 hours; in what time will both fill it?

43. A boy hired to a mechanic for twenty weeks on condition that he should receive $20 and a coat. At the end of twelve weeks the boy ceased work, when it was found that he was entitled to $9 and the coat. What was the value of the coat?

44. An aeroplane with a velocity of 25 miles an hour in still air has a journey of 85 miles to make. How long will it take if the wind blows steadily 8 miles an hour in the direction the aeroplane is sailing? How long will it require to return the same distance, if the velocity of the wind has fallen to $5\frac{1}{2}$ miles an hour?

45. If my motor boat moves through still water with a velocity of 22 miles per hour, how far may I go down a stream whose current is 5 miles an hour, that I may return up stream to the starting point and be gone 14 hours, 40 minutes?

# MEASURES

## MEASURES OF VALUE

**282.** U. S. Money

| | | | | | |
|---|---|---|---|---|---|
| | | *m* | *c* | *d* | *h* |
| 10 Mills | = 1 Cent | 1 = | 10 = | 100 = | 1000 = 10000 |
| 10 Cents | = 1 Dime | | 1 = | 10 = | 100 = 1000 |
| 10 Dimes | = 1 Dollar | | | 1 = | 10 = 100 |
| 10 Dollars | = 1 Eagle | | | | 1 = 10 |

**283.** English Money

| | | £ | *s* | *d* | *f* |
|---|---|---|---|---|---|
| 4 Farthings | = 1 Penny | 1 = | 20 = | 240 = | 960 |
| 12 Pence | = 1 Shilling | | 1 = | 12 = | 48 |
| 20 Shillings | = 1 Pound | | | 1 = | 4 |

£1 = $4.8665* (Gold)

**284.** French Money

| | | *fr.* | *cent.* |
|---|---|---|---|
| 100 Centimes | = 1 Franc | 1 = | 100 |

1 Franc = $0.1930* (Gold)

**285.** German Money

| | | *M* | *Pf* |
|---|---|---|---|
| 100 Pfennige | = 1 Mark | 1 = | 100 |

1 Mark = $0.238* (Gold)

### Other Foreign Moneys

Notes.—The Latin Union countries: France, Belgium, Switzerland, Greece and Italy, have the same monetary unit, called the franc in the first three, the drachma in Greece and the lira in Italy. The standard gold and silver coins circulate freely in the different countries.

Finland, Spain and Venezuela have monetary units of the same value as the franc of France ($0.193), called respectively mark, peseta and bolivar.

Denmark, Norway and Sweden have a like monetary unit called the crown, value $0.268.

Mexico, Guatemala, Honduras, Nicaragua, Salvador, Chile, Uruguay, Argentine Republic and the Philippine Islands have each a monetary unit known as the peso, varying in value according to the pure silver or gold contents of the coins of these countries, the lowest being that of Chile, worth $0.365, and the highest, that of Uruguay, worth $1.034. The silver coins fluctuate in value, while the gold coins do not.

Besides the United States, the following six countries or colonies have the dollar as a monetary unit, the value of which is appraised in each case: British Honduras $1.00; Canada $1.00; Cuba $1.00; Hong-Kong $0.463; Liberia $1.00; Newfoundland $1.014. These coins are all of gold, except that of the British colony of Hong-Kong, which is a silver subsidiary coin, as above on April 1, 1916.

* These monetary unit values are determined by the Director of the United States Mint and proclaimed in a circular issued every three months by the Secretary of the Treasury. They represent the intrinsic value of the precious metal in the coins and are used in estimating the value of imported merchandise upon which duties are collected at the various custom houses. They have nothing to do with the commercial rates at which bills of exchange are bought and sold, the latter fluctuating with the supply and demand in the foreign exchange market. The commercial rates may, therefore, be higher or lower than these intrinsic or par value rates.

## MEASURES OF EXTENSION AND SURFACE

**386.** LONG MEASURE

| | | *Mi.* | *rd.* | *yd.* | *ft.* | *in.* |
|---|---|---|---|---|---|---|
| 12 Inches | = 1 Foot | 1 = | 320 = | 1760 = | 5280 = | 63360 |
| 3 Feet | = 1 Yard | | 1 = | 5½ = | 16½ = | 198 |
| 5½ Yards | = 1 Rod | | | 1 = | 3 = | 36 |
| 320 Rods | = 1 Mile | | | | 1 = | 12 |

39.37 Inches = 1 Meter

A *knot* is equal to 6087 feet.

The *hand*, used in measuring the height of horses, equals 4 inches.

A *pace* is equal to three feet, and 5½ paces approximate a rod.

A *rod* is sometimes called a perch or pole.

A *statute mile* in the United States and England is 5280 feet, and is distinguished from the *geographic mile* which is 6087 feet.

The *yard* is the standard unit of extension of the United States and also of Great Britain.

**387.** SURVEYORS' LONG MEASURE

| | | |
|---|---|---|
| 7.92 | Inches | 1 Link |
| 25 | Links | 1 Rod |
| 100 | Links (4 rods) | 1 Chain |
| 80 | Chains | 1 Mile |

The unit of *surveyors' long measure* is the *Gunter's chain*, which is 4 rods, or 66 feet in length. Since the chain has 100 links, links may be written as hundredths of a chain. Thus 18 chains, 22 links = 18.22 chains.

**388.** SQUARE MEASURE

| | | *A.* | *sq. rd.* | *sq. yd.* | *sq. ft.* | *sq. in.* |
|---|---|---|---|---|---|---|
| 144 Square Inches | = 1 Square Foot | 1 = | 160 = | 4840 = | 43560 = | 6212640 |
| 9 Square Feet | = 1 Square Yard | | 1 = | 30¼ = | 272¼ = | 39182 |
| 30¼ Square Yards | = 1 Square Rod, or Perch | | | 1 = | 9 = | 1296 |
| 160 Square Rods | = 1 Acre | | | | 1 = | 144 |
| 640 Acres | = 1 Square Mile | | | | | |

1.196 Square Yards = 1 Square Meter

With the exception of the acre, the units of square measure are derived from the corresponding units of long measure.

**389.** SURVEYORS' SQUARE MEASURE

| | |
|---|---|
| 625 Square Links | = 1 Square Rod |
| 16 Square Rods | = 1 Square Chain |
| 10 Square Chains, or 160 Square Rods | = 1 Acre |
| 640 Acres | = 1 Square Mile |
| 36 Square Miles | = 1 Township |

The unit of land measure is the acre.

In surveying public lands, a square mile, 640 acres, is called a "section."

## MEASURES OF VOLUME OR CAPACITY

### 390. Solid Measure

| | *cu. yd.* | *cu. ft.* | *cu. in.* |
|---|---|---|---|
| 1728 Cubic Inches = 1 Cubic Foot | 1 = | 27 = | 46656 |
| 27 Cubic Feet = 1 Cubic Yard | 1 = | | 1728 |

128 Cubic Feet = 1 Cord

40 Cubic Feet = 1 Ton (for freight)

24¾ Cubic Feet = 1 Perch of Stone

($16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24\frac{3}{4}$)

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1.308 Cubic Yards = 1 Cubic Meter

1 Cubic Meter or Stere is the metric unit of Wood Measure.

The unit of Lumber Measure is a Board Foot, which is $\frac{1}{12}$ of a Cubic Foot.

### 391. Liquid Measure

| | *gal.* | *qt.* | *pt.* | *gi.* |
|---|---|---|---|---|
| 4 Gills = 1 Pint | 1 = | 4 = | 8 = | 32 |
| 2 Pints = 1 Quart | | 1 = | 2 = | 8 |
| 4 Quarts = 1 Gallon | | | 1 = | 4 |

The ordinary or Wine Gallon contains 231 cubic inches.

For estimating, a barrel contains 31½ gallons, and a hogshead 2 barrels.

Commercially, the exact contents of barrels are found by *gauging*.

1.05668 Liquid Quarts = 1 Liter (used for milk and wine)

### 392. Dry Measure

| | *bu.* | *pk.* | *qt.* | *pt.* |
|---|---|---|---|---|
| 2 Pints = 1 Quart | 1 = | 4 = | 32 = | 64 |
| 8 Quarts = 1 Peck | | 1 = | 8 = | 16 |
| 4 Pecks = 1 Bushel | | | 1 = | 2 |

A *bushel*, stricken measure (the *Winchester bushel*) contains, approximately, 2150.4 cubic inches, and is used in measuring grain, such as wheat, corn (shelled), beans; also, small fruits, chestnuts, etc.

A *heaped bushel* contains, approximately, 2747.7 cubic inches, and is used in measuring large fruits (apples, pears, etc.), vegetables, corn in the ear, coal, coke, lime, etc.

The *British Imperial bushel* contains, approximately, 2218.2 cubic inches.

The *dry gallon* or *half peck*, contains 268.8 cubic inches, or ⅛ of 2150.4.

.9081 Dry Quarts = 1 Liter  100 Liters = 1 Hectoliter

2.8375 Bushels = 1 Hectoliter

## MEASURES OF WEIGHT

### 393. Troy Weight

| | | *lb.* | *oz.* | *pwt.* | *gr.* |
|---|---|---|---|---|---|
| 24 Grains | = 1 Pennyweight | 1 = | 12 = | 240 = | 5760 |
| 20 Pennyweights | = 1 Ounce | | 1 = | 20 = | 480 |
| 12 Ounces | = 1 Pound | | | 1 = | 24 |

The Troy pound, 5760 grains, is the unit of weight. Troy weight is used in weighing gold, silver and precious stones.

The carat, for weighing diamonds, is 3.2 Troy grains.

The carat, indicating fineness of gold, means $\frac{1}{24}$ part. Gold 20 carats fine has 20 parts gold and 4 parts alloy.

### 394. Apothecaries' Weight

| | | ℔ | ℥ | ʒ | ℈ | *gr.* |
|---|---|---|---|---|---|---|
| 20 Grains | = 1 Scruple | 1 = | 12 = | 96 = | 288 = | 5760 |
| 3 Scruples | = 1 Dram | | 1 = | 8 = | 24 = | 480 |
| 8 Drams | = 1 Ounce | | | 1 = | 3 = | 60 |
| 12 Ounces | = 1 Pound | | | | 1 = | 20 |

This weight is used in weighing small quantities of drugs and medicines. For large quantities avoirdupois weight is used.

### 395. Avoirdupois Weight

| | | *T.* | *cwt.* | *lb.* | *oz.* |
|---|---|---|---|---|---|
| 16 Ounces | = 1 Pound | 1 = | 20 = | 2000 = | 32000 |
| 100 Pounds | = 1 Hundredweight | | 1 = | 100 = | 1600 |
| 20 Hundredweights | = 1 Ton | | | 1 = | 16 |

The *gross* or *long ton* used at U. S. Custom Houses, and by wholesale dealers in coal and iron, is 2240 pounds, the hundredweight 112 pounds, the quarter 28 pounds.

1 Pound Avoirdupois = 7000 Troy Grains

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| | |
|---|---|
| 15.432354 Troy Grains | = 1 Gram. |
| 2.2046 Pounds Av. (approximately) | = 1 Kilogram. |
| 2204.6 Pounds " " | = 1 Ton or Tonneau. |
| 1000 Grams | = 1 Kilogram. |
| 1000 Kilos | = 1 Ton or Tonneau. |

### 396. Weights of Grains

| Article | Measure | Pounds | Exceptions | Article | Measure | Pounds | Exceptions |
|---|---|---|---|---|---|---|---|
| Barley | bushel | 48 | Pa. 47 | Hemp | bushel | 44 | |
| Beans | bushel | 60 | | Millet | bushel | 50 | |
| Buckwheat | bushel | 50 | Pa. 48 | Oats | bushel | 32 | N. J. 30; Md. 26 |
| Clover Seed | bushel | 60 | N. J. 64 | | | | |
| Corn in ear | bushel | 70 | | Peas | bushel | 60 | |
| Corn, shelled | bushel | 56 | | Rye | bushel | 56 | |
| Flaxseed | bushel | 56 | N. J. 55 | Timothy Seed | bushel | 45 | |
| | | | | Wheat | bushel | 60 | |

The above table and the one following contain the weights of the common grains, fruits, and vegetables, as used by a majority of the States. Exceptions are given in Pa., Md. and N. J.

**397.** WEIGHTS OF FRUITS, VEGETABLES, ETC.

| Article | Measure | Pounds | Exceptions | Article | Measure | Pounds |
|---|---|---|---|---|---|---|
| Apples | bushel | 50 | N. J. 48 | Beef | barrel | 200 |
| Peaches, dried | bushel | 33 | | Pork | barrel | 200 |
| Potatoes | bushel | 60 | | Flour | barrel | 196 |
| Sweet Potatoes | bushel | 54 | | Fish | quintal | 100 |
| Peanuts | bushel | 22 | | Grain | cental | 100 |
| Corn Meal | bushel | 50 | | Nails | keg | 100 |
| Salt coarse | bushel | 70 | Pa. 85 | | | |
| Lime unslaked | bushel | 80 | Pa. 70 | | | |
| Onions | bushel | 57 | Pa. 50 | | | |
| Apples dried | bushel | 25 | | | | |

**398.** TIME MEASURE

| | | |
|---|---|---|
| 60 Seconds | = | 1 Minute |
| 60 Minutes | = | 1 Hour |
| 24 Hours | = | 1 Day |
| 7 Days | = | 1 Week |
| 365 Days | = | 1 Year |
| 12 Calendar Months | = | 1 Year |

A day is added for Leap Years. All years exactly divisible by 4, excepting Centennial Years, are Leap Years. Centennial Years exactly divisible by 400 are Leap Years.

**399.** CIRCULAR MEASURE

| | | |
|---|---|---|
| 60 Seconds | = | 1 Minute |
| 60 Minutes | = | 1 Degree |
| 360 Degrees | = | 1 Circle |

This table is used for measuring angles and arcs and for latitude and longitude. The signs are degrees, minutes, seconds.

LONGITUDE AND TIME

**400.** As the circle of the earth (360 degrees) passes under the sun in 24 hours, in one hour $\frac{1}{24}$ of the circle, or 15°, would pass before. 15° of longitude make a difference of 1 hour in time before, to find difference in longitude multiply difference in time by 15; and to find difference in time, divide difference in longitude by 15.

**401.** MISCELLANEOUS

| | | | |
|---|---|---|---|
| 20 Units | = 1 Score | 6 Feet | = 1 Fathom |
| 12 Units | = 1 Dozen | 1¼ Miles | = 1 Knot |
| 12 Dozen | = 1 Gross | 3 Knots | = 1 League |
| 12 Gross | = 1 Great Gross | 69½ Knots or Geographical Miles | = 1 Degree |
| 24 Sheets | = 1 Quire } Stationers | 69½ Statute Miles | |
| 20 Quires | = 1 Ream } | | |
| 25 Sheets | = 1 Quire } Commercial Paper | | |
| 20 Quires | = 1 Ream } | | |

# DENOMINATE NUMBERS

**402. Denominate numbers** are concrete numbers expressing divisions of money, weight, measure, etc. They may be reduced, added, subtracted, multiplied and divided.

**403. To reduce to lower denominations.**

EXAMPLE.—Reduce £13 9*s*. 10*d*. to pence

| £ | *s*. | *d*. |
|---|---|---|
| 13 | 9 | 10 |
| 20 | | |
| 269 | | |
| 12 | | |
| 3238 | | |

£13 9*s*. 10*d*. = 3238*d*.

**404.** RULE.—*Multiply the number of higher denomination by the number of the next lower denomination in a unit of the higher, adding the number of the lower denomination to the product. Proceed until the required denomination is reached.*

### WRITTEN EXERCISE

**405.** *1*. Reduce 18 lb. 11 oz. 3 pwt. 16 gr. to grains.
*2*. Reduce 12 lb. 5℥ 6℥ 1℈ 14 gr. to grains.
*3*. Reduce 77 bu. 1 pk. 1 pt. to pints.
*4*. Reduce 12 mi. 30 rd. 5 yd. 2 ft. 10 in. to inches.
*5*. Reduce 365 da. 5 hr. 48 min. 49 sec. to seconds.
*6*. Reduce 77° 26′ 45″ to seconds.
*7*. Change 9 tons 12 cwt. 60 lb. 12 oz. to ounces.
*8*. Reduce 145 mi. 124 rd. 4 yd. to feet.
*9*. Change 215 gal. 3 qt. 1 pt. to pints.
*10*. How many pence in £472 10s.?

**406. To reduce or change to higher denomination.**

EXAMPLE.—Reduce 7534 pt. to bushels

2)7534
8)3767
4)470+7 qt.
117+2 pk.

7534 pt. = 117 bu. 2 pk. 7 qt.

**407.** RULE.—*Divide the lower denomination by the number in a unit of the next higher denomination, the remainder, if any, being put down as a part of the result. Proceed until the required higher denomination is obtained.*

### WRITTEN EXERCISE

**408.** *1.* Change 1213 pt. to higher denominations.
*2.* Change 478260 gr., apothecaries' weight, to higher denominations.
*3.* Change 95346 gr., Troy weight, to higher denominations.
*4.* Change 288692 oz., avoirdupois weight, to higher denominations.
*5.* Change 4841 far. to higher denominations.
*6.* Change 1456 gi. to gallons.
*7.* Change 436614 in. to miles.
*8.* Change 18500 sq. ft. to higher denominations.
*9.* Change 108000 seconds of time to higher denominations.
*10.* Change 160000 seconds of circular measure to higher denominations.

### 408. To reduce denominate fractions and decimals.

EXAMPLE.—(1) Reduce $\frac{1}{8}$ mi. to feet; (2) $\frac{7}{8}$ ton to lower denominations

$\frac{1}{8}$ of 5280 = 880 ft.
Result, 880 ft.

$\frac{7}{8}$ of 20 = $17\frac{1}{2}$ cwt.
$\frac{1}{2}$ of 100 = 50 lbs.
Result, 17 cwt. 50 lbs.

### WRITTEN EXERCISE

**410.** *1.* Change $\frac{7}{8}\frac{7}{0}\frac{1}{0}$ of a mile to rods.
*2.* Change $\frac{1}{1}\frac{3}{7}\frac{1}{4}\frac{1}{0}$ of a ton to pounds.
*3.* Reduce $\frac{1}{8}\frac{7}{0}$ of a gallon to lower denominations.
*4.* Reduce $\frac{1}{3}\frac{1}{0}$ of a square mile to lower denominations.
*5.* Change $\frac{1}{4}\frac{1}{0}$ of a bushel to pints.
*6.* Reduce $\frac{3}{1}\frac{3}{1}$ of an acre to lower denominations.
*7.* Reduce $\frac{3}{4}$ of a year to lower denominations.
*8.* Change £$\frac{3}{8}$ to pence.
*9.* Reduce $\frac{5}{6}$ of a Troy pound to lower denominations.
*10.* Change $\frac{3}{4}$ of a peck to pints.

EXAMPLE.—(2) Reduce .255 bu. to lower denominations

.255 bu. × 4 = 1.02 pk.
.02 pk. × 8 = .16 qts.
.16 qts. × 2 = .32 pts.
Result, 1 pk. .32 pt.

*11.* Change .02 of a gross to the decimal of a dozen.
*12.* Change .007 of an acre to the decimal of a square rod.
*13.* Reduce £.00625 to lower denominations.
*14.* Reduce .46875 of a bushel to lower denominations.
*15.* Change £.2685 to lower denominations.
*16.* Reduce .00125 of a ton to lower denominations.

EXAMPLE 3.—Reduce $\frac{3}{4}$ ft. to fraction of a rod

$$\frac{\cancel{3}}{\cancel{4}_{2}} \times \frac{1}{\cancel{3}} \times \frac{\cancel{2}}{11} = \frac{1}{22} \text{rd.}$$

*17.* Reduce $\frac{1}{8}$ qt. to the fraction of a bushel.

*18.* Reduce $\frac{3}{7}$ pt. to the fraction of a gallon.

*19.* Reduce $\frac{3}{7}\frac{3}{8}$ in. to the fraction of a mile.

*20.* Reduce $\frac{5}{8}$ cwt. to the fraction of a ton.

*21.* Change $\frac{1}{5}$ of a pint to the fraction of a bushel.

EXAMPLE 4.—Change .3759 pt. to decimal of a gallon

2).3759
4).18795
.0469875 gal.

*22.* Change .25 of a pint to the decimal of a gallon.

*23.* Change .625 of an ounce to the decimal of a ton.

*24.* Change 430 cubic inches to the decimal of a cubic yard.

*25.* What decimal of an acre is $6\frac{5}{72}$ square feet?

*26.* Change .4 of a foot to the decimal of a rod.

EXAMPLE 5.—Reduce 8 oz. 7 pwt. 12 gr. to fraction of a pound

| oz. | pwt. | gr. |
|---|---|---|
| 8 | 7 | 12 |
| 20 | | |
| 167 | | |
| 24 | | |
| 680 | | |
| 334 | | |
| 4020 gr. | | |

1 lb. = 5760 gr.

$$\frac{4020}{5760} = \frac{201}{288} = \frac{67}{96} \text{lb. or } .6979\frac{1}{6}$$

*27.* Reduce 3 qts. 1 pt. to the fraction of a gallon.

*28.* Reduce 5 oz. 17 pwt. 6 gr. to the fraction of a pound.

*29.* Reduce 4 oz. 10 pwt. 20 gr. to the fraction of a pound.

*30.* Change 10s. 6d. to the decimal of a pound sterling.

*31.* Change 14 hr. 50 min. 30 sec. to the decimal of a day.

*32.* What part of 3 pk. 6 qt. is 4 qt. 1 pt.?

*33.* What part of 6 score is 14 dozen?

*34.* What part of 1.25 miles is 150 rd. 2 yd. 6 in.?

*35.* What part of $8\frac{1}{3}$ bu. is 2 pk. $6\frac{1}{2}$ qt.?

*36.* What decimal part of a cubic yard is 2 cu. ft. 280 cu. in.?

**411. To add denominate numbers.**

EXAMPLE Find the sum of 26 bu 3 pk 4 qt, 18 bu 1 pk 3 qt, and 17 bu 5 qt.

| bu | pk | qt | |
|---|---|---|---|
| 26 | 3 | 4 | |
| 18 | 1 | 3 | Result, 62 bu 1 pk 4 qts. |
| 17 | 0 | 5 | |
| 62 | 1 | 4 | |

412. RULE *Find the sum of the lowest denomination, reduce this sum to next higher, setting down remainder, if any; carry the number of next higher denomination, find sum, and reduce in like manner, so continue until addition is completed.*

**412. To subtract denominate numbers.**

EXAMPLE From 31 gal 2 qt 1 pt take 15 gal 3 qt.

| gal | qt | pt | |
|---|---|---|---|
| 31 | 2 | 1 | |
| 15 | 3 | 0 | Result, 15 gals 3 qts 1 pt |
| 15 | 3 | 1 | |

414. RULE *Find the difference of lowest denomination, if the subtrahend number is greater than the minuend number, borrow one of the next higher denomination, adding to the minuend number as many of the lower denomination as make one of the higher, then find difference. After borrowing, diminish the minuend number of the next higher denomination by one, then find difference. So continue until the subtraction is completed.*

**415. To multiply denominate numbers.**

EXAMPLE Multiply 3 yd 2 ft 5 in by 130

| yd | ft | in | |
|---|---|---|---|
| | | | 5 × 130 = 650 in = 54 ft |
| 3 | 2 | 5 | 2 × 130 = 260 + 54 = 314 ft |
| | | 130 | = 104 yd 2 ft |
| | | | 1 × 130 = 130 + 104 = 234 yd |
| 434 | 2 | 0 | Result 434 yd 2 ft |

416. RULE *Multiply as in whole numbers and reduce to next higher denominations, setting down remainder, if any, and carrying the number of higher to product of next higher. Proceed in like manner until full product is obtained.*

**417. To divide denominate numbers.**

EXAMPLE Divide 176° 42′ 12″ by 12

| | | | | |
|---|---|---|---|---|
| 12 | 176° | 42′ | 12″ | 176° ÷ 12 = 14 + 8° |
| | 14° | 42′ | 12″ | 8° × 60 = 480′ + 42′ = 522′ |
| | | 43′ | 12″ | 522′ ÷ 12 = 43 + 6 |
| | | | 12″ | 6′ × 60 = 360″ + 12″ = 372″ |
| | | | 31″ | 372″ ÷ 12 = 31 |

418. RULE *Divide the number of highest denomination and set down the quotient; reduce the remainder, if any, to next lower denomination, add to result the number in the dividend of same denomination, divide, set down quotient and proceed in like manner until division is completed. If the lowest denomination dividend has a remainder it may be expressed fractionally.*

NOTE The methods of adding, subtracting, multiplying and dividing here considered are not much used in commercial operations. Custom usually designates some denominations as the ones to be used, and quantities are named in it and fractional parts of it.

## WRITTEN PROBLEMS

**419.** *1.* Change 192000 oz. to tons.

*2.* A haystack contains 23000 lbs.; what is its value at \$15 per ton?

*3.* How many grains Troy in 12 lbs. 4 oz. 12 pwt.?

*4.* What is the cost of three bushels of tomatoes at 12 cents a quart?

*5.* Reduce 15 gal. 1 qt. 1 pt. to gills.

*6.* How many 2-dram powders of magnesia can be put up from 13 lb. 5 oz.?

*7.* How many half-acre lots in a piece of ground containing 8200 sq. rd. 30 sq. yd.? State the surplus, if any, in sq. ft.

*8.* What is the value of a diamond weighing $\frac{7}{16}$ of a carat at \$75 a carat?

*9.* What is the value of the pure gold in an ornament weighing 4 pwt., 18 carats fine, at \$20 an ounce for pure gold?

*10.* Reduce 80 lb. avoirdupois to denominations of Troy weight.

*11.* Change 224 oz. Troy to denominations of avoirdupois weight.

*12.* If $\frac{3}{8}$ of a lb. of gold is worth \$174.25, what is the value of $\frac{5}{8}$ of a pennyweight?

*13.* A man had $10\frac{3}{8}$ tons of coal after buying 2.5 tons; how many pounds had he at first?

*14.* If a man earns 80 cents in 2 hr. 40 min., what will he earn in 6 days, if he works 8 hours a day?

*15.* What is the value of £11 5s. 4d. at \$4.8665 a pound sterling?

NOTE.—A business rule for changing shillings and pence to £ is to multiply shillings by 5, and point off 2 places; the pence by $4\frac{1}{6}$, point off 3 places.

SUGGESTION

1s. = £ $\frac{1}{20}$ or .05

1d. = £ $\frac{1}{240}$ or .004$\frac{1}{6}$

£11 — 5s. — 4d.

5 — 4⅙

.25 — 016⅔

.016⅔

11.266⅔

4.8665

56330

67596

67596

9012 8

45064

32443

54.829233 3 Res. \$54.83.

*16.* Change to United States money

| | | | | | |
|---|---|---|---|---|---|
| *1.* £1 1s. 1d. | | | *13.* £10 10s. 10d. | | |
| *2.* £2 2s. 2d | | | *14.* £11, 11s. 11d. | | |
| *3.* £3 3s. 3d. | | | *15.* £12 12s. 1d. | | |
| *4.* £4 4s. 4d. | | | *16.* £13 13s. 3d. | | |
| *5.* £5 5s. 5d. | | | *17.* £13 5s. 9d. | | |
| *6.* £6 6s. 6d. | | | *18.* £14 14s. 4d. | | |
| *7.* £7 7s. 7d. | | | *19.* £14 17s. 5d. | | |
| *8.* £8 8s. 8d. | | | *20.* £15 15s. 6d. | | |
| *9.* £9 9s. 9d. | | | *21.* £16 16s. 6d. | | |
| *10.* £16 16s. 3d. | Exchange | $4.86. | *22.* £25 13s. 4d. | Exchange | $4.88. |
| *11.* £17 17s. 5d. | " | 4.87. | *23.* £8 19s. 8d. | " | 4.87. |
| *12.* £18 18s. 8d. | " | 4.866. | *24.* £229 3s. 4d. | " | 4.85. |

*17.* Find the cost in United States money of

| | | |
|---|---|---|
| *1.* 120 yd. Silk @ £1 5s. 3d. per yd. | Exchange | $4.87. |
| *2.* 144 yd. Silk @ £1 1s. 1d. per yd. | " | 4.88. |
| *3.* 326 yd. Silk @ £1 5s. 2d. per yd. | " | 4.86. |
| *4.* 546 yd. Carpet @ £1 15s. 4d. per yd. | " | 4 87. |
| *5.* 579 yd. Silk, @ £2 2s. 9d. per yd. | " | 4.85. |
| *6.* 500 doz. Knives @ £1 17s. 3d. per doz. | " | 4.86. |
| *7.* 820 tons Iron @ £3 14s. 3d. per ton | " | 4.87. |
| *8.* 869 acres Land @ £32 17s. 11d. per acre | " | 4.8665. |
| *9.* 564 tons Iron @ £4 17s. 8d. per ton | " | 4.8665. |
| *10.* 1856 bushels Wheat @ 8s. 9d. per bu. | " | 4.8665. |

SUGGESTION.—These exercises are arranged for practise in conversion of Foreign money to United States money.

*18.* Bought 180 meters of silk at 8.75 francs per meter and sold at $1.80 per yd. Find the gain.

*19.* Bought 240 meters of lace at 6.5 francs per meter and sold at $1.40 per yd. Find the gain.

*20.* Bought 150 meters of ribbon at 4.75 francs per meter and sold at $1.20 per yd. Find the gain.

*21.* Bought 324 meters of velvet at 6.75 francs per meter and sold at $1.50 per yd. Find the gain.

*22.* Bought 600 meters of cloth at 4.32 francs per meter and sold at $1 per yd. Find the gain.

*23.* Bought 426 meters of lace at 7.19 francs per meter and sold at $1.50 per yd. Find the gain.

*24.* Bought 356 meters of silk at 6.24 francs per meter and sold at $1.85 per yd. Find the gain.

*25.* Bought 295½ meters of taffeta at 5.18 francs per meter and sold at $1.28½ per yd. Find the gain.

*26.* Bought 356.35 meters of silk at 8.75 francs per meter, and sold at $2.15 per yd. Find the gain.

*27.* Bought 516.5 meters of cloth at 6.85 francs per meter, and sold at $1.85 per yd. Find the gain.

## REVIEW PROBLEMS

**420.** *1.* Find the time by compound subtraction from the date of the Declaration of Independence until to-day. Reduce to minutes, counting 30 days to the month.

*2.* Buy pens at $1.25 a gross and sell them at a cent apiece. What will be the gain in a sale of a great gross?

*3.* How long has a sum of money been on interest that was placed July 4, 1876? (Time by compound subtraction.)

*4.* What is the difference in time between a place in longitude 69° 50′ west and a place 120° 25′ west?

*5.* Find what fractional part of a bushel 3 pk. 6 qt. is, and change the result to a decimal.

*6.* Bought 172.5 meters silk at 5.18 francs per meter and sold it at $1.93 a yard. How much was gained?

*7.* Buy wheat at 12.35 francs a hectoliter and sell it at a dollar a bushel. How much is gained on a hectoliter?

*8.* What would be gained by buying wine at 1 franc a liter and selling it for 20 cents a quart?

*9.* Bought 100 kilos French candy at 5.18 francs a kilo and sold it for half a dollar a pound. What was my gain or loss on a pound?

*10.* What part of a tonneau would be gained by buying coal by the long ton and selling by the tonneau at the same price?

*11.* What part of a ton would be lost by buying coal by the tonneau and selling it by the long ton at the same price?

*12.* How many square rods in a piece of land containing 7 square chains?

*13.* If the circumference of the earth is 24899 miles, how many kilometers is it?

# PRACTICAL MEASUREMENTS

**421.** A **line** has one dimension only—length.

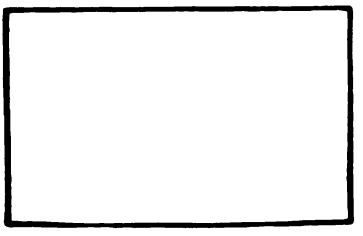
**422.** A **surface** has but two dimensions—length and width.

**423.** A **square unit** is a square in which the units of length and width are alike.

**424.** A **solid** has three dimensions—length, width and thickness.

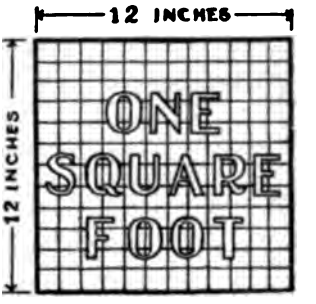
**425.** A **cubic unit** is a cube in which the units of length, width and thickness are alike.

## SURFACE MEASUREMENTS



RECTANGLE

NOTE.—Under this head come such practical measurements as carpeting, plastering, papering, painting, roofing, flooring, paving, land measurements, etc.




**426. To find the area of rectangular surfaces.**

EXAMPLE.—How many square feet in a tight board fence 6 feet high and 40 rods long?

40 rods = 660 ft.
660 ft. × 6 = 3960 sq. ft.

NOTE.—When the dimensions are expressed in different denominations, they should be reduced to units of the same denomination before multiplying; as, inches to feet, or feet to inches.

**427.** RULE.—*Multiply the length by the width; the product will be the area.*

### WRITTEN PROBLEMS

**428.** *1.* What is the value of a lot 32 feet front, 120 feet deep, at 25 cents a square foot?

*2.* How many square yards of linoleum will be required to cover a floor 22½ feet long, 14¼ feet wide?

*3.* What will be the cost of painting a wainscoting 40 feet in length, 3 feet 6 inches high, at 25 cents a square yard?

*4.* A railroad 60 feet wide passes half a mile through a tract of land. If the damage is assessed at $150 an acre, what must the company pay?

*5.* What will it cost, at $2.25 a square yard, to pave a street with asphalt 50 feet wide for a length of 1600 feet? What must a property owner pay who has a frontage of 40 feet?

*6.* How many square feet of tin will be required to cover a flat roof 40 feet long and 20 feet wide? What will be the cost at $5.25 per square of 100 square feet?

*7.* At $67.50 per thousand square feet, what will it cost to floor and ceil a room 60 feet long and 25 feet wide?

*8.* A ceiling 18 feet 10 inches long, 14 feet 7 inches wide may be of hard wood, costing 12½c. a square foot, or of corrugated iron at $1.35 a square yard. Which is the cheaper and how much?

## ROOFING AND PAVING

**429. To find the number of bricks, shingles, slates, etc., required to cover a given surface.**

EXAMPLES.—(1) Find the number of bricks, 4 x 8 inches, required to lay a walk 5 feet wide and 40 feet long. (2) Find the number of pieces each 2 inches square required to inlay a vestibule floor, 6 feet 8 inches by 5 feet 6 inches.

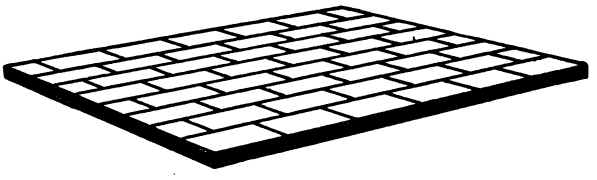
$\frac{5 \times \cancel{40}^{10} \times \cancel{144}^{18}}{\cancel{4} \times \cancel{8}} = 900$

Result 900 bricks.

SUGGESTION FOR EXAMPLE 2
6 ft. 8 in. = 80 in. long
5 ft. 6 in. = 66 in. wide

$\frac{\cancel{80}^{40} \times \cancel{66}^{33}}{\cancel{2} \times \cancel{2}} = 1320$ pcs.

Result 1320 pieces.



**430.** RULE.—*Divide the given surface by the surface of a unit to find the required number.*

### WRITTEN PROBLEMS

**431.** *1.* How many shingles, averaging 4 inches in width, laid five inches to the weather, will be required to cover two sides of a roof 40 feet long, each side 18 feet wide? What will the shingles cost at $12 per M?

NOTE.—The ordinary shingles are 16 in. by 4 in. and are put up in bundles of 250 each. Nothing less than an entire bundle should be considered if purchased by the bundle.

*2.* At $12.50 per M, what will the shingles cost for a roof 60 ft. long, 18 ft. from eaves to ridge on one side, and 24 ft. on the other, the shingles being 4 inches wide, laid $4\frac{1}{2}$ inches to the weather? At $3 per bundle?

*3.* How many slates each 10 inches wide, laid $5\frac{1}{2}$ inches to the weather, will be required for a roof 50 ft. long and 20 ft. from eaves to ridge?

*4.* How many bricks, $8 \times 4$, laid flat, will be required to pave a 6-foot walk 800 feet long? What will the bricks cost at $14 a thousand? What will the laying cost at 60 cents a square yard?

*5.* What will it cost to pave a section of street 40 feet wide, 600 feet long, with vitrified brick laid on edge, $9 \times 2\frac{1}{2}$ in., at $11 per thousand and 72 cents a square yard for laying?

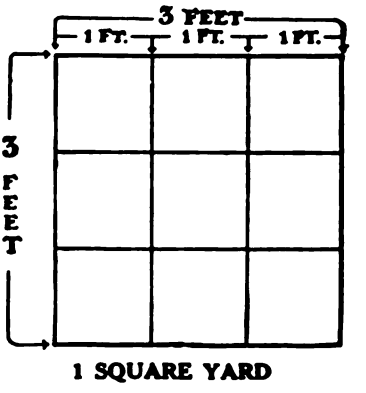
*6.* How many wooden blocks each $2\frac{1}{2}$ in. thick, 6 in. wide and 10 in. long, placed on end, will it take to pave a street 50 ft. wide for a distance of $2\frac{1}{4}$ miles? What will the paving cost at $4.75 a sq. yd. for material and labor?

## PLASTERING, PAPERING AND PAINTING

**432. To find entire surfaces of rooms, boxes, etc.**

FORMULA: (Length + Width) = ×2 = Perimeter
P. × Height = Surface of Walls
L. × W. = Surface of top or bottom

EXAMPLE.—How many square yards of plastering in the walls and ceiling of a room 24 feet long, 16 feet wide and 12 feet 6 inches high above the baseboard, no allowance for doors and windows?



1 SQUARE YARD

24 ft. long
16 " wide
40 × 2 = 80 ft. around room
12½ " high
960
40
1000 sq. ft. in walls
384 " " ceiling
1384 " " w. & c.

24 ft.
16 "
384 sq. ft. in ceiling

9)1384
153 7/9 sq. yards

**433.** RULE.—*Multiply the perimeter (distance around) by the height, and to this result add twice the product of the length by the width.*

### WRITTEN PROBLEMS

**434.** *1.* How many square feet of surface has a box that is 5 ft. 6 in. long, 3 ft. 6 in. wide and 4 ft. high?

*2.* How many square feet of surface has a room that is 18 feet long, 14 feet wide, 9½ feet high? What would be the cost of plastering the walls and ceiling at 35 cents a square yard, no allowance for doors and windows?

*3.* At 42 cents a square yard, what will it cost to plaster the walls and ceiling of a hallway 80 ft. long, 9 ft. wide and 15 ft. high, allowing for half the space occupied by 12 doors, each 5 ft. 4 in. wide and 9 ft. high?

*4.* Find the cost of lathing, plastering and flooring a hall 30 ft. by 50 ft. and 20 ft. high at 85 cents a sq. yd. for the lathing, 35 cents a sq. yd. for plastering, and $4.50 per 100 sq. ft. for the flooring; allowance to be made in the lathing for the full

space occupied by 12 windows, each 5 ft. wide and 8 ft. 6 in. high, and 2 doors 6 ft. 6 in. by 12 ft., and $\frac{2}{3}$ of the door and window space to be allowed in the plastering.

*5.* At 82 cents a square yard, what will it cost to plaster the walls and ceiling of a room 35 ft. long, 20 ft. wide, and 12 ft. 6 in. high?

*6.* How many rolls of paper 8 yards long, 18 inches wide, would be required for the sides, ends and ceiling of a room 15 ft. 6 in. long, 14 ft. 3 in. wide and 9 ft. high?

NOTE.—Single rolls of wall paper are 8 yards long; double rolls, 16 yards long. Paper-hangers estimate the number of rolls required by measuring the unbroken wall space; it is found that the irregular spaces above doors and windows can be covered from the remnants of the rolls.

*7.* What will be the cost of papering the sides and ends of a room 18 feet long, 15 feet wide, 10 feet high, deducting 10 feet from the perimeter for doors and windows, if the paper costs 50 cents a double roll?

*8.* Find the cost of weather boarding a house 40 ft. 3 in. long, 18 ft. 6 in. wide and 32 ft. high, allowing 20 square yards for windows and doors; adding 350 square feet for gables, lumber costing $21 per M.

*9.* Find the cost of painting the walls and ceiling of a room 28 ft. long, 17 ft. wide and 11 ft. high, at 82 cents a square yard.

*10.* At 40 cents a square yard, what will it cost to plaster the walls and ceiling of a room 32 ft. 6 in. long, 22 ft. 9 in. wide and 11 ft. 4 in. high?

*11.* At 32 cents a square yard, what will it cost to plaster the walls and ceiling of a room 29 ft. long, 16 ft. 6 in. wide and 12 ft. high, allowing for half the space occupied by 2 doors 6 ft. by 3 ft. and 4 windows 8 ft. 6 in. by 3 ft.?

*12.* Find the cost at $78\frac{3}{4}$ cents a sq. yd. of plastering the walls and ceiling of a room 63 ft. 3 in. long, 22 ft. 9 in. wide and 32 ft. high, allowing for the full space occupied by 3 doors 6 ft. by 3 ft. 6 in. and 10 windows 5 ft. by 3 ft.

*13.* At 40 cents a square yard, what will it cost to paint the walls and ceiling of a room 62 ft. long, 38 ft. wide and 12 ft. high; allowing for the full space occupied by 2 doors 6 ft. by 3 ft. 6 in., 4 windows 8 ft. by 3 ft., 1 skylight 8 ft. by 6 ft. 3 in. and a wainscot around the room 4 ft. 6 in. high?

*14.* Find the cost of lathing, plastering and flooring a room 60 ft. 3 in. long, 22 ft. 9 in. wide and 18 ft. 6 in. high, allowance to be made in the lathing for the full space occupied by 2 doors 7 ft. 6 in. by 4 ft., 10 windows 7 ft. by 3 ft. 3 in. and $\frac{1}{2}$ of the space occupied by the windows and doors for plastering. The lathing is 70 cents a square yard, plastering 42 cents a square yard and the flooring $4.20 per C.

## CARPETING

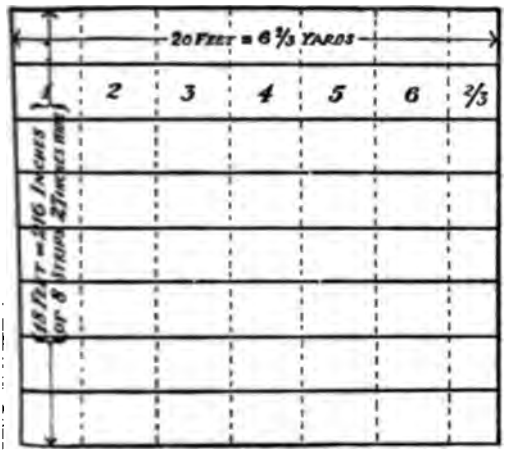
**435.** Carpet is sold by the yard, a yard of carpet being 3 feet long, regardless of its width. Linoleum and oil cloth are generally sold by the square yard.

In estimating the number of yards of carpet for a room it is always necessary to find the number of strips required, and to do this we must know whether they are to run lengthwise or crosswise. They are generally laid lengthwise, but if the room is nearly square they may run either way, and sometimes there is a considerable saving in laying them crosswise.

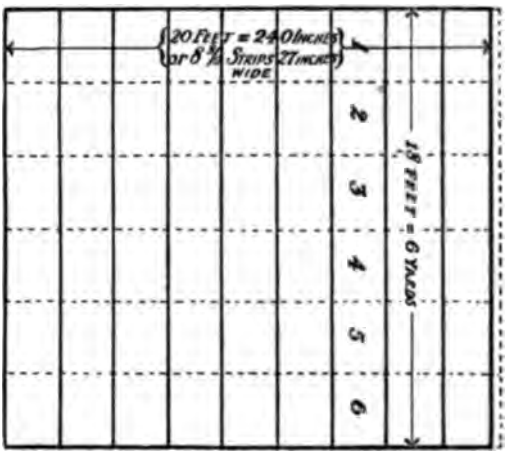
**436. To find the number of yards of carpet required for a room.**

EXAMPLE.—Which will be the cheaper, and how much, to lay the strips lengthwise or crosswise on a floor 18 ft. by 20 ft., carpet 27 in., costing $1.50 a yard, if there is no waste in matching either way?

Carpet laid lengthwise




Carpet laid crosswise




$18 \times 12 = 8$ strips, each 20 ft. 6⅔ yds. long.
$27$ 6⅔ yds. $\times 8 = 53\frac{1}{3}$ yds.

Or, 8 strips each 20 ft. = 160 ft.
160 ft. ÷ 3 = 53⅓ yds.

$20 \times 12 = 8\frac{4}{9} = 8\frac{1}{3}$ strips, or 9 strips, each 18 ft., or 6 yds. long = 54 yds.
$27$
$9$

54 yds. − 53⅓ yds. = ⅔ yds. less lengthwise

⅔ yds. at $1.50 = $1.00, the result

NOTE.—The heavy lines in the diagrams indicate the direction of the strips, and the dotted lines the number of yards in each strip.

**437.** RULE.—*If the strips are to be laid lengthwise, divide the width of the room in inches by the width of the carpet in inches; the result will be the number of strips, counting any fraction as a whole strip.*

*If the strips are to be laid crosswise, divide the length of the room in inches by the width of the carpet in inches, and count any fraction left over as a whole strip.*

*Multiply the length of the strips in feet, plus any allowance for waste in matching, by the number of strips, and divide by 3.*

### WRITTEN EXERCISE

**438.** Find the number of yards and the cost of each of the following:

| | Length of Room | Width of Room | Width of Carpet | Strips to Run | Waste in Matching | Price of Carpet |
|---|---|---|---|---|---|---|
| *1.* | 25 ft. | 19 ft. | 27 in. | Lengthwise | None | $1.50 |
| *2.* | 40 ft. | 31 ft. | 27 in. | Lengthwise | 6 in. | 3.25 |
| *3.* | 28 ft. | 27 ft. | 27 in. | Lengthwise | 1 ft. | 2.50 |
| *4.* | 20 ft. | 18 ft. | 27 in. | Crosswise | None | 1.25 |
| *5.* | 35 ft. | 22 ft. | 36 in. | Lengthwise | 9 in. | .95 |
| *6.* | 19 ft. 6 in. | 15 ft. 9 in. | 27 in. | Cheaper way | None | 1.10 |
| *7.* | 72 ft. 9 in. | 40 ft. 6 in. | 36 in. | Lengthwise | 1 ft. 3 in. | 1.12½ |
| *8.* | 50 ft. | 40 ft. | 27 in. | Cheaper way | 1 ft. | 1.50 |
| *9.* | 13 ft. 6 in. | 12 ft. | 27 in. | Crosswise | 10 in. | 1.75 |
| *10.* | 16 ft. | 14 ft. | 31½ in. | Cheaper way | 3 in. | 1.33⅓ |
| *11.* | 30 ft. 8 in. | 21 ft. 11 in. | 36 in. | Lengthwise | 4 in. | 1.25 |
| *12.* | 22 ft. 6 in. | 20 ft. 10 in. | 27 in. | Cheaper way | None | 1.16⅔ |
| *13.* | 24 ft. | 18 ft. | 31½ in. | Lengthwise | 1 ft. | 1.35 |
| *14.* | 27 ft. | 24 ft. 6 in. | 27 in. | Cheaper way | None | 1.50 |
| *15.* | 11 ft. 3 in. | 10 ft. 9 in. | 36 in. | Crosswise | 6 in. | 2.25 |

NOTE.—Exercises of a similar character adapted to other surfaces or solids will furnish excellent practise.

### WRITTEN PROBLEMS

**439.** 1. How many yards of carpet ¾ yd. (27 in.) wide will be required for a room 15 ft. wide and 22 ft. long, if the strips are laid lengthwise and there is a waste of 1 foot on each strip in matching?

*2.* What will it cost at 87½ cents a yard to carpet a room 17 ft. by 22 ft. 6 in., the carpet being 1 yd. wide, and laid the more economical way, no allowance for waste in matching?

*3.* Find the cost at $1.12½ a yard to carpet a room 17 ft. 6 in. wide and 30 ft. 9 in. long with carpet 27 in. wide, the strips to be laid lengthwise and 9 in. to be allowed on each strip for waste in matching.

*4.* At 97½ cents a yard for the carpet and 5 cents a square yard for lining, what would it cost to cover a floor 30 ft. wide and 50 ft. long with carpet 31½ in. wide, no allowance for waste in matching?

*5.* At $1.50 a yard, what will it cost to make a rug, the body of which is to be 3 yards wide and 6 yards long, made of 27-inch Axminster, the border surrounding it to be 24 inches wide, with no waste except that caused by the diagonal folds at the corners of the border?

*6.* A lady desiring to cover the floor of her parlor, 20 feet long and 17 feet wide, has the choice of three kinds of carpet, viz.: 27 inches wide, with an allowance of 6 inches for matching, at 62½c. a yard; ⅞ of a yard wide, with 8 inches allowance for matching, at 80 cents a yard; or 1 yard in width, with 10 inches allowance for matching, at $1 a yard. Laying the strips lengthwise, which is the most economical?

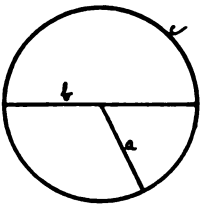
## THE CIRCLE

**440.** A **circle** is a plane figure bounded by a curved line every point of which is equally distant from a point within called the *center*.

**441.** The **circumference** of a circle is its boundary line.

**442.** The **diameter** of a circle is any line passing through the center, ending in the circumference.

**443.** The **radius** of a circle is one-half the diameter, or any line drawn from the center to the circumference.




NOTE.—A circle may be converted into a plane surface, the length of which is ½ of the circumference, and the width of which is the radius.

**444. To find the circumference of a circle.**

**445.** RULE.—*Multiply the diameter by 3.1416.*

**446. To find the diameter of a circle.**

**447.** RULE.—*Divide the circumference by 3.1416.*

**448. To find the area of a circle.**

**449.** RULE.—*Multiply the circumference by ¼ of the diameter, or the diameter by ¼ of the circumference.*

### WRITTEN PROBLEMS

**450.** *1.* What is the circumference of a wheel whose diameter is 60 inches?

*2.* What is the diameter of a circle whose circumference is 628.32 rods?

*3.* A carriage wheel whose diameter is 3½ feet made 1400 revolutions in going a certain distance. Find the distance.

*4.* What is the area of a fish-pond whose diameter is 25 ft.?

*5.* Find the cost of building a wall around a circular garden 2500 ft. in diameter, at $5.50 per linear yard.

*6.* What is the area of a circular pond which shall contain 45 times as much area as one 35 rods in diameter?

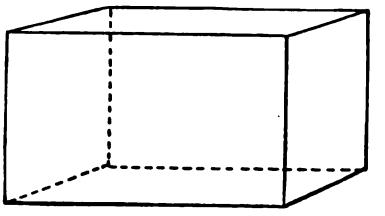
*7.* What will it cost, at 18 cents a square yard, to gravel a walk 6 ft. wide outside a circular plot of ground whose diameter is 80 yards?

*8.* At $2 a square (100 sq. ft.) what will it cost to paint the surface of a smoke-stack 31 ft. 6 in. in circumference and 120 ft. high?

*9.* If a horse is tied to a post in one corner of a square field (fenced) by a rope 50 feet long, and can reach 3 feet beyond the end of the rope, over what decimal part of an acre can he graze?

## SOLID MEASUREMENTS

**451.** Under this head come such practical measurements as masonry, brickwork, excavations, lumber, contents of heavy timbers, wood measure, capacities of tanks and bins.



**452.** A **volume** has length, breadth, and thickness or height.

**453.** A **cube** is a rectangular solid, the six faces of which are equal.

**454.** Students should note carefully the following facts:

*1.* 27 cu. ft. = 1 cu. yd. or load; $24\frac{3}{4}$ cu. ft. = a perch of stone.

*2.* In estimating the amount of material required for a wall, deductions are made for all openings and, in a building, for the corners, the latter being four times the thickness of the wall.

*3.* In estimating the value of masons' labor on walls, no allowance is made for corners and only part allowance for the openings.

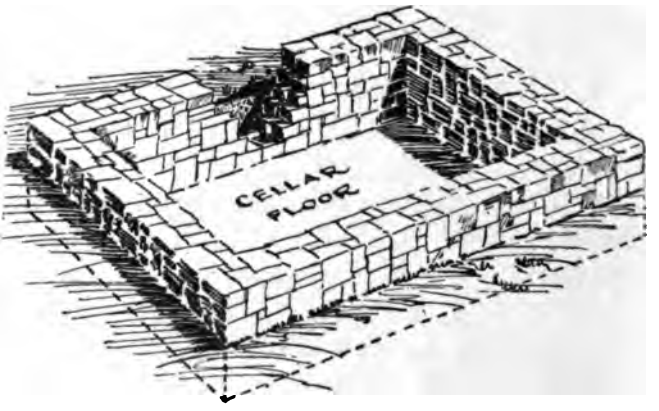
*4.* Because of the fact that the number of cubic feet in a perch varies so much in different localities, the practise of estimating by the perch is being discontinued, and all kinds of heavy masonry, whether brick, stone or concrete, is now quite generally estimated by the cubic yard.

*5.* Since $2\frac{1}{4}$, the difference between 27 cubic feet and $24\frac{3}{4}$ cubic feet, is $\frac{1}{11}$ of the latter or $\frac{1}{12}$ of the former, cubic yards may be changed to perches by adding $\frac{1}{11}$, or perches may be reduced to cubic yards by subtracting $\frac{1}{12}$.

*6.* In southeastern Pennsylvania and adjacent parts of New Jersey it is customary, in the absence of a special contract, to allow only 22 cubic feet to the perch in measuring walls to estimate either labor or material.

**455. To find the contents of walls or other solids in cubic feet, cubic yards, or perches.**

EXAMPLE. — How many cubic feet in a stone wall 81 ft. long, 22 in. thick and 8 ft. high? How many cubic yards? How many perches?




NOTE.—Only like dimensions should be multiplied, hence, write 22 in. as $\frac{22}{12}$ ft.

SUGGESTION.—$24\frac{3}{4} = \frac{99}{4}$ which, being the divisor, is inverted in the solution, and the work shortened by cancelation.

**456.** RULE.—*Multiply together the three dimensions—length, width (or thickness) and height (or depth); the result will be the contents in cubic units of the given denomination.*

$$\frac{81 \times 22 \times 8}{12} = 1188 \text{ cu. ft.}$$

$$\frac{81 \times 22 \times 8}{12 \times 27} = 44 \text{ cu. yds.}$$

$$\frac{81 \times 22 \times 8 \times 4}{12 \times 99} = 48 \text{ perches}$$

### WRITTEN PROBLEMS

**457.** *1.* How many perches of stone in a wall 57 ft. long, 7 ft. high, 2½ ft. thick? What will be the cost of laying at 75 cents a perch?

*2.* How many perches of stone in a cellar wall 18 in. thick, measuring 36 ft. by 24 ft. outside, 8 ft. deep?

*3.* What will it cost to lay a stone wall 150 yards long, 4 ft. high, 20 in. thick, at 95 cents a perch?

*4.* What will it cost to excavate a reservoir 50 meters square, 2 meters deep, at 65 cents a cubic meter?

*5.* What will it cost to cover the bottom of the reservoir in preceding problem with concrete 1 inch thick, at 45 cents a square yard?

*6.* How many cubic feet of earth must be removed in digging a ditch 120 rods long, 30 inches wide, 3½ feet deep? What will be the cost of the work at 55 cents a cubic yard?

*7.* Find the cost of digging a cellar 24 feet wide, 54 feet long and 6 feet deep at 35c. a cubic yard for ⅔ of it, and 45c. a cubic yard for the remainder.

*8.* Find the cost of the material for the walls of a cellar 25 ft. 6 in. long, 18 ft. 9 in. wide, 6 ft. high and 1½ ft. thick at $3.75 a perch.

*9.* What will it cost to construct a retaining wall 90 feet long, 28 in. thick, and of an average height of 7 ft., at $2.70 a perch for material and 60 cents a perch for labor? At $3.60 a cubic yard for both?

*10.* At $3.25 a perch for the material and $1.30 a perch for the labor, what will it cost to build a stone house 60 ft. long, 42 ft. 6 in. wide, 28 ft. high, the walls being 16 inches thick, allowing 524 cu. ft. for doors and windows?

*11.* A stone wall is 65 ft. long, 2 ft. thick and 16½ ft. high. How many perches does it contain? How many cubic yards?

*12.* The foundation of a house is to be 40 ft. long and 24 ft. wide. If built 20 inches thick and 7½ ft. high, what will it cost at $3 a perch for the material (net measure) and $2.50 a perch for the mason work (outside measure), no allowance being made for openings?

*13.* Find the cost of digging a cellar 108 ft. long, 46 ft. wide and 10 ft. deep at 44c. a cubic yard.

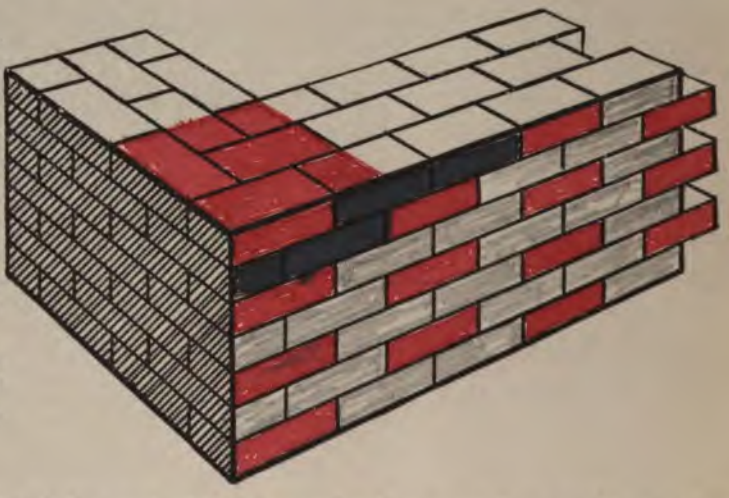
*14.* What will it cost to build a foundation wall in the cellar of the preceding problem, the wall to be 2½ ft. thick and 12 ft. high, at $3.75 a perch for the material, and $2.75 a perch for the labor, if in estimating the material full allowance is made for 2 openings each 6 ft. × 8 ft. and 10 openings each 3 ft. × 6 ft., and ⅔ of these openings is deducted in estimating the masonry?

*15.* A foundation wall 2 ft. thick and 8 ft. high for a building 60 ft. long and 28 ft. 6 in. wide contains 2 openings for doors, each 7 ft. by 4 ft., and 10 windows each 6 ft. by $3\frac{1}{2}$ ft. Find the cost of the material at \$3.20 a perch, allowing for all the openings; also find the cost of the labor at \$3.50 a perch, allowing for half the openings.

*16.* How many bricks 8 in. × 4 in. × 2 in. will be required for a wall 60 feet long, 24 feet high and three bricks thick?

NOTE.—The number of bricks required for a wall is usually found by calculating the number needed for a course and multiplying by the number of courses. An allowance is made for mortar, usually from $\frac{1}{8}$ of an inch to $\frac{1}{4}$ of an inch, to each dimension.

This allowance need not be considered in working these problems.



*17.* What would be the cost of bricks required in preceding problem at \$8.50 per M?

*18.* In a brick-yard there is a pile of common bricks, 72 feet 8 inches long, 32 feet 4 inches wide and 12 feet high. What are the bricks worth at \$8.50 per M?

*19.* How many common bricks (8 in. × 4 in. × 2 in.) are required for the walls of a house 48 ft. long, 30 feet 6 inches wide, 25 feet high and 2 feet thick, deducting 132 cubic feet for windows and doors?

*20.* How many bricks will be required to build a house, the walls of which are 50 ft. 8 in. long, 25 ft. 6 inches wide, 42 ft. high and 2 ft. thick, allowing 21 bricks to a cu. ft.? Allow for 4 doors 8 ft. high, 3 ft. 6 inches wide, and 10 windows 7 ft. long, 3 ft. wide. What will the bricks cost at \$8.50 per M?

*21.* A brick house measures on the outside 40 ft. 9 in. long, 34 feet wide and 32 ft. 6 in. high, the walls are 24 inches thick. Making an allowance for 12 windows 6 ft. 3 in. long, 3 ft. wide, and 6 doors 8 ft. long, 4 ft. wide, what would be the cost of the bricks at \$8 per M, allowing 21 bricks to the cu. ft.?

*22.* How many perches of stone in three bridge piers, each 24 ft. long, 40 ft. high, 12 ft. thick at the base and 5 ft. thick at the top? How many cubic yards?

## LUMBER

**458.** An amount of lumber is usually given in board feet, except in the case of a few rare hardwoods, such as black ebony, lignum vitæ, German black walnut and vermilion, which are sold by the pound; and mouldings, which are sold by the running foot at so much per hundred.

**459. A board foot** is a **square foot** of board **one inch thick** or **less.**

NOTE.—That is, in lumber dealers' usage, a foot of lumber is (or is the equivalent of) a board twelve inches long, twelve inches wide, and one inch thick or less—surface alone being considered; but if the board be more than one inch thick, a square foot of it makes proportionately more than a board foot. Thus if 1¼, 1½, or 2 inches thick, a square foot of board equals one and one-fourth, one and one-half, or two board feet; and so on with greater thicknesses.

**460. A moulding-foot** is 1 inch wide, 1 inch thick and 1 foot long. Mouldings are estimated by the size of the pieces from which they are cut, as shown by the thick edge and back; that is, if the moulding is made from 2 in.×3 in. lumber, every foot in length is counted 6 moulding-feet.

**461. To find the amount of lumber.**

EXAMPLE.—How many feet of lumber (board feet) in 26 pieces, 3 inches thick, 4 inches wide, 12 feet long?

NOTE.—In a lumber bill this is written 26 pc., 3 × 4 — 12.

$$\frac{12 \times 4 \times 3 \times 26}{12} = 312 \text{ feet}$$

**Formula** (when width is in inches):

$$\frac{\text{Length} \times \text{Width} \times \text{Thickness} \times \text{Pieces}}{12} = \text{board feet}$$

NOTE.—The operation may frequently be shortened by cancelation.

### WRITTEN EXERCISE

**462.** *1.* How many board feet in 200 inch boards 10 in. wide, 16 feet long?

*2.* How many board feet in 150 2-inch planks, 9 inches wide, 18 feet long?

*3.* What amount of lumber in 80 joists 3×7—20?

*4.* How many feet in 140 scantlings 3×4—18?

*5.* How much lumber in 36 sills 6×8—30?

*6.* What amount of lumber in 36 plates 4×6—30?

*7.* Find the amount of lumber in 180 pieces 2×4—16.

*8.* Find the amount of lumber in 225 planks 2×9—14.

*9.* How many feet in 75 rafters 3×5—22?

*10.* How many feet in 24 girders 3×10—28?

*11.* Find amount of lumber in following order:

| | Solution | |
|---|---|---|
| 18 pc. 3×7—20 | 18 pc. 3×7—20 | 630 |
| 36 " 3×4—18 | 36 " 3×4—18 | 648 |
| 50 " 2×10—12 | 50 " 2×10—12 | 1000 |
| | | 2278 |

*12.* Find the amount of lumber in the following bill:

40 pc. 3×4—20
12 " 4×6—30
8 " 4×6—20
60 " 2×4—20
140 " 2×10—12

*13.* Find board feet in the following order:

50 pc. 10×12—36
120 " 6× 8—30
170 " 4× 6—20
2500 ft. 2 in. plank

*14.* What amount of lumber in the following bill?

65 pc. 10×12—44
150 " 6× 8—36
224 " 4× 6—24
880 " 3× 7—36
880 " 2× 6—36

*15.* What is the value of 280 pc. 3×4—16 @ $14 per M?

Solution

$$\frac{280\times\overset{1}{\cancel{3}}\times\overset{1}{\cancel{4}}\times 16}{\underset{1}{\cancel{12}}} = 4480 \text{ ft.}$$

4.480
14
17920
4480
Result, $62.720

*16.* What is the cost of the following bill of hemlock at $27.50 per M?

60 pc. 4× 6—24
75 " 3× 4—18
120 " 2×10—12

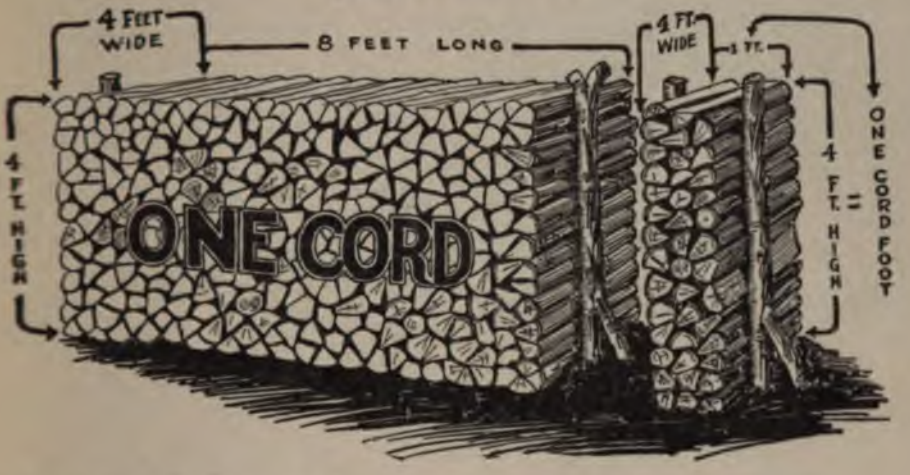
*17.* What is the value of the following pine lumber at $55.50 per M?

20 pc. 14×20—45
62 " 12×16—40

*18.* What is the cost of the following bill of oak at $56 per M?

50 pc. 4×6—20
50 " 3×4—10
100 " 2×6—10
1200 ft. boards

## WOOD MEASURE




**463.** The **cord** is the unit of measure for rough wood.

**464.** A **cord of wood** is a pile 8 feet long, 4 feet high and each stick 4 feet long. It contains 128 cubic feet.

**465.** A **cord foot** is a portion of the cord 1 foot long.

**466.** RULE.—*Multiply together the length, height, and width (each expressed in feet) and divide by 128 to get cords.*

### WRITTEN PROBLEMS

**467.** *1.* A pile of wood is 84 ft. long, 12 ft. high, and the sticks are 6 ft. long. What is it worth at $3.25 a cord?

*2.* A train of 18 cars, each 44 ft. long, 7½ ft. wide, is piled with wood to a height of 7 ft. Of how many cords does the load consist?

*3.* A shed 32 ft. long and 14 ft. wide is high enough to hold 35 cords. How much will it cost to paint the outside at 4½ cents a square foot?

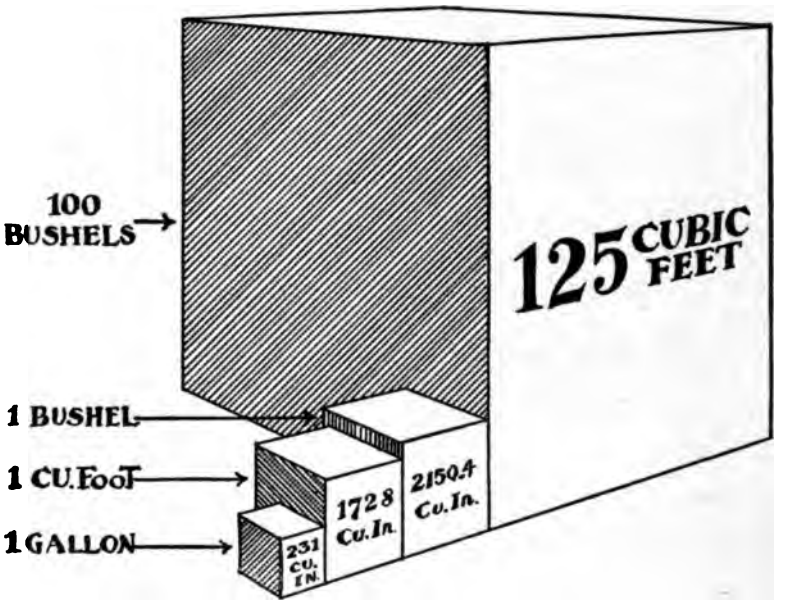
*4.* A roof projecting 18 in. on all sides, covers a pile of wood 27 ft. wide. The boards for the roof cost $47.56½ at $22.65 per M. What is the value of the wood piled under it to a height of 15 ft. 8 in., at $3.78 a cord?

*5.* A pile of wood 25 ft. wide, 64 ft. long and 15 ft. high contains how many cords?

*6.* What is the value of a pile of wood 30 ft. long, 8 ft. wide and 7½ ft. high, at $4.50 a cord?

*7.* From a pile of pulp-wood 84 ft. long, 8 ft. wide and 12 ft. high, 48 cords were sold. What was the length of the pile remaining?

## CAPACITIES




**468. To find the capacity of boxes, bins, or tanks, in gallons or bushels.**

**469.** RULE.—*Find contents in cubic inches and divide by number of cubic inches in a gallon (Liquid 231) to get gallons; by 2150.4 cubic inches to get stricken bushels, or by 2747.7 cubic inches to get heaped bushels.*

NOTE.—See Mensuration for fuller discussion of surfaces and solids.

### WRITTEN PROBLEMS

**470.** *1.* A bin 7 ft. 6 in. long, 4 ft. 9 in. wide and 5 ft. 6 in. deep is filled with wheat. What is it worth at 97½ cents a bushel?

*2.* A rectangular tank is 8 ft. long, 5 ft. wide and 3 ft. 4 in. deep. How many gallons of water will it hold? How many barrels?

*3.* If the tank in the preceding problem were converted into a bin, how many bushels of wheat would it hold? How many bushels of potatoes?

*4.* How many bushels of wheat will a bin 6 feet square and 4 feet deep contain?

*5.* What is the capacity, in barrels, of a tank 8 feet square, 10 feet deep?

*6.* A farmer has a bin 24 ft. long, 8 ft. 4 in. wide and 9 ft. high. How many bushels of corn in the ear will it hold? How many bushels of wheat in it when it is filled to within 18 inches of the top?

*7.* A bin 15 ft. long, 8 ft. 4 in. wide and 6 ft. 3 in. deep was filled with apples, from which enough cider was pressed to fill a tank 9 ft. 4 in. long, 4 ft. 6 in. wide and 3 ft. deep. If the apples were bought at 37½ cents a bushel and the cider was sold at 33⅓ cents a gallon, how much was gained, if the charge for making the cider was 75 cents a barrel?

## GAUGING

**471. Gauging** is a process of finding the capacity or volume of tanks, casks, barrels, etc.

**472. To find the capacity, in gallons, of a round tank of uniform diameter.**

**473.** RULE.—*Multiply the square of the diameter in inches by the height or depth of the tank in inches, and this product by .0034.*

### WRITTEN PROBLEMS

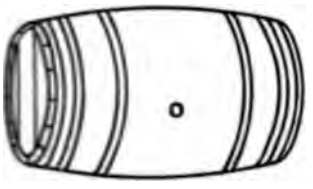
**474.** *1.* What is the capacity of a tank 8 feet 4 inches in diameter and 12 feet 6 inches deep?

*2.* Find the capacity of a stand-pipe 10 feet in diameter and 180 feet high.

*3.* An oil tank 5 ft. 8 in. in diameter and 24 ft. 2 in. long (inside measurement) is filled with oil. What is it worth at 6½ cents a gallon?

*4.* A well 3 ft. 4 in. in diameter has 12 feet of water in it. How many barrels of water are there in the well?

**475. To find the capacity of a cask or a barrel in gallons.**



A cask is equivalent to a cylinder, having the same length and a diameter equal to the mean diameter of the cask.

**476.** RULE.—*To find the mean diameter of a barrel or cask, add to the head diameter ⅔ of the difference between the head and the bung diameters, expressed in inches; or, if the staves are but little curved, ⅗.*

*Multiply the square of the mean diameter by the length (both expressed in inches), and this product by .0034; the result will be the capacity in gallons.*

### WRITTEN PROBLEMS

**477.** *1.* How many gallons in a cask whose head diameter is 24 inches, bung diameter 30 inches and its length 34 inches?

*2.* What are the contents of a cask 3 feet 6 in. long, the head diameter being 26 inches and the bung diameter 30 inches?

*3.* A merchant received ten casks of New Orleans molasses, having the following dimensions: head diameter, 30 inches; bung diameter, 38 inches and length, 42 inches, at an average cost of 38 cents per gallon. He retailed the same at 12 cents per quart. What was his gain on the entire lot?

*4.* How much did a farmer realize from the sale of 7 barrels of vinegar at 12½ cents a gallon, if the staves were slightly curved, the head diameter being 2 ft., the bung diameter 2 ft. 3 in. and the length 2 ft. 10 in.?

*5.* The bung diameter of a hogshead is 45 inches, its head diameter 40 inches and its length 55 inches; what is its capacity?

*6.* How many gallons in a cask of slight curvature, 3 ft. 6 in. long, the head diameter being 26 in. and the bung diameter 31 in.?

## GENERAL REVIEW PROBLEMS

**478.** *1.* How many yards will it take to make a rug 4 yds. wide and 9 yds. long, including a border 18 inches wide, if the strips in the body of the rug are made of 27-inch Brussels and 9 in. are lost on each strip in matching?

*2.* What weight of water is contained in a tank 14 in. × 9½ in. × 8¼ in., a 6-inch cube of water weighing 125 oz.?

*3.* Find the cost of slating a board 23 ft. 6 in. long and 2 ft. 8 in. wide at 9 cents per square foot.

*4.* Divide ⅓ of 25.08⅓ miles by $\frac{1}{9}$ and change to proper denominations.

*5.* Find the value in American money of express money orders amounting to £17 9s. 6d. (Exchange $4.8665.)

*6.* What is the total cost of 24 tons (2240 lbs.) of wheat at 90 cents per bushel and 140 centals of wheat at 87½ cents per bushel?

*7.* What is the depth of a tank 7 feet long and 4 feet wide that will hold 42 barrels?

*8.* What is the weight of shelled corn that will exactly fill a bin 8 ft. long, 5 ft. wide, 6 ft. deep?

*9.* How many Philadelphia bricks (8¼ × 4⅛ × 2⅜) in a pile 80 feet × 6 feet × 12 feet? What is their value at $8 per M?

*10.* A tank 14 feet long, 3 feet 8 inches wide and 11 feet deep is filled with petroleum. What is its value at 15 cents per gallon?

*11.* If 48 gallons of wine and 12 gallons of water are mixed, how much wine is there in 4 qts. 1 pt. of the mixture?

*12.* If a miller takes $\frac{1}{8}$ for toll, and a bushel of wheat produces 40 pounds of flour, how many bushels must be taken to mill to obtain 10 barrels of flour?

*13.* What will a tract of land 270 rods long and 495 yards wide cost at $62½ per acre?

*14.* It requires 24 men 30 days to do a piece of work. After they have worked 8 days, 6 men leave. In what time can the entire work be completed?

*15.* The sum of two numbers is 367 and their difference is 11. What are the numbers?

*16.* What will the following lumber cost at $56.25 per M?

75 pc. 5×7—28
75 " 3×7—14
150 " 2×8—14
300 " 1×4—14

*17.* A man wishing to build a tank in his barn found that he could not safely place there a weight of more than 2½ tons of water. How long may he make a tank 4 feet wide and 4 feet deep, water weighing 62½ pounds per cubic foot?

*18.* What will it cost to pump the water out of a flooded cellar, 30 feet by 18 feet, the water being 4½ feet deep, at 8 cents a hogshead?

*19.* How many bricks 8 in. × 4 in. × 2 in. will be required to pave a street 1½ miles long and 30 feet wide, if the bricks are laid edgewise?

*20.* If $\frac{3}{4}$ of A's money equals $\frac{4}{5}$ of B's, and $\frac{2}{3}$ of B's equals $\frac{5}{8}$ of C's, and together they have $23300, how much money has each?

*21.* What is the cost of 550 inch boards, each 18 feet long and 4 inches wide, at $18$\frac{1}{2}$ per M?

*22.* A farmer sold 45 bushels of potatoes and 55 bushels of rye for $55.50, receiving 10 cents a bushel more for the rye than for the potatoes. What was the price of each per bushel?

*23.* Find the cost of carpeting two rooms, each 21 feet by 18 feet, with Brussels carpet $\frac{3}{4}$ yard wide at $1.12$\frac{1}{2}$ per yard, carpet to be laid lengthwise and 1 foot allowed to each strip for waste.

*24.* Divide .06$\frac{3}{4}$ acres by 1.66$\frac{2}{3}$, and multiply the quotient by 9.3$\frac{1}{2}$.

*25.* A grain elevator is 40 feet long, 18 feet wide and 25 feet deep; it is $\frac{7}{8}$ full of wheat. How many bushels are in it? What is the weight of the wheat in it?

*26.* If a bin would hold 1000 bushels of grain, how much water would it contain?

*27.* What is gained or lost by buying 15 bushels of chestnuts at $5 per bushel, dry measure, and selling them at 25 cents per quart, liquid measure?

*28.* Bought in Paris, 325.5 meters of silk at 15.60 francs per meter. Find cost in United States money.

*29.* If the driving wheels of a locomotive are 10 feet 6 inches in circumference, and average 8 revolutions per second, how long will it require to go 76 mi. 108 rd. 3 yd.?

*30.* A rectangular field 140 rods long produces 1764 bushels of wheat at the rate of 32 bushels per acre. Find its width.

*31.* At 25 cents per square foot, find the cost of a walk 3 feet wide around a grass plat 81 feet long and 60 feet wide; also the cost of a drive 10 feet wide around the walk at the same price per square foot.

*32.* Bought 36500 tons of iron at £14 10s. 6d. per ton. What is its value in United States money? At what price should it be sold per ton in United States money to gain $2$\frac{1}{2}$ per ton, and allow $2$\frac{1}{2}$ per ton duty?

*33.* A bought 320 long tons of hay at $15 per ton and sold it at $16.25 per short ton. Find his gain.

*34.* A rectangular iron tank, including the top, is made of 2-inch plates. If the inside dimensions of the tank are 8 feet long, 5 feet wide and 10 feet deep, how many cubic feet of iron in the material?

*35.* Find the weight of the tank in the preceding problem, when filled with water, if iron is 7 times heavier than water.

*36.* How many feet of siding would be required for a house 37 feet 6 inches long, 25 feet wide and 18 feet high, with two gables each 12 feet high, adding $\frac{1}{8}$ for lap and waste?

*37.* How many perches of stone in a wall 396 feet long, 4$\frac{1}{2}$ feet thick and 6.3 feet high? What will it cost to build the wall at $3.25 per perch?

*38.* When wheat is quoted at $1.25 per cental, what are 250 bushels worth?

*39.* 45 men agreed to do a piece of work, but 25 of them did not come and the work was prolonged 5 days. In what time could the 45 have done the work?

*40.* What should be paid for a piece of cloth $35\frac{1}{3}$ yards long and $1\frac{1}{2}$ yards wide, if \$52.50 is paid for a piece of the same quality $15\frac{3}{4}$ yards long and $1\frac{1}{3}$ yards wide?

*41.* Find the cost of painting the walls, floor and ceiling of a room 32 feet by 21 feet and 15 feet 6 inches high at $19\frac{1}{2}$ cents per square yard.

*42.* Find the cost of the following, hemlock being worth \$40.50 per M, and pine \$24.50 per M:

80 pc. Hemlock 6×8—32
160 " " 4×6—24
240 " " 2×10—24
12000 ft. Pine boards

*43.* What will 18 two-inch planks cost at $\$13\frac{1}{2}$ per M, each 14 feet long, 10 inches wide at one end, 8 inches at the other?

*44.* A grocer purchased eggs at 20 cents per dozen and sold them at the rate of 6 for 19 cents. How many must he sell to gain \$1.20?

*45.* A pharmacist bought 210 pounds of drugs at 72 cents a pound, avoirdupois weight, and sold them at 72 cents a pound, apothecaries' weight. Find his gain.

*46.* Find the cost of a lot 176 feet front and 88 feet deep, at \$375.50 per acre.

*47.* A bin 15 feet long, 6 feet wide and 9 feet deep, is $\frac{3}{4}$ full of wheat. What is the value at 90 cents per bushel?

*48.* How many board feet in a board 26 feet long, 15 inches wide and $\frac{1}{2}$ inch thick?

*49.* What will be the cost of 9.37125 miles of insulated wire at $3\frac{1}{4}$ cents a foot?

*50.* If a contractor receives \$7425 for grading a roadbed $1\frac{1}{4}$ miles long, what should he get for 325.9 feet?

*51.* A man bought 12 acres of land at \$325 per acre and laid it out in town lots 1 chain 50 links long by $1\frac{1}{2}$ chains deep. He sold the lots at \$825 each. What was his gain?

*52.* Certain city lots are 24 feet by 100 feet. How many such lots in an acre?

*53.* At $2\frac{1}{4}$ cents a square inch, what will it cost to bronze a cube the edge of which is $3\frac{1}{2}$ feet?

*54.* A farmer sowed 3 bu. 1 pk. 1 qt. of seed and harvested 87 bu. 2 pk. and 3 qt. How many bushels did he harvest from a bushel of seed?

*55.* What would it cost, at 16 cents per square yard, to paint the outside of a house 56 feet by 24 feet by 23 feet 6 inches in height?

*56.* A can plow $\frac{3}{7}$ of a field in 3 days; B $\frac{3}{4}$ of it in 6 days. How long will it require A and B working together to plow it?

*57.* How many meters in a mile? How many yards in 565.25 meters?

*58.* If a five-cent piece is $\frac{1}{50}$ of a meter in diameter, how many of them will extend a yard?

52. How many rods of fence will be required to fence a railway $\frac{1}{2}$ of a mile long? How many posts each 6 feet apart? How many feet of wire, if the fence is 5 wires high?

53. Reduce $\frac{1}{2}$ of an ounce avoirdupois to the decimal of a ton.

54. If a lady weighs 125 pounds avoirdupois, what is her weight Troy?

55. The height of a flight of stairs is 18 feet. How many steps, each 8 inches high? How many yards of carpet will be required if the tread is ten inches, allowing 1 yard for waste?

56. Divide .0078 of a mile by 2.5, and multiply the quotient by $63\frac{1}{2}$. Change the results to proper denominations.

57. How many boards, each 18 feet long and 6 inches wide, will be required to make a cubical box 6 feet long?

58. Find the cost of the masonry of a building at $7.25 per perch, the dimensions of which are 31 feet 6 inches by 21 feet 6 inches and 7 feet high, the walls being 18 inches thick.

59. What will be the thickness of a piece of timber 30 feet long, 10 inches wide, to contain 240 board feet?

60. If $\frac{1}{4}$ of $\frac{4}{5}$ will buy $3\frac{1}{2}$ dozen oranges how many dozens can you buy for $\frac{83\frac{1}{2}}{6}$?

61. What will it cost, at $16 per M, to erect a tight board fence 6 feet high around a lot 60 yards long and 76 feet wide?

62. Multiply 193.875 miles by .063, and divide the result by .055.

63. Find the cost at $60 per acre of a field 15 chains 75 links long and 9 chains 25 links wide.

64. A paper manufacturer paid $357 at $8.50 a cord for a pile of pulp-wood 32 feet long, $5\frac{1}{2}$ feet wide. How high was it?

65. If 8 men or 12 boys can do a piece of work in 10 days, how long will it require 6 men and 15 boys to do the work?

66. If a blackboard 21 feet long contains 9 sq. yd. 6 sq. ft., how wide is it?

67. Add $0\frac{1}{2}$ sq. yd., $63\frac{1}{2}$ sq. ft. and 18 sq. in. and multiply the sum by $12\frac{1}{2}$.

68. Which will be the cheaper and how much, to lay carpet 27 inches wide lengthwise or crosswise in a room 20 feet long and 21 feet wide, at $1.25 per yard, allowing one foot to each strip for waste?

69. A ton 9 feet long, 5 feet wide contains 360 bricks. Allowing $1\frac{1}{2}$ cu. ft. to a hundred, how deep is it?

70. Allowing 21 bricks to a cubic foot, how many bricks would be required for the main walls of a building 40 feet long, 28 feet 9 in. less wide, 24 feet 6 in. less high and 20 inches thick? What would be the cost of the bricks at $10.50 per M?

71. Divide 155 inches by .0625, and change to higher denominations.

72. If 8 barrels of potatoes and 9 barrels of apples are worth $30, and 10 barrels of potatoes and 12 barrels of apples are worth $40, what is the price of one barrel of each?

*80.* How many bundles of lath will be required for the walls and ceiling of a room 28 feet long, 15 feet 6 inches wide and 10 feet high, each bundle being estimated to cover 5 sq. yd.?

*81.* A case of cranberries containing $3\frac{3}{16}$ bushels was bought at \$3 per bushel and retailed at 10 cents per quart, liquid measure. How much was gained thereby?

*82.* If $\frac{\frac{2}{3}}{\frac{3}{4}}$ of a property cost \$700, what should be given for $\frac{\frac{3}{4}}{\frac{2}{3}}$ of a property at the same rate?

*83.* Find the cost of

300 inch boards, 14 ft. long, 8 in. wide, at \$32.50 per M
50 joists 12 ft. long, 4 by 5 in. " 28.00 " "
60 planks, 20 ft. long, 10 by $2\frac{1}{2}$ in. " 9.50 " C

*84.* A grain dealer paid \$118 for 22 barrels of flour, giving \$6 for the first grade and \$4 for the second grade. How many barrels did he purchase of each?

*85.* At \$.56 a load or cubic yard, what will the excavation of a cellar cost, the dimensions of which are 29 feet 6 inches by 22 feet 8 inches and 6.3 feet deep?

*86.* How many centals of grain will a rectangular box 9 feet long, 4 feet 2 inches wide and 18 inches deep hold, allowing 60 pounds to a bushel? How many gallons of water?

*87.* From £17 8s. 6d. deduct .05 of itself.

*88.* How many sheets of tin 20 inches by 14 inches will cover a roof 60 feet long and 23 feet from eaves to ridge?

*89.* A railroad passes through $1\frac{1}{4}$ miles of A's farm. If the road is 60 feet wide, what is the cost of the right of way at \$66 per acre?

*90.* If 45 T. 15 cwt. of coal are worth \$222.25, what is the value of 23 T. 12 cwt.?

*91.* The floor of a pavilion 87 yards long and 26 feet wide is made of $2\frac{1}{2}$ inch oak. What is it worth at $\$37\frac{1}{2}$ per M?

*92.* Find weight in long tons of the ice $6\frac{1}{2}$ inches thick on a pond, the dimensions of which are 180 yards by 36 feet, if water expands $\frac{1}{10}$ of its volume in freezing and a cubic foot of water weighs 1000 ounces.

*93.* If 8 barrels of flour cost £15 8s. 6d., what will 5 barrels cost at the same rate? (Result in English and United States money.)

*94.* Find the cost, at 8 cents a pound, of lining with zinc a cistern (including the top) 9 feet 6 inches by 4 feet 3 inches by 9 feet, if for every 5 sq. ft. 12 pounds are required.

*95.* How many half-acre lots can be laid out in a tract of land containing 105 A. 30 sq. rd.?

*96.* At \$15 an acre, what is the cost of a tract of land $3\frac{1}{2}$ miles square?

*97.* How many feet of inch boards will be required to build a fence around a lot 35 rods long and 22 rods wide, if the fence is made 6 feet high? What will be the cost of the boards at \$12.50 per M?

## APPROXIMATE MEASUREMENTS

**479.** For practical purposes the following rules approximate accuracy. They will be found convenient for use by farmers and others in cases in which only approximately accurate results are required.

### GRAIN

**480. To find the number of stricken bushels of grain in a bin or box.**

Note. The relation of 2150.4 cu in. to 1.25 cu ft. is approximately as 5 to 4; cu ft. are multiplied by 4/5 to get cubical contents, the number of cu ft. diminished by $\frac{1}{5}$ will give an equivalent in bushels, and the bushels increased by $\frac{1}{4}$ will give an equivalent in cu ft.

**481.** Rule. *Multiply together the three dimensions expressed in feet, and three fourths of that by* $\frac{4}{5}$ *or* .8.

Note 1. For greater accuracy add $\frac{1}{50}$ (.02) for every 100 cu ft.

Note 2. If the bin or box is only partly filled use the depth of the grain for the third dimension.

**482. To find the number of heaped bushels in a box or bin.**

**483.** Rule. *Find the contents in cubic feet and multiply by* .8.

**484. To find the number of bushels of well-seasoned corn in the ear, in a bin or crib.**

**485.** Rule. *Multiply the cubic feet by* .4, *and divide by* 3 *if the corn be of good quality, by* 4 *if of inferior quality; or simply multiply the cubic feet by* .3.

Note 1. The above rule is based on the fact that 3 cu ft. of good corn in the ear = 2 cu ft. of shelled corn = 1 bushel (1.244 cu ft.) of shelled corn, etc.

Note 2. If the crib be a flaring one, wider at the top than at the bottom, take for the third dimension the width at half the height to which it is filled with corn, the corn being heaped for measuring.

### HAY

**486.** Owing to the variety of conditions affecting the kind it is difficult to make an accurate estimate of the quantity of hay in a given space or stack. Under ordinary conditions, however, the following facts will give a fair approximate. A ton of hay compressed in a bale or loft is 500 cu ft.; in a common covered hay barn or in a low stack, 600 cu ft.; if timothy hay, in mow compressed with grain or in bottle of large stack, 1200 cu ft.; in well-settled large stack or mow, 450 cu ft.; if clover hay, 540 cu ft.

**487.** Rule. *Find contents in cu ft. and divide by, the number of cubic feet in ton of hay required.*

### COAL

**488.** Lehigh white ash coal, egg size, 40 cu. feet = a ton. Schuylkill white ash coal, 35 cubic feet, red ash stove coal, 36 cu. feet.

**489.** Rule. *Find contents in cubic feet and divide by the number of cubic feet in a ton.*

## LIQUIDS

**490. To find approximately the capacity in gallons or barrels of a rectangular tank or cistern.**

**491.** RULE.—*Multiply the contents in cubic feet by 7½ for gallons, or divide the cubic feet by 4⅕ (4.2) for barrels.*

NOTE.—For greater accuracy in gallons, deduct 1 gallon for every 50 cubic feet.

**492. To find the approximate capacity of a round tank, well, cistern, etc., the dimensions of which are given in feet.**

**493.** RULE.—*Multiply the square of the diameter in feet by the depth in feet and this product by 5⅞. The result will be the capacity in gallons.*

NOTE.—In a flaring or sloping tank (one smaller at the top than at the bottom), the average or mean diameter is, approximately, ½ the sum of the top and bottom diameters.

## BRICKS AND SHINGLES, OR SLATES

**494. To find the approximate number of bricks required for a wall.**

NOTE.—22 common bricks laid in mortar equal 1 cu. ft.

**495.** RULE.—*Multiply the number of cubic feet in the wall by 22.*

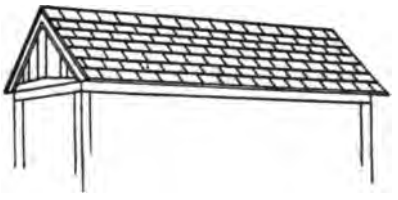
NOTE 1.—The dimensions of common bricks are 8 in. × 4 in. × 2 in.; of Philadelphia and Baltimore bricks, 8¼ × 4⅛ × 2⅝ in.; of North River bricks, 8 × 3½ × 2½ in.; of Milwaukee bricks, 8½ × 4⅛ × 2⅝ in.; and of Maine bricks, 7½ × 3⅝ × 2⅝ in.

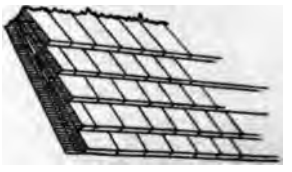
NOTE 2.—To find the number of fancy bricks for outside courses of walls, multiply the square feet of surface by the number of bricks required for 1 sq. ft. of surface, which number is obtained by dividing 144 by the exposed surface of 1 brick, making proper allowance for mortar. The number of fancy bricks multiplied by the number of bricks in thickness back of them will give the approximate number of common bricks required for the rest of the wall.

**496. To find the approximate number of shingles or slates for a roof.**

NOTE 1.—If laid 5 inches to the weather it requires 7⅕ shingles to make a square foot of roof, or 720 to the square of 100 sq. ft.; if laid 4 inches to the weather, it requires 8 to the square foot, or 800 to the square. To make proper allowance for waste, builders usually estimate 800 to 1000 shingles (or about 4 bundles) to every 100 square feet of roof.

NOTE 2.—Roofing-slate is sold by the square (100 sq. ft.), the price depending on the color and the quality, as well as on the number of pieces required. Sizes vary from 6″×12″ to 14″×24″—the length always being in even inches, but the width being in either even or odd inches. The number of pieces to the square depends, of course, on the exposure, which dealers usually reckon as ⅓ of the remainder after deducting 3 from the length of the slate. Thus the exposure of a slate 18 in. long is $7\frac{1}{2}$ in. $\left(\frac{18-3}{2}\right)$.





## LOGS

**497.** In estimating the amount of lumber in square-edged inch boards that can be sawed from a log, lumbermen and others make use of the following rules:

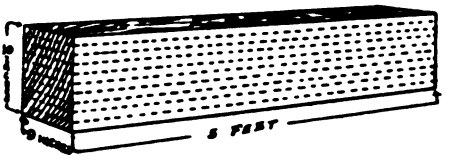
**498.** DOYLE'S RULE.—*From the diameter in inches, subtract 4; the square of the remainder will be the number of board feet yielded by a log 16 feet long. Or, from the diameter in inches, subtract 4, square ¼ of the remainder and multiply the product by the length in feet.*

**499.** ROPP'S RULE.—*From the square of the diameter in inches, subtract 60, multiply the remainder by ¼ the length in feet, and point off the right-hand figure of the product.*

**500.** TWO-THIRDS' RULE.—*From the diameter deduct ⅓ for saw kerf (cut) and slab; square the remainder, multiply by the length of the log, and divide this product by 12.*

**501.** THREE-FOURTHS' RULE.—*Same as preceding rule, except that ¼ is deducted for saw kerf and slab.*

EXAMPLE.—How many square feet of inch boards can be cut from a log 24 in. in diameter and 20 ft. long?




### SOLUTIONS

| DOYLE'S RULE | ROPP'S RULE | TWO-THIRDS' RULE | THREE-FOURTHS' RULE |
|---|---|---|---|
| | | 24 − 8 = 16 | 24 − 6 = 18 |
| 24 − 4 = 20 | 24 × 24 = 576 | 16 × 16 = 256 | 18 × 18 = 324 |
| ¼ of 20 = 5 | 576 − 60 = 516 | 256 × 20 = 5120 | 324 × 20 = 6480 |
| 5 × 5 × 20 = 500 | 516 × 10 = 516.0 | 5120 ÷ 12 = 426⅔ | 6480 ÷ 12 = 540 |
| Result, 500 ft. | Result, 516 ft. | Result, 426 ft. | Result, 540 ft. |

NOTE 1.—The term "diameter" is understood to mean the diameter inside the bark at the top or smaller end of the log.

NOTE 2.—No rule can be relied upon for absolute accuracy under all conditions. Doyle's rule, though in general use, is too favorable to the buyer of small logs and to the seller of large ones. "The Woodman's Handbook," issued by the Bureau of Forestry, Department of Agriculture, Washington, D. C., gives a complete list of rules.

### WRITTEN PROBLEMS

**502.** *1.* How many bushels of wheat or other small grain can be stored in a granary 24 ft. 6 in. long, 7 ft. wide and 6 feet high?

*2.* How many bushels of apples, potatoes or turnips can be put into a bin 10 ft. 6 in. long, 4 ft. wide, and 3 feet 9 in. deep?

*3.* A box car 28 ft. 6 in. long and 8 ft. wide (inside measurement) is filled with lime to a depth of 4 ft. 3 in. How many bushels does it contain?

*4.* A corn crib 30 ft. long, 9 ft. high, 5 ft. wide at the bottom and 8 ft. wide at the top is filled with corn of good quality. How many bushels, approximately, are there in the crib, and what should be its weight when shelled?

*5.* How many tons, approximately, of clover hay are there in a stack 20 ft. long, 15 ft. wide and 12 ft. high, and what is the hay worth at $15 per ton?

*6.* How many tons of Lehigh stove coal can be put into a bin 14 ft. long, 10 ft. wide and 8 ft. deep? How many tons of Schuylkill white ash stove coal? How many tons of red ash stove coal?

*7.* A dealer has a bin filled with Schuylkill white ash stove coal that cost him $405, at $4.50 per ton. The bin is 25 ft. long and 15 ft. wide. What is the depth of the coal?

*8.* A farmer sold the timothy hay in a mow 30 ft. long, 18 ft. wide and 11 ft. high, at $10.25 a ton of 324 cu. ft. How much did he receive for it?

*9.* A contractor paid $150 for a stack of hay. It was 30 ft. long, 10 ft. wide and 12 ft. high. What was the price of the hay per ton, if 450 cubic feet were taken as the equivalent of a ton?

*10.* A rectangular tank 8 ft. long, $3\frac{1}{2}$ ft. wide and $2\frac{1}{2}$ ft. deep will contain how many gallons when filled?

*11.* A round tank 4 ft. 8 in. in diameter and 9 ft. long will hold how many gallons? How many barrels?

*12.* A well 3 ft. 6 in. in diameter and 30 ft. deep is filled with water to within 14 ft. of the top. How many hogsheads of water in it, approximately?

*13.* Find the capacity in barrels of a tank 9 feet 4 inches in diameter at the top, 11 feet 8 inches at the bottom and 18 feet 4 inches deep.

*14.* Find the contents in board feet of two logs each 18 in. in diameter at the top and 24 ft. long. (Results by different rules.)

*15.* How many bricks will it take to build a wall 7 ft. high, 17 inches (or 4 bricks) thick, on three sides of a garden 50 ft. square, allowing for a gate 6 feet wide?

*16.* How many fancy bricks, 8 in. $\times$ 2 in. on the face, will it take for the outside course of a building, 45 ft. long and 28 ft. wide, the walls being 21 ft. 6 in. high from the foundation to the cornice, deducting 4 doorways, each 5 ft. by 10 ft. and 16 windows, each $4\frac{1}{2}$ ft. by 8 ft.? What will they cost at $27.50 per M?

*17.* If the outside walls in the preceding problem are 5 bricks thick, and 18 in. all around the top of them is made entirely of common bricks, besides a total length of 150 ft. of 3-brick partition walls in the building, averaging 20 ft. high, and 80 ft. of 2-brick partitions 10 ft. high, how many common bricks will be required? What will they cost at $12.50 per M?

*18.* A farmer hauled to a sawmill 20 straight, smooth logs of the following dimensions: 2 logs 48 in. in diameter, 12 ft. long; 2 logs 36 in. diameter, 14 ft. long; 2 logs 30 in. in diameter, 16 ft. long; 3 logs 24 in. in diameter, 18 ft. long; 3 logs 18 in. in diameter, 20 ft. long; 2 logs 15 in. in diameter, 22 ft. long; 2 logs 12 in. in diameter, 24 ft. long; 4 logs 10 in. in diameter, 28 ft. long. What did it cost him to have these logs sawed into lumber at $6 per thousand board feet? (Results by different rules.)

# PERCENTAGE

**503. Percentage** is a method of computation by *hundredths*.

**504. Per cent.** is an abbreviation of the Latin phrase *per centum*, and means *by the hundred*. The sign for per cent. is %.

NOTE.—10% means 10 of every hundred, or ten hundredths ($\frac{10}{100}$).

**505.** There are three principal elements considered in percentage: the *Base*, the *Rate* and the *Percentage*.

**506.** The **Base** is the number of which a number of hundredths are taken.

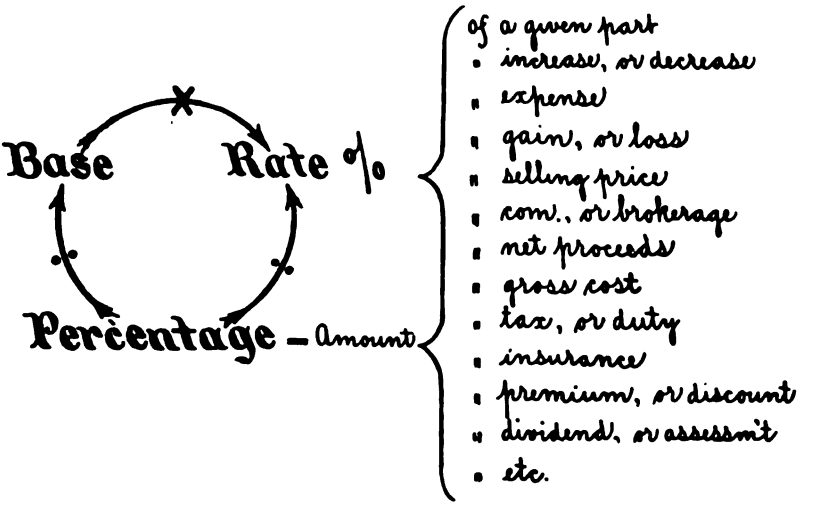
**507.** The **Rate** is the number of hundredths.

**508.** The **Percentage** is the result obtained by taking a certain number of hundredths of the base.

**509.** In computing percentage, it is frequently more convenient to use the equivalent common fraction than the decimal. The following equivalents should be memorized by the student:

| | | | | |
|---|---|---|---|---|
| 50 % = $\frac{1}{2}$ | $14\frac{2}{7}\%$ = $\frac{1}{7}$ | 10% = $\frac{1}{10}$ | $22\frac{1}{2}\%$ = $\frac{9}{40}$ | $33\frac{1}{3}\%$ = $\frac{1}{3}$ |
| 25 % = $\frac{1}{4}$ | 15 % = $\frac{3}{20}$ | 20% = $\frac{1}{5}$ | 35 % = $\frac{7}{20}$ | $66\frac{2}{3}\%$ = $\frac{2}{3}$ |
| $12\frac{1}{2}\%$ = $\frac{1}{8}$ | $7\frac{1}{2}\%$ = $\frac{3}{40}$ | 30% = $\frac{3}{10}$ | 45 % = $\frac{9}{20}$ | $16\frac{2}{3}\%$ = $\frac{1}{6}$ |
| 75 % = $\frac{3}{4}$ | $2\frac{1}{2}\%$ = $\frac{1}{40}$ | 40% = $\frac{2}{5}$ | $7\frac{1}{7}\%$ = $\frac{1}{14}$ | $83\frac{1}{3}\%$ = $\frac{5}{6}$ |
| $37\frac{1}{2}\%$ = $\frac{3}{8}$ | $13\frac{1}{3}\%$ = $\frac{2}{15}$ | 60% = $\frac{3}{5}$ | $22\frac{2}{9}\%$ = $\frac{2}{9}$ | $11\frac{1}{9}\%$ = $\frac{1}{9}$ |
| $62\frac{1}{2}\%$ = $\frac{5}{8}$ | $3\frac{1}{3}\%$ = $\frac{1}{30}$ | 70% = $\frac{7}{10}$ | $44\frac{4}{9}\%$ = $\frac{4}{9}$ | $9\frac{1}{11}\%$ = $\frac{1}{11}$ |
| $87\frac{1}{2}\%$ = $\frac{7}{8}$ | $18\frac{3}{4}\%$ = $\frac{3}{16}$ | 80% = $\frac{4}{5}$ | $28\frac{4}{7}\%$ = $\frac{2}{7}$ | $8\frac{1}{3}\%$ = $\frac{1}{12}$ |
| 5 % = $\frac{1}{20}$ | $17\frac{1}{2}\%$ = $\frac{7}{40}$ | 90% = $\frac{9}{10}$ | $6\frac{2}{3}\%$ = $\frac{1}{15}$ | $6\frac{1}{4}\%$ = $\frac{1}{16}$ |

GENERAL PERCENTAGE FORMULA



**510. To find the percentage, the base and rate being given.**

FORMULA.—Base × Rate (expressed decimally) = Percentage

EXAMPLE.—A has a farm containing 375 acres; 60% of it is in wheat. How many acres are in wheat?

375 Base
.60 Rate
225.00 Percentage

**511.** RULE.—*To find the percentage, multiply the base by the rate expressed decimally.* Or, *take the equivalent fractional part.*

### MENTAL PROBLEMS

**512.** *1.* A merchant having a stock of 75 barrels of flour sold 20 per cent. of them; how many barrels were left?

SOLUTION.—Sold 20 per cent. or $\frac{1}{5}$ of 75 barrels, or 15 barrels, hence there remain 75 minus 15 or 60 barrels.

*2.* What is the difference between 20 per cent. of $60 and 25 per cent. of $48?

*3.* On a house costing $5000, repairs to the amount of 5 per cent. are made yearly; what is the amount spent for repairs?

*4.* On an importation of watches invoiced at $40 each, a duty of 20 per cent. is charged; how much per watch is added to the cost for duty?

*5.* On a debt of $500, monthly instalments of 10 per cent. are promised until settled; how much must be paid each month?

*6.* There must be added for waste 12½% in matching carpet; how much must be added for a room requiring 80 yards?

*7.* I buy a bill of goods of $360, agreeing to pay in cash, 33⅓ per cent.; what is the amount of cash to pay?

*8.* I own one-half interest in a business enterprise and sell 33⅓ per cent. of my interest; what part of the business do I own after the sale?

*9.* If cloth, when sponged, shrinks 5 per cent., what will be the loss from this cause in sponging 40 yards?

*10.* An employer withholds 20 per cent. of the wages of his employees as a guarantee. What would be withheld from the wages of a person receiving $60 a month?

*11.* The duty on imported tobacco is 15 per cent.; what amount of duty would an importer have to pay on an invoice of $600?

*12.* A merchant reckons his yearly loss on furniture by depreciation at 12½ per cent.; if the last inventory showed $1200, what must the new one, taken at the end of the year, show?

### WRITTEN EXERCISE

**513.** *1.* Find 2% of 12694
*2.* 3% of $289.63
*3.* 7% of $2439.22
*4.* 14% of $7896
*5.* 19¾% of $4293.85
*6.* What is 117% of 32648?
*7.* 43% of $874.75?
*8.* 138% of $2550.80?
*9.* 77% of 7777?
*10.* 29% of $32560?

## WRITTEN PROBLEMS

**514.** *1.* A merchant pays his debts at the rate of 35 cents on the dollar; how much does a creditor receive whose account is $5270?

*2.* From a debt of $175.75, 15% was deducted for cash; what amount of cash was received?

*3.* A broker bought bonds whose face value was $5500 and received $\frac{1}{4}\%$ for buying; how much did he receive?

*4.* The marked price of an article of furniture was $300, but in a reduction sale the price was marked down $16\frac{2}{3}\%$; what was the reduced price?

*5.* A clerk was directed to mark goods that cost $50, 35% above cost; what was the marked price?

*6.* Goods marked 20% above their cost price of $10 were sold 5% below the marked price; what was received for them?

*7.* A merchant had marked his stock 40% above cost. He opened a reduction sale and marked down the goods 50%; what part of the cost price did he receive?

*8.* A, B and C each invest $1500 in business. If their gain during a business season is $22\frac{1}{2}\%$, what total amount of gain will they have to share?

*9.* A merchant bought a bill of goods amounting to $1980, for which he received a commission of $2\frac{1}{2}\%$; what was the amount of his commission?

*10.* A bill collector having bills amounting to $897.62 to collect, gets the money for 78% of the whole amount, and receives for his services 2% of the amount collected. How much does he receive?

*11.* In a town of 2748 inhabitants, $66\frac{2}{3}\%$ of the population are white and the remainder colored. How many colored persons are there in the town?

*12.* A mine produces 380 tons of metal; 70% of the metal is silver and 30% lead. Find number of pounds of each.

*13.* In 1875 the population of a certain town was 1864. In 1885 it had gained 25% over 1875; and in 1895 it had gained 20% over 1885. What was the population in 1895?

*14.* In 1895 a certain official's salary was $1800. In 1896 his salary was raised $11\frac{1}{9}\%$, and in 1897 it was raised 10%. What did his salary average for the three years?

*15.* A man having received a legacy of $42318 invested 50% of it in stocks and bonds, paid 25% of the remainder for a house and lot, loaned on a mortgage $33\frac{1}{3}\%$ of what he then had left, spent 20% of the balance, and then deposited the rest of his money in bank. How much did he deposit?

*16.* A grocer bought 75 sacks of salt at $2.40 per sack and 65 barrels of flour at 25% more per barrel than the price of the salt per sack; for which did he pay the greater amount, and how much?

*17.* A man owing a debt amounting to $186.20, paid 20% of the debt, and afterward 35% of what remained unpaid; how much does he still owe?

*18.* A farmer raised 90 bushels of potatoes per acre and had planted 45 acres. The next year he planted 20% more acres, but the yield was 10% less per acre. How many bushels did he raise the second year?

*19.* A farmer planted an orchard of 300 apple-trees, 25% more peach-trees than apple-trees, 20% fewer plum-trees than peach-trees, 40% fewer cherry-trees than apple and peach-trees together; how many trees in the orchard?

*20.* A merchant begins business with a capital of $16000; the first year he gains 20%, the second year he loses 25%, the third year he gains 2%, and the fourth year he gains 10%. If no money has been withdrawn, what is his capital at the end of the fourth year?

*21.* A's sales for a certain year are 20% less than B's; B's sales are 40% more than C's; C's sales are 25% less than D's. If D's sales are $6000, what are the sales of each of the others?

*22.* How many gallons of linseed oil can be extracted from 1270 pounds of flaxseed, if flaxseed contains 11% of oil and a pint of oil weighs $\frac{3}{4}$ of a pound?

*23.* 18% of a man's wealth is in real estate, 24% in bank stock, 26% in railroad bonds, and the remainder in money. What amount is invested in each, and what amount has he in money if his wealth amounts to $10288?

*24.* A, B and C are partners. A's investment is 30% more than B's, and C's investment is 40% less than A's. If B's investment is $3500, what is the capital of the firm?

*25.* Two railroad companies transported 5300 lbs. of freight at the through rate of 35 cents per cwt. If one company receives 45% of the through rate, how much should each receive?

**515. To find the base, the percentage and rate being given.**

FORMULA.—Percentage ÷ Rate (expressed decimally) = Base

EXAMPLE.—A's farm is worth $4050, which is 35% more than B's farm is worth. What is the value of B's farm?

EXPLANATION.—If A's farm is worth 35% more than B's, then 100% or B's + 35% of B's or 135% of B's = $4050 or A's farm. If 135% of B's is $4050, 100%, or B's, is $3000.

R.
1.35)4050.00 P.
3000 B.

**516.** RULE.—*To find the base, divide the percentage by the rate expressed decimally.*

NOTE.—It is frequently convenient to change the rate to its equivalent common fraction and solve by analysis.

**MENTAL PROBLEMS**

**517.** *1.* A wagon was sold for $90, which was 10 per cent. below its value; what would have been gained by selling it for $125?

SOLUTION.—10% equals $\frac{1}{10}$ below value; then $90 is $\frac{9}{10}$ of value; $\frac{1}{10}$ of the value is $\frac{1}{9}$ of $90 or $10, and the value is 10 × $10 or $100; therefore the gain would have been $125 − $100 or $25.

*2.* I received $300 or $16\frac{2}{3}$ per cent. of an amount owed to me; what was the amount of the debt?

*3.* A merchant marks goods at an advance of 20 per cent.; what is the cost of goods marked $360?

*4.* A man offers to pay $500 for a 25 per cent. interest in a business; what is the valuation of the business?

*5.* A man received a cash discount of $60 on a bill at the rate of 2%; what was the amount of the bill?

*6.* A man bought a table at a discount of 12½% from the marked price and saved $4; what was the price of the table?

*7.* Jones and Smith together own $24500, and Smith owns 16⅔% less than Jones; how much does each own?

*8.* A sold his horse for $7840, which was 16½% more than it cost. What did it cost?

*9.* A broker's commission, at ⅛%, amounted to $62.50; what was the amount of the transaction?

### WRITTEN EXERCISE

**518.** *1.* $425 is 5% of what?

*2.* 720 lbs. is 9% of what?

*3.* 740 is 7½% less than what number?

*4.* $817.32 is 6¼% of what amount?

*5.* What number increased by 12½% of itself equals 896?

*6.* $1252.16 is 8% more than what amount?

*7.* $32.87 is ⅜% of what sum?

*8.* What sum diminished by 3% of itself equals $1763.24?

*9.* $9378.25 is 87½% of how much?

*10.* 12 yds. is ⅛% of what distance?

*11.* 5 bu. 2 pk. is 11% of what quantity?

*12.* 12½% of an amount of money is $250; what is the amount?

*13.* If 12½% of the distance from Philadelphia to New York is 11¾ miles, what is the whole distance?

*14.* Through a reduction of 28¼% from list price, I gain $40 on a purchase. What is the list price?

### WRITTEN PROBLEMS

**519.** *1.* If 24% of a contract can be executed in 108 days, in what time can the whole contract be executed?

*2.* A man owing a bill pays 35% or $83.65. How much does he still owe?

*3.* A farm was sold for $18000, which was 15% less than the price asked for it; what was asked for it?

*4.* If I sell $30000 worth of property, what is left will be worth 85% of the value of the whole property; what is the value of the property I have left?

*5.* $1980 is 20% of the cost of B's farm; and the cost of B's farm is 90% of the cost of A's farm. How much more did A's farm cost than B's?

*6.* Brown has his property insured for 80% of its value. If his property is damaged 25% by fire, and the insurance company pays him $2340, being 25% of his policy, how much does he lose by the fire?

*7.* By cutting down his expenses 3½% a merchant saves $482.18. What were and what are his expenses?

*8.* A man owning ⅜ of a ship sells 27½% of his share for $680. What is the value of the whole vessel at that rate?

*9.* 17½% of Jones's stock of goods was destroyed by fire, and he received $2162.18 insurance, which was ¾ of the loss. What was his whole stock worth?

*10.* After deducting 3% for prompt payment, a merchant receives $1284.92 for a bill of goods. What was the whole amount of the bill?

*11.* A's sales for the year are 13% less than B's, and B's sales are 11¾% more than C's. A's sales amount to $6289.17. Find B's and C's sales.

*12.* Smith's share of a certain claim was 35%. After paying 5% for having his claim collected, Smith received $2182.93. What was the amount of the entire claim?

*13.* In a certain orchard there are 25% more cherry-trees than peach-trees, 20% fewer plum-trees than cherry-trees, 25% more pear-trees than plum-trees, and 20% more apple-trees than pear-trees. There are 60 apple-trees. How many trees are there in all?

*14.* A dealer marked his goods 30% above cost. The goods becoming damaged, he sold them for 20% less than the marked price; his customer failed and he lost 20% of his selling price. If he received $800, find his net loss.

*15.* A bought a house, paying $1610 cash and giving his note for the remainder, which was 12½% of the cost. What did the house cost, and what was the amount of the note?

*16.* In building a house, 45% of the cost was for brickwork, 25% for the carpenter work, 15% for the mason work, and the remainder, amounting to $3840, was for painting and plastering; what did the house cost, and what was the amount paid for each part of the work?

*17.* Of a cargo, 35% was sold to one buyer, 60% of the remainder to another, and the remainder, amounting to 1716 tons, to a third. How many tons in the cargo?

*18.* A, B and C formed a partnership. A invested $25200, which is 20% more than B invested, and 20% less than C invested. Find the capital of the firm.

*19.* A owns 15% of a business; B, 25%; C, 28%; and D, the remainder. What is the value of each one's share, if D's share is $34464?

*20.* A paid $3402 for a house, which was 20% less than it cost to build it; what did it cost to build the house?

*21.* Mr. Jones withdrew 30% of his deposit from bank, and invested 30% of what he withdrew in a house which cost $3000. What amount of money has he remaining in bank?

*22.* A bought 75 bbls. of flour, and B increased his stock 22% by buying 12% fewer than A. How many barrels has B now?

*23.* A merchant's sales increased 16⅔ per cent. the second year, 25% the third year, but decreased 10% the fourth year, and increased 22½% the fifth year; the total sales for the five years is $398562.75. What was the decrease the fourth year?

*24.* A wishing to sell a cow and a horse to B, asked 150% more for the horse than for the cow; he then reduced the price of the cow 25% and the price of the horse 33⅓%, at which price B took them, paying $580. What was the price paid for each?

**520. To find the rate, the percentage and base being given.**

FORMULA.—Percentage ÷ Base = Rate (expressed decimally)

EXAMPLE.—Bread made from 120 pounds of flour weighs 162 pounds. What per cent. more than the flour does the bread weigh?

EXPLANATION.—*More than* follows the words *per cent.* and precedes the *base* (the flour). Hence the flour, 120 pounds, is the base. Subtracting the weight of the flour from the weight of the bread to find how much more the bread weighs than the flour, gives 42 pounds as the percentage. Dividing the percentage 42 pounds by the base 120 pounds, carrying the quotient to hundredths, will give .35 or 35%.

162
120
1.20)42.00
.35 or 35%

**521.** RULE.—*To find the rate divide the percentage by the base.* Or, *find the equivalent fraction and change to a decimal.*

## MENTAL PROBLEMS

**522.** *1.* A boy having 50 cents, spent 20 cents; what per cent. remained?

SOLUTION.—There remained 30 cents or $\frac{3}{5}$ of the entire amount; $\frac{1}{5}$ of 100 per cent. equals 60 per cent.

*2.* I own property worth $5000, the yearly repairs of which cost me $150; what per cent. of the value is expended in repairs?

*3.* To close out a stock of goods a merchant offers $500 worth for $400; what is the per cent. of discount?

*4.* The marked price on chairs is $2 each, and they are sold at $1.75; what is the per cent. of discount?

*5.* On a debt of $750 a man paid $375; what per cent. of his debt did he pay?

*6.* A merchant buys a bill of goods for $400, and receives a discount of $20 for cash; what is the rate of cash discount?

*7.* An agent sells a consignment of goods for $1200; if his commission is $30, what is the rate of commission?

*8.* A stockholder of a bank, who owns $1200 worth of stock, is assessed $240; what is the rate of assessment?

*9.* A stockholder of a railroad company, owning $12000 worth of stock, received a dividend of $360; what is the rate of dividend?

*10.* In a roll of carpet of 40 yards, 4 yards are wasted in matching; what is the per cent. of waste?

*11.* Linoleum costing 60 cents a yard is sold for 54 cents a yard; what is the loss per cent.?

*12.* Overcoats marked at $25 have been reduced to $15; what is the per cent. of reduction?

*13.* A merchant who has a stock of $6000 gains $2000 per year upon his stock; what is his average rate of gain?

*14.* A house valued at $4000, rents for $500 a year; what per cent. of its value is received in rent?

### WRITTEN EXERCISE

**523.** *1.* \$309.68 is what % of \$13272?

*2.* 49 lb. 8 oz. is what % of 450 lbs.?

*3.* \$823 is what % more than \$712?

*4.* What % less than 1262 is 928?

*5.* 554⅜ lbs. is what % of 3 long tons?

*6.* 10 hr. 57 min. is what % of a year?

*7.* \$854.44 is what % of \$6252?

*8.* \$2906.86 is what % less than \$3620?

*9.* \$32899.40 is what % more than \$24390?

*10.* 19 yd. 2 ft. 4½ in. is what % of a mile?

### WRITTEN PROBLEMS

**524.** *1.* A baseball team lost 12 games out of 32. What per cent. did it win?

*2.* Jones pays \$152.32 for \$4760 insurance. What rate does he pay?

*3.* The population of a certain town was 2764 in 1880. In 1890 it was 3173. What was the per cent. of increase?

*4.* Imported 1230 yards of dress goods at \$1.32, and paid \$649.44 duty. What per cent. was the duty?

*5.* Of 435 crates of fruit, 60 crates spoiled. What per cent. must the price of the remainder be advanced to cover the loss?

*6.* If 12340 lbs. of an alloy containing 4% nickel is mixed with 4560 lbs. of another alloy containing 7⅜% nickel, what per cent. of nickel does the mixture contain?

*7.* If A receives a commission of 2¼% for selling goods, and B receives \$54.16⅔ more than A on each \$5000 worth sold, what is the rate of B's commission?

*8.* What per cent. more than \$3875.60 is 93⅓% of \$8972.10?

*9.* A house costs \$4000, and rents for \$25 per month; the expenses for the year are: water tax, \$15; city taxes, \$80; and repairs, \$25. What net yearly rate per cent. on the investment does the house pay?

*10.* A has \$6500, B has 28% more money than A. What per cent. less than B has A?

*11.* A father receives a yearly salary of \$1250 and his son a yearly salary of \$1000. What per cent. does the father get more than the son?

*12.* The list price of a piano is \$550. It can be bought for \$539. What per cent. is deducted from the list price?

*13.* A man's salary is \$2750 per year; he pays \$343.75 for rent and \$412.50 for other expenses. What per cent. of his salary does he save?

*14.* A jeweler bought a watch for \$160 and asked such a price for it that, after falling \$18, he made 20 per cent. What per cent. did he deduct from his asking price?

*15.* A house which costs \$9000 rents for \$50 per month. What is the net annual rate per cent. on the investment if the expenses are \$130 for taxes, \$25 for insurance, \$175 for ground rent and \$45 for repairs?

*16.* Bought 450 bushels of wheat at \$1.25 a bushel. If 10 bushels spoiled, and I sold the remainder at \$1.40 a bushel, what was my gain per cent.?

# PROFIT AND LOSS

**525.** Calculations in **Profit and Loss** are computations to determine the gain or loss resulting from buying merchandise at one price and selling at another.

**526.** The **cost** of goods is the amount paid to purchase them, or to produce them.

**527.** The **prime cost** of goods is the first or original cost.

**528.** The **gross cost** is the prime cost increased by all incidental expenses, such as freight, commission, insurance, duty, packing, drayage, etc.

**529.** The **selling price** of goods is the price at which they are sold.

**530.** The **net selling price** is the original selling price diminished by any deductions or allowances made to the customer.

**531. Profit** is the difference between cost and selling price, the selling price being *more* than the cost.

**532. Loss** is the difference between cost and selling price, the selling price being *less* than the cost.

**533. Gain** or **loss** in commercial transactions is reckoned as a certain *per cent. of the gross cost.*

NOTE.—Some businesses reckon the profit or loss on the selling price. In such cases the selling price is the base, and must be divided into the profit or loss to find the per cent. of profit or loss.

**534. To find the profit or loss when the cost and rate are given.**

EXAMPLE.—For how much must goods that cost \$34 be sold to realize a profit of 15%?

| | | | | | |
|---|---|---|---|---|---|
| \$34 | | 10% of \$34 | = | \$3.40 | \$34 cost |
| .15 | or | 5% of \$34 | = | 1.70 | 5.10 profit |
| \$5.10 | | 15% | | \$5.10 | \$39.10 selling price |

**535.** RULE.—*Multiply the cost by the rate expressed decimally.*

### MENTAL PROBLEMS

**536.** *1.* A desk costing \$60 was sold at 33⅓ per cent. profit; what was the amount of gain?

SOLUTION.—A gain of 33⅓ per cent. is a gain of ⅓ of the cost, or \$20.

*2.* A watch costing \$20 was sold at a gain of 20 per cent.; what was the selling price?

*3.* A horse costing \$80 was sold at a loss of 25 per cent.; what was the amount of loss?

*4.* Land costing \$70 an acre increases in value 14⅔ per cent.; what is it worth an acre?

*5.* A clock costing $36 is sold at a loss of $16\frac{2}{3}$ per cent.; what is the amount of loss?

*6.* Bought a horse for $100 and asked for him 20 per cent. above cost; what would I gain by accepting 10 per cent. less than I ask?

*7.* I buy flour at $5 a barrel and sell for 20 per cent. more; what do I get per barrel?

*8.* Silk bought at $2 a yard is marked 50 per cent. above cost and sold $33\frac{1}{3}$ per cent. below marked price; what is gained on a yard?

*9.* Lamps costing $5 each are marked 60 per cent. above cost and sold at a discount of 25 per cent. off marked price; what is gained on a lamp?

*10.* Cloth marked $6 a yard is bought at a discount of $16\frac{2}{3}$ per cent. and sold at a gain of 20 per cent.; what is the gain on a yard?

*11.* Tea costing 45 cents a pound is sold 20 per cent. above cost; what is the selling price of a pound?

*12.* Wheat quoted at 80 cents a bushel drops 5 per cent.; if I buy, and sell again when it reaches 80 cents, how much do I make on a bushel?

*13.* Bought coffee worth 32 cents a pound at a discount of $12\frac{1}{2}$ per cent.; what is saved on a pound?

*14.* A piano listed at $300 is sold at $33\frac{1}{3}$ per cent. below list; what is received for it?

*15.* A piano costing $200 to build is listed at $400 and sold 40 per cent. below list; what is the amount of gain?

### WRITTEN PROBLEMS

**537.** *1.* Fish bought for $640 were sold at an average gain of 24%; what was received for them?

*2.* A fur dealer marked down his stock 15%; what would be the reduced price of a set formerly marked $117.50?

*3.* The list price of a lot of goods is $960; if I buy at 10% off and sell at a profit of 30%, what do I gain on the goods?

*4.* A vessel cost its owner to build $21000, and was sold by him at a loss of 15%; the buyer sold it again at 15% above what he paid for it; how did the price he received for it compare with its original cost?

*5.* A merchant began business with a capital of $17500 and gained $16\frac{2}{3}$%, which he added to his capital; the second year he lost 2%; what is his capital at the end of the second year?

*6.* A man doing a business of $6500 a year finds his expenses are 35% of his business; what is his net business?

*7.* A man's inventory of furniture and fixtures at the beginning of his business year is $1550; at the end of the year he marks off $16\frac{2}{3}$% for wear and tear; what is the new inventory?

*8.* A man buys merchandise to the amount of $72500 and averages a profit of 15%; what is the amount of sales?

*9.* A company having a capital of $250000 has gained 4% on its capital and pays a dividend of 2½% on its capital; how much of the profit is undivided?

*10.* Goods that cost $375.84 were marked 30% more than cost and were sold for 15% less than the marked price. What was the net gain?

*11.* What must be the selling price of goods that cost $3642.70, to net a profit of 18%? To net 18% of the selling price?

*12.* A merchant bought goods for $4286.91 and sold them at a loss of 22½%. How much did he receive for the goods?

*13.* Sold goods that cost $237.28 at a profit of 25%, but my customer failed and paid only 75 cents on the dollar; did I gain or lose on the goods, and how much?

*14.* Bought three lots of goods for $2780.14, $3628.32 and $4245.79 respectively. The first I sold at a profit of 12½%, the second at a loss of 37½% and the third at a profit of 30%. What was my gain or loss on the whole?

*15.* A hardware dealer bought 6 gross of locks at $4.80 a dozen and marked them 40% above cost. If he sells them at 12% less than the marked price, what will be his profit on the lot?

*16.* A bought a house and a lot for $6500 and sold them to B at a loss of 12½%; B sold them for 22¼% more than he paid for them; how much did he receive?

*17.* A dry goods merchant bought 348 yards of cloth for $131.37 and sold ⅓ of it at a profit of 11¼ cents a yard; he then sold the remainder at such a price as to gain 35% on the whole. At what price per yard did he sell the remainder?

*18.* A bought goods for $2138.75 and sold them to B at a profit of 18¾%; B sold them to C at a profit of 14⅔%; C sold them to D at a loss of 20⅛% and D sold them at a profit of 8%. How much did D receive for them?

*19.* A cargo of wheat weighing 632897 lbs. was bought for 68¾ cents a bushel. One-third of it was sold at a profit of 8%, three-eighths of the remainder at a profit of 12¼% and the balance at a loss of 3⅓%. The proceeds were invested in coffee, which was sold at a profit of 18¾%. What was the entire gain?

*20.* Jones bought a house for $3285 and sold it to Smith at a profit of 9⅛%. Smith spent $394.28 for repairs and then sold the house at a profit of 15%. How much did he gain?

*21.* A merchant was making 28% profit by selling at 10% below his marked price. He later sold at 25% below the same marked price, and thus doubled his sales. What were his profits for a week during the reduced sale if his total sales were $48564.85?

*22.* A merchant wishes to make 15% profit on his goods; he allows ½% for an average credit of 3 months on all sales, and loses 3% in collection. What should he receive for a piano that cost him $215?

**538. To find the rate per cent. when the profit or loss and the cost are given.**

EXAMPLE.—What is the rate per cent. of profit on goods bought for $6520 and sold for $7458.88?

$7458.88 selling price
6520. cost
$ 938.88 profit

$$6520\overline{)938.88}\quad 14\tfrac{2}{5} \text{ or } 14\tfrac{2}{5}\%$$

6520
28688
26080
2608
6520

$$1304 \mid \frac{2608}{6520} = \frac{2}{5}$$

**539.** RULE.—*Divide the profit or loss by the cost.*

## MENTAL PROBLEMS

**540.** *1.* Damaged merchandise costing $240 is sold for $180; what is the per cent. of loss?

SOLUTION.—There is a loss of $60, or $\frac{60}{240}$, or $\frac{1}{4}$ of the value, which is equivalent to 25 per cent.

*2.* What per cent. is gained by selling an article costing 50 cents for 60 cents? What per cent. of the selling price is gained?

*3.* What per cent. is lost by selling an article costing 60 cents for 50 cents?

*4.* If wheat is bought for 80 cents and sold the same day for 84 cents, what is the rate of gain?

*5.* If wheat is bought at 80 cents and sold the same day for 76 cents, what is the rate of loss? What is the rate of loss if based on the selling price?

*6.* If an article costing 10 cents is marked 50 per cent. above cost and sold for 3 cents less than the marked price, what is the per cent. of gain on the sale?

*7.* A horse costing $200 is sold for $250; what is the gain per cent.?

*8.* A horse costing $200 is sold for $150; what is the loss per cent.?

*9.* What per cent. is gained by selling an article costing 70 cents for 77 cents?

*10.* If one-half of an invoice is sold at 25 per cent. gain and the other half at 20 per cent. loss, what is the rate per cent. of loss or gain on the whole invoice?

*11.* What is the gain or loss per cent. by asking 30 per cent. more than cost for goods and selling them for 10 per cent. less than the asking price?

*12.* What do I gain per cent. by selling at $3 each chairs which cost $2.50 each?

*13.* If an article is marked to gain 20 per cent. and a discount of 10 per cent. be allowed, what will be the rate per cent. of gain?

*14.* If a share of stock, par value ($100), is bought at $70 and sold at $80, what is the gain per cent.?

*15.* What do I gain per cent. by selling stock at 60 which I bought at 56?

*16.* What do I earn per cent. as commission agent, by selling $120 worth of goods, if I received $6 commission?

### WRITTEN PROBLEMS

**541.** *1.* Merchandise bought for $6250 was sold $782.50 above cost; what was the per cent. of gain? What is the gain per cent. on the selling price?

*2.* Merchandise bought for $6250 was sold for $5467.50; what was the per cent. of loss? What per cent. of loss if based on the selling price?

*3.* A business takes in $45200 a year; its expenses are $6780; what is the rate per cent. of expense?

*4.* From a tank of oil containing 2250 gallons there is a leakage of 45 gallons a week; what per cent. is the leakage?

*5.* A creditor gave a collector $50 to collect $1000; what was the rate of commission?

*6.* Carpets bought at 85 cents a yard are marked down to 68 cents; what is the per cent. of loss? What per cent. of the selling price was loss?

*7.* What is gained per cent. by marking at 33⅓% above list and selling at 25% off the marked price?

*8.* What is the gain per cent. in buying at 10% off list price and selling at 20% above list price?

*9.* A house was offered for sale at 25% of its cost, but there being no takers at that price, was sold at 25% below the price first asked; what was the rate per cent. of gain or loss?

*10.* A man paid $650 for a diamond and sold it for $910; what did he gain per cent.? What per cent. of the sale price was gain?

*11.* Goods bought for $3786 were marked 25% above cost, but afterward sold for 25% less than the marked price. What per cent. was gained or lost?

*12.* If an investment of $7463.80 yields annually $559.78, what is the rate per cent. of income?

*13.* Bought goods for $4987.36, and sold them at an advance of 28¼%, but was unable to collect $374.50 of the sales. What net per cent. did I gain?

*14.* What per cent. is gained by buying cigars at $65 a thousand and selling them at three for a quarter of a dollar?

*15.* Merchandise costing $6387.50 was sold as follows: ⅓ at a profit of 42%, $1846.25 worth (cost price) at a profit of 27%, and the remainder at cost. What per cent. was gained?

*16.* What per cent. is gained by buying coal at $4.87½ per ton of 2240 lbs. and selling it at $5.50 per ton of 2000 lbs.?

*17.* What per cent. is gained by buying flour at $5.60 a barrel and selling it at 85 cents per sack of 24 lbs.?

*18.* What per cent. profit does a druggist make by buying a powder at 38 cents a pound avoirdupois and selling it at 10 cents an ounce apothecary?

**542. To find the cost when the profit or loss and the rate are given.**

EXAMPLE.—If $249.36 is gained by selling goods at 4% profit, what is the cost?

.04)249.36
$6234

Or, .04 = $\frac{1}{25}$ of cost,
and $249.36 × 25 = $6234 cost

**543.** RULE.—*Divide the profit or loss by the rate expressed decimally; or reduce rate to equivalent fraction and solve by analysis.*

## MENTAL PROBLEMS

**544.** *1.* A furniture dealer gained $1 by selling chairs at a profit of 20 per cent.; what did the chairs cost?

SOLUTION.—A profit of 20 per cent. is equal to $\frac{1}{5}$ of the cost, which equals $1; the entire cost is 5 times $1, or $5.

*2.* A man sells a lot of goods at $4 gain, which is 25 per cent. of the cost; what is the cost?

*3.* A loss of $20 is sustained by selling a horse at 10 per cent. below cost; what is received for the horse?

*4.* A boat builder gains $20 on the sale of a boat, which is equal to 16$\frac{2}{3}$ per cent. of the cost; what does he get for the boat?

*5.* A carriage is sold at a gain of $25, which is 12$\frac{1}{2}$ per cent. of the cost; what is the selling price of the carriage?

*6.* I gain 10 per cent., or $15, on the sale of a watch; what did the watch cost me?

*7.* A jeweler gained $10 on the sale of a ring, which was 20 per cent. of the cost; what was the ring marked, if it was marked 10 per cent. above the selling price?

*8.* A cow was sold at a loss of $6, which was equal to 5 per cent. of its value; what should have been obtained for the cow so as to neither gain nor lose?

*9.* A loss of 50 cents on a barrel of flour is equivalent to a loss of 12$\frac{1}{2}$ per cent.; what is the flour worth a barrel?

*10.* A man gained $25 by selling a boat at 25 per cent. above its cost; what would he have gained by selling it at 15 per cent. above its cost?

*11.* $60 less was asked for a piano at a discount of 30 per cent. from list; what was the list price?

*12.* $32 is 4 per cent. of a young man's money, whose amount is equal to 50 per cent. of his companion's money; what amount have both?

*13.* A dealer gains 20 per cent., or $12, by the sale of a bicycle; what would he have obtained for it if he had sold it at a gain of $18?

*14.* A watch sold at $50 more than cost, gives a gain of 25 per cent.; what was its cost?

### WRITTEN PROBLEMS

**545.** *1.* By selling goods at an advance of $16\frac{2}{3}\%$, a merchant gained \$324.68; what did the goods cost him?

*2.* James Clark bought a carriage and sold it at a loss of $27\frac{1}{2}\%$. With the amount received he bought another carriage, which he sold at a profit of 30%, gaining \$27 on this sale. What did he pay for the first carriage?

*3.* Sold a property for \$287.17 less than it cost, losing thereby 13%. What was the cost? If the loss were 13% of the selling price, what was cost?

*4.* A dealer sold a quantity of goods, $\frac{1}{2}$ at a profit of 20% and the other $\frac{1}{2}$ at a loss of 18%. His net gain was \$322. Find the cost of the goods.

*5.* Brown sold a cargo of lumber at a profit of 29%. 47 per cent. of his gain was \$4211.67. How much did he pay for the cargo?

*6.* Goods sold at an advance of \$350, realized a gain of 35%; what did the goods cost?

*7.* A house was sold for \$270 less than its value, at a loss of 3%; what was the price obtained for it?

*8.* A property was sold at a gain of \$550, which was 18% of its cost; what was obtained for it?

*9.* Merchandise sold during a year showed an average gain of 30%; if the amount gained was \$3250, what was the amount sold?

*10.* A corporation set aside a dividend fund of \$12500, which was $2\frac{1}{2}\%$ of the capital stock; what was the capital stock?

**546. To find the cost when the selling price and the rate per cent. of profit or loss are given.**

EXAMPLE.—By selling goods for \$5239.36, 12% was gained. What did the goods cost?

$$1.12)5239.36 = 4678 \text{ cost}$$

**547.** RULE.—*Divide the selling price by 1 (100%) plus the rate of gain, or by 1 minus the rate of loss.*

### MENTAL PROBLEMS

**548.** *1.* A horse sold at \$110 yields a gain of 10%; what was the cost of the horse?

SOLUTION.—A gain of 10 per cent. is a gain of $\frac{1}{10}$ of the cost; the cost and $\frac{1}{10}$ of the cost or $\frac{11}{10}$ of the cost equals \$110; $\frac{1}{10}$ of the cost equals $\frac{1}{11}$ of \$110, or \$10; $\frac{11}{11}$ or the cost is \$100.

*2.* By selling a horse at \$90, a loss of 10 per cent. is sustained; what was the cost of the horse?

*3.* By selling merchandise for \$60, a gain of 20 per cent. is realized; what is the cost?

*4.* Chairs sold for \$30 a dozen realize a gain of 20 per cent.; what did they cost?

*5.* Coffee sold at 35 cents a pound yields a gain of $16\frac{2}{3}$ per cent.; what was its cost per pound?

*6.* $33\frac{1}{3}$ per cent. is gained by selling a wheel for \$40; what did it cost?

*7.* Furniture inventoried at \$180 shows a depreciation of 10 per cent.; what did it cost?

*8.* \$105 in money returned to a lender includes 5 per cent. interest; what was the amount loaned?

*9.* Land sold at \$66 an acre shows a gain of 10 per cent.; what was its cost?

*10.* \$160 is obtained for a carriage, which is a loss of 20 per cent.; what was the cost?

### WRITTEN PROBLEMS

**549.** *1.* Two horses were sold for \$100 each, one at a gain of 20%, the other at a loss of 20%. How much was gained or lost on both?

*2.* A sold a house to B for $33\frac{1}{3}\%$ less than it cost him. B sold it to C for 25% more than he paid for it. C paid \$2500 for the house. What did it cost A?

*3.* Sold a lot of merchandise at a gain of $21\frac{7}{8}\%$, and with the sum received purchased another lot which I sold for \$4535.17, at a loss of 3%. Find the cost of the first lot.

*4.* A jobber offered a lot of goods at 18% more than they cost him, but afterward sold them for \$1237.76, which was $7\frac{1}{2}\%$ less than his first offer. How much had the jobber paid for the goods? What rate of profit did he make on them?

*5.* If goods which were marked 31% above cost, were afterward sold for 90% of the marked price, yielding a profit of \$9392.89, what was their cost?

*6.* A merchant's capital at the end of a business year is \$16619.80; during the year he lost 8%; what was his capital at the beginning of the year?

*7.* A shipment realized \$6887.50, at a loss of 5%; what was the value of the shipment?

*8.* Goods are insured for \$8287.50, which is $2\frac{1}{2}\%$ less than their value; what are the goods worth?

*9.* An agent obtained \$956.25 for goods, which was a gain of $12\frac{1}{2}\%$; what was the cost of the goods?

*10.* Goods marked \$1620 were sold for \$1350, which was a loss of 10%; how much were they marked above cost?

*11.* A dealer purchased 250 dozen eggs, which he marked to sell at 30 cents a dozen, and gain 25%; 4 dozen were unsalable; he sold the remainder at an advance of 10% on his marked price; find his gain or loss.

## REVIEW PROBLEMS

**550.** *1.* A hardware dealer buys locks at $4.80 per dozen, list price, less 25% and 20%, and sells at the same list price less 20% and 16⅔%. What is his gain on 125 dozen, and his gain per cent.?

*2.* A clothier gains 25% by selling cloth at $5 per yard, but a bale of 80 yards being damaged, he has to reduce the selling price 10%. What is his profit on this damaged bale, and his gain per cent.?

*3.* If green coffee costs 25 cents a pound, and ½ cent a pound for roasting, for what must a pound of roasted coffee sell to gain 16⅔%, if green coffee shrinks 15 per cent. in roasting?

*4.* A drover sold a horse for $180 and lost 25%; with this money he bought another horse, which he sold at a gain of 25%. What was his gain or loss on the transaction and gain or loss per cent.?

*5.* A man bought a horse and carriage for $450 paying 25% more for the horse than for the carriage. He sold the horse at a gain of 40%, and the carriage at a loss of 30%. What did he gain on the transaction?

*6.* A merchant bought dry goods for $6000; he sold 20% of them at a gain of 10%; 40% of them at a gain of 20%; 40% of the remainder at a gain of 5%, and the remainder at cost. What did he gain or lose?

*7.* My retail price of fans is $2.50 each; by selling them at this price I would gain 25%. I sell the same fans wholesale, at $40 per dozen, less 12½% and 20%. Do I gain or lose at the wholesale price, and how much a fan?

*8.* A merchant marked his goods 60% above cost. He gave one of his customers a discount of 15% off the marked price; what was his gain on $6.80 received from that customer?

*9.* A merchant sold 20% of an invoice at 30% profit; 25% of the remainder at 20% profit; what was his total gain, if the cost of the goods unsold is $3600?

*10.* How should goods be marked that cost $5 so as to offer a discount of 20% and 10% and still make a profit of 25%?

*11.* A man bought a lot of apples and lost 25% of them. What must be his per cent. of gain on the remainder to net a gain of 20% on the cost of the lot?

*12.* A man sold a house at 30% profit, and with this money bought another house which he sold at 25% profit. What did he pay for each house if the total gain was $2500?

*13.* A merchant buys cloth at $2.50 a yard. How should it be marked to gain 25% if the merchant loses 5% of his sales in bad debts?

*14.* I sold a lot for $102.25 more than cost and gained 5%. I sold another lot which cost the same for $3000. What was my gain per cent. on the second sale?

*15.* The retail price of an article was \$262.50, which was 25% more than cost to the retailer. The retailer bought of a jobber who sold it at a gain of 20% above the manufacturer's selling price and the manufacturer made a profit of $16\frac{2}{3}\%$. What did it cost to manufacture this article, and what was each one's profit?

*16.* A bought 80 yards of broadcloth at \$3.40 per yard, and 74 yards of cassimere at \$2.50 per yard. He sold the cassimere at a loss of 20%. What should he ask per yard for the broadcloth to net a gain of 25% on the cost of both?

*17.* A sold a wagon to B and lost 25%. B sold it to C at a loss of 5%; C spent \$80 for repairs and sold it for \$472.25, gaining 20%. What did the wagon cost A, B and C respectively?

*18.* A real estate dealer bought three houses at \$1800 each, and sold them at a profit of 8%, 12% and 15% respectively. What was his gain on the three houses?

*19.* A real estate agent sold three houses for \$2400 each, at a gain of 20%, $12\frac{1}{2}\%$ and 15% respectively. What was his entire gain?

*20.* How much should be asked for coffee which costs 18 cents a pound to gain 10%, allowing 10% for loss in roasting?

*21.* A merchant spent equal sums of money in cotton, linen and woolen goods, and gained 10% on the cotton, 8% on the linen, but lost 25% on the woolens; the whole amount of his sales was \$5684.20. What did he pay for each kind of goods? What was his gain or loss?

*22.* By selling apples at \$2.50 per barrel, I gained \$400. If I had sold them at \$2.75 per barrel, I would have gained $37\frac{1}{2}\%$. How many barrels did I sell?

*23.* Sold two properties for \$10500, receiving $\frac{1}{3}$ more for one than for the other; the more expensive was sold at $14\frac{2}{7}\%$ loss, the other at 20% profit. What was the per cent. of profit or loss on the sale of both?

*24.* Smith has 30% more money than Jones; Gray has 25% less than Brown; 65% of Brown's money equals Smith's; if Gray has \$960 more than Smith, how many acres of land costing $7\frac{1}{2}\%$ less than \$140 per acre can be purchased with their total money?

*25.* A merchandise account shows the following: inventory at opening, \$3874.55; purchases, \$26785.90; sales, \$32648.75; inventory at closing, \$4728.92. What per cent. profit was made?

*26.* A bookseller buys books at 20% and 10% from catalogue prices, and sells at 10% below catalogue prices. Find his gain on a lot of books which he sells for \$4920.

*27.* Marked goods to gain 18%; sold 60% of the goods at this price, then reduced the price 10% and sold the remainder. If 8% of the sales is charged as expense, what is the per cent. of gain on an invoice?

*28.* Sold 20% of a purchase at 15% gain; sold 40% of the remainder for \$950, losing $7\frac{1}{2}\%$. All sales were made through an agent who charged 2%. What was my gain, and gain per cent.?

89. A merchant, having 3750 lbs. of coffee to sell, finds he is selling at 15% loss, so has sold 2000 lbs at this price. At what per cent profit must he sell the remainder to make a clear gain of 30 per cent?

90. A and B form a partnership. A invests 35 per cent of his money, and B 65% less. During the year the business netted 25% gain, and was worth $10531.25 at the end of the year. If, at first, A's money was 60% of B's, how much will each be worth after getting his profits for the year?

91. I sold two houses for the same price. On the first house my gain was 33⅓% and on the second house my loss was 25%. If my total loss was $400, what did I pay for each house?

92. A sold his house and farm for $18,250, receiving 80% as much for his house as for his farm. On the farm he gained 7%, and on the house he lost 5%, what was the cost of each? What was A's loss or gain?

93. A invested $4000 in oil and lost 20%, he then invested the remainder in gas and gained 25%, he then invested his money in fancy stock and lost 30%. Did he gain or lose, and how much?

94. If on 60, of my sales, sold at a gain of 33⅓%, I gain $2400, and on the remainder of my sales gain 10%, what are my total sales and my total gain?

95. A man sold a horse at 33⅓% profit. He added $100 to the sum received for the horse and bought a piano, which he sells at 20% profit, if his total profits were 25% what was the cost of the horse?

96. A merchant failed in business with resources of $15,748 and liabilities of $26,246. What per cent will the creditors lose and how much will A receive to whom he owes $6,248?

97. In closing out my business, I am willing to sell at a loss of 10%. If I allow discounts of 20%, and 5%, what must be the marked price of an article costing $7.20?

98. Coffee which cost 24c per lb. was marked for sale at 32c per lb. The price was reduced 5%, at which price 300 bags of 150 lbs. each were sold. Before the remnant was purchased failed and finally paid 80 cents on the dollar. Did the seller gain or lose, how much, and what per cent?

99. A merchant marked his goods at an average advance of 25%, on cost. He sold 75% of his goods at these prices. He then closed out the stock to Bush at 81 cents on the dollar. Bush sold the stock he bought for 15%, off the merchant's marked price and gained $2400. What was the merchant's gain or loss on his entire stock?

100. I have a house which I offered at $6750. I gave it to an agent charging 2½% commission and lowered the price 7½%. The agent sold it and deducted his commission. How much more would I have had if I had sold it for $6750?

# INVOICE EXTENSION AND TRADE DISCOUNT

**551. Invoice Extension,** or **Bill Work,** consists in multiplying the items of a bill or invoice by their respective prices and placing the results in the money-column to the right.

**552. A discount** is any deduction from the face of a bill or debt. It is usually reckoned at a certain rate per cent.

**553. Commercial discounts** are of two kinds: (1) **Trade discounts,** which are deductions from the fixed or list price of goods, allowed to the "trade"—(those in the same line of business) by manufacturers, jobbers and wholesalers; (2) **Cash discounts**—deductions for immediate payment, made from the net amount of a bill for payment within a definite time.

It will thus be seen that trade discounts are *absolute*, while cash and time discounts are *conditional*—the first being deducted from the list price when the goods are billed, and the others from the amount of the bill when payment is made in accordance with the expressed conditions.

**554. The list price** is also called the **marked price, asking price, offering price,** or **gross price;** and the price after the trade discount has been deducted, the **net price** or simply the **"net."**

**555.** Business men usually announce their terms on their bill-heads; as, "Terms: Net 3 months, or 5% off for cash"; "Terms: Net 60 days, or 2% off in 10 days."

**556. Trade discounts** arise principally from two different causes:

*1.* In many lines of business it is customary for merchants to issue catalogues and price-lists of their goods, with the different articles listed at fixed prices higher than the actual selling prices. They then issue discount sheets from time to time, varying the discounts as the market prices change, instead of altering the list prices.

*2.* In almost all lines of business, the larger quantity a dealer buys at one time the lower price he can get; and in many cases this concession is made in the form of an extra discount.

**557. A series of discounts** consists of two or more discounts to be taken off successively, the first discount being reckoned on the *list price*, the second on the remainder left after subtracting the first, the next on the remainder left after subtracting the second, and so on.

NOTE.—Observe that \$100 less 20% is \$80; but \$100 less 10% and 10% is \$81. \$100 less 40% is \$60; but \$100 less 20% and 20% is \$64.

**558. To find the net amount of a bill.**

EXAMPLE 1.—Find the net amount of a bill of $467.50, subject to discounts of 1%, 3% and 2%.

| | |
|---|---|
| 467.50 | |
| 32.72 | 50 |
| 434.78 | |
| 13.04 | 34 |
| 421.74 | |
| 8.43 | 48 |
| 413.31 | net |

EXPLANATION.—1. To get 1%, 2%, 3%, 4%, 5%, 6%, 7%, 8%, 9%, 11%, 12%, or any discount not an easy aliquot part of 100%, multiply and reject two right-hand figures.

**559.** RULE.—*Deduct the first discount from the list price, and each subsequent discount from the respective remainders.*

NOTE.—In taking off discounts, usage regarding fractions of a cent differs. For uniformity, students should drop all such fractions.

Before beginning to discount, add two ciphers for cents to sums expressed in dollars only.

**WRITTEN EXERCISE 1**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *1.* | $382.75 | less | 9%, 6% and 2% | *9.* | $388.51 | less | 4%, 5% and 6% |
| *2.* | 862.64 | " | 8%, 6% and 4% | *10.* | 629.87 | " | 12%, 6% and 5% |
| *3.* | 687.54 | " | 11%, 7% and 3% | *11.* | 457.86 | " | 8%, 3% and 5% |
| *4.* | 732.84 | " | 6%, 4% and 1% | *12.* | 462.68 | " | 9%, 7% and 11% |
| *5.* | 789.62 | " | 8%, 5% and 2% | *13.* | 572.83 | " | 6%, 4% and 2% |
| *6.* | 832.45 | " | 6%, 3% and 2% | *14.* | 789.26 | " | 1%, 3% and 7% |
| *7.* | 752.84 | " | 9%, 6% and 8% | *15.* | 872.86 | " | 11%, 12% and 4% |
| *8.* | 892.63 | " | 7%, 8% and 11% | *16.* | 923.84 | " | 12%, 6% and 9% |

EXAMPLE 2.—Find the net amount of $467.50, less 25%, 14⅔% and 8⅓%.

| | |
|---|---|
| 25 | 467.50 |
| | 116.87 |
| 4⅔ | 350.63 |
| | 50.09 |
| 8⅓ | 300.54 |
| | 25.04 |
| | 275.50 net |

EXPLANATION.—2. To get 50%, 33⅓%, 25%, 20%, 16⅔%, 14⅔%, 12½%, 11⅑%, 10%, 9 1/11%, 8⅓%, divide by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 respectively.

**WRITTEN EXERCISE 2**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *1.* | $789.62 | less | 20%, 10%, and 25% | *9.* | $732.84 | less | 14⅔%, 16⅔% and 20% |
| *2.* | 832.45 | " | 25%, 12½% and 8⅓% | *10.* | 382.75 | " | 8⅓%, 16⅔% and 12½% |
| *3.* | 852.47 | " | 33⅓%, 16⅔% and 8⅓% | *11.* | 463.87 | " | 10%, 20% and 16⅔% |
| *4.* | 574.82 | " | 50%, 25% and 12½% | *12.* | 295.52 | " | 9 1/11%, 11⅑% and 8⅓% |
| *5.* | 375.82 | " | 33⅓%, 25% and 20% | *13.* | 735.68 | " | 14⅔%, 20% and 25% |
| *6.* | 382.54 | " | 33⅓%, 25% and 20% | *14.* | 792.87 | " | 12½%, 50% and 20% |
| *7.* | 972.63 | " | 25%, 20% and 16⅔% | *15.* | 722.12 | " | 33⅓%, 16⅔% and 8⅓% |
| *8.* | 758.75 | " | 50%, 12½% and 20% | *16.* | 856,75 | " | 25%, 12½% and 14⅔% |

EXAMPLE 3.—Find the net result of \$467.50, less 22⅞%, 62½% and 18¾%.

| | | |
|---|---|---|
| 22⅞ | 467.50 | 9)93500 |
| | 103.88 | |
| 62½ | 363.62 | 8)181810 |
| | 227.26 | |
| 18¾ | 136.36 | 4)40908 |
| | 25.56 | 4)10227 |
| | 110.80 net | |

EXPLANATION.—3. To get 22⅞% (⅞), multiply by 2 and ÷ 9; 37½% (⅜), multiply by 3 and ÷ 8; 62½% (⅝), multiply by 5 and ÷ 8; 44⅜% (⅞), multiply by 4 and ÷ 9; 18¾% (³⁄₁₆), multiply by 3 and ÷ 16.

**WRITTEN EXERCISE 3**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | \$489.62 | less | 37½%, 18¾% and 25% | 6. | \$876.42 | less | 22⅞%, 18¾% and 37½% |
| 2. | 982.67 | " | 22⅞%, 11⅛% and 18¾% | 7. | 954.62 | " | 37½%, 44⅜% and 62½% |
| 3. | 842.63 | " | 62½%, 37½% and 12½% | 8. | 984.36 | " | 62½%, 18¾% and 22⅞% |
| 4. | 756.75 | " | 44⅜%, 22⅞% and 11⅛% | 9. | 928.63 | " | 18¾%, 44⅜% and 37½% |
| 5. | 932.56 | " | 37½%, 18¾% and 22⅞% | 10. | 879.26 | " | 37½%, 62½% and 22⅞% |

EXAMPLE 4.—Find the net result of \$467.50, less 2½%, 3⅓% and 6¼%.

| | | |
|---|---|---|
| 2½ | 467.50 | |
| | 11.68 | |
| 3⅓ | 455.82 | |
| | 15.19 | |
| 6¼ | 440.63 | 4)110 15 |
| | 27.53 | |
| | 413.10 | |

EXPLANATION.—4. To get 5% ($\frac{1}{20}$), divide by 2 and set quotient one place to the right.
To get 3⅓% ($\frac{1}{30}$), divide by 3 and set quotient one place to the right.
To get 2½% ($\frac{1}{40}$), divide by 4 and set quotient one place to the right.
To get 6¼% ($\frac{1}{16}$), divide by 16, or by 4 twice.

**WRITTEN EXERCISE 4**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1. | \$634.89 | less | 10%, 5% and 2½% | 9. | \$962.37 | less | 5%, 10% and 2½% |
| 2. | 584.67 | " | 10%, 3⅓% and 2½% | 10. | 846.75 | " | 3⅓%, 2½% and 5% |
| 3. | 564.37 | " | 6¼%, 5% and 3⅓% | 11. | 962.54 | " | 10%, 5% and 5% |
| 4. | 675.84 | " | 6¼%, 3⅓% and 2½% | 12. | 954.85 | " | 3⅓%, 2½% and 5% |
| 5. | 922.36 | " | 5%, 3⅓% and 2½% | 13. | 634.89 | " | 10%, 3⅓% and 2½% |
| 6. | 892.53 | " | 5%, 2½% and 2½% | 14. | 536.43 | " | 3⅓%, 2½% and 5% |
| 7. | 642.33 | " | 3⅓%, 2½% and 5% | 15. | 895.44 | " | 5%, 10% and 2½% |
| 8. | 892.64 | " | 10%, 2½% and 3⅓% | 16. | 784.92 | " | 3⅓%, 5% and 10% |

EXAMPLE 5.—Find the net result of \$467.50, less 15%, 7½% and 6⅔%.

| | | |
|---|---|---|
| 15 | 467.50 | 2)140258 |
| | 70.12 | |
| 7½ | 397.38 | 4)119214 |
| | 29.80 | |
| 6⅔ | 367.58 | 3)73514 |
| | 24.50 | |
| | 343.08 net | |

EXPLANATION.—5. To get 6⅔% ($\frac{2}{30}$), multiply by 2, reject right-hand figure and divide by 3.
To get 7½% ($\frac{3}{40}$), multiply by 3, reject right-hand figure and divide by 4.
To get 13⅓% ($\frac{4}{30}$), multiply by 4, reject right-hand figure and divide by 3.
To get 15% ($\frac{3}{20}$), multiply by 3, reject right-hand figure and divide by 2.
To get 35% ($\frac{7}{20}$), multiply by 7, reject right-hand figure and divide by 2.

### WRITTEN EXERCISE 5

| | | | | | |
|---|---|---|---|---|---|
| 1. | $897.56 | less 6¼%, 7½% and 13⅓% | 6. | $542.36 | less 7½%, 15% and 6¼% |
| 2. | 695.84 | " 15%, 13⅓% and 6¼% | 7. | 875.63 | " 15%, 7½% and 13⅓% |
| 3. | 957.76 | " 35%, 15% and 7½% | 8. | 986.54 | " 35%, 15% and 7½% |
| 4. | 872.45 | " 13⅓%, 7½% and 6¼% | 9. | 563.87 | " 13⅓%, 6¼% and 15% |
| 5. | 938.56 | " 35%, 6¼% and 7½% | 10. | 436.58 | " 7½%, 7½% and 6¼% |

**Example 6**—Find the net result of $467.50, less 87½%, 75% and 50%.

87½ | 467.50
75 | 58.43
50 | 14.60
7.30 net

Explanation.—6. To get net results if discounts are 50% (½), 66⅔% (⅔), 75% (¾), 80% (⅘), 83⅓% (⅚), 85⅞ %(⅞), 87½% (⅞), 90% (⁹⁄₁₀), which leave but ½, ⅓, ¼, ⅕, ⅙, ⅐, ⅛, ⅑, ⅒, divide the sum by 2, 3, 4, 5, 6, 7, 8, 9, 10 respectively.

### WRITTEN EXERCISE 6

| | | | | | |
|---|---|---|---|---|---|
| 1. | $4385.72 | less 87½%, 75% and 50% | 6. | $5896.72 | less 91⅔%, 75% and 50% |
| 2. | 5892.65 | " 50%, 66⅔% and 83⅓% | 7. | 6287.96 | " 66⅔%, 75% and 50% |
| 3. | 5486.72 | " 88⅛%, 50% and 75% | 8. | 7382.64 | " 87½%, 50% and 66⅔% |
| 4. | 6389.50 | " 85⅞%, 66⅔% and 50% | 9. | 7056.79 | " 88⅛%, 75% and 50% |
| 5. | 4482.96 | " 90⅛%, 75% and 50% | 10. | 6489.86 | " 87½%, 50% and 75% |

**Example 7.**—Find the net result of $832, less 70%, 60% and 30%.

70 | 832.00
60 | 249.60
30 | 99.84
69.88 net

Explanation.—7. To get 30%, 40%, 60%, 70%, multiply by left-hand figure (3, 4, 6, 7) and reject right-hand figure in product; or, to get the "net," use as a multiplier the left-hand figure of the difference between the rate and 100%.

### WRITTEN EXERCISE 7

| | | | | | |
|---|---|---|---|---|---|
| 1. | $3245.89 | less 70%, 30% and 20% | 6. | $3642.55 | less 70%, 20% and 20% |
| 2. | 2256.37 | " 80%, 40% and 30% | 7. | 4384.62 | " 40%, 20% and 80% |
| 3. | 3489.62 | " 60%, 50%, and 20% | 8. | 5283.64 | " 60%, 40% and 50% |
| 4. | 4287.63 | " 40%, 20% and 90% | 9. | 3876.42 | " 20%, 30% and 90% |
| 5. | 3857.90 | " 20%, 30% and 60% | 10. | 6273.89 | " 40%, 30% and 30% |

**Example 8.**—Find the net result of $652.70, less 40%, 33⅓% and 25%.

40 | 100%
40
33⅓ | 60%
20
25 | 40%
10
30% net

30 | 652.70
195.81 net

Explanation.—8. A series may have an easy equivalent; as, 40%, 33⅓% and 25% equals 70% off; 33⅓%, 25% and 20% equals 60% off.

Note.—This method is used when a series of discounts can be reduced to an easy single equivalent discount.

**560. Form of invoice showing extensions, discounts and net extensions.**

Philadelphia, Pa., May 12, 1914

M (Name of student)

BOUGHT OF Merchant & Co.

Terms: Net cash. 917-919 Chestnut Street

| | | | | | | |
|---|---|---|---|---|---|---|
| 263' yds. | Cassimere | @ $1.95² | 514 | 65 | | |
| | Less 10% | 5% and 2½% | 85 | 61 | 429 | 04 |
| 357⅔ " | Broadcloth | @ $2.18³ | 782 | 40 | | |
| | Less 25% | 12½% and 6¼% | 301 | 04 | 481 | 36 |
| 196³ " | Blue Chevriot | @ $5.16⅔ | | | | |
| | Less 33⅓%, | 16⅔% and 8% | | | | |
| 462⅜ " | Foulard Silk | @ $1.33⅓ | | | | |
| | Less 50%, | 20% and 10% | | | | |
| 287² " | Blk Taffeta | @ $4.87² | | | | |
| | Less 14⅔% | 5% and 3⅓% | | | | |

NOTE.—The small figures at the right in the quantity and price are fourths of a yard and fourths of a cent, the numerator alone being written. All other fractions have both numerator and denominator expressed.

The operations of items 1 and 2 illustrate the method and form to be followed.

1

263¼
1.95½
1315
2367
263 ⅛ = 1
131½ = 4
48¾ = 6/11/8 = 1⅜

| | | |
|---|---|---|
| 10 | 514.65 | |
| | 51.46 | |
| 5 | 463.19 | |
| | 23.15 | |
| 2² | 440.04 | |
| | 11.00 | |
| | 429.04 net | |

OPERATION 1

(1) 263×1.95
(2) ½ of 263 = 131½
(3) ¼ of 195 = 48¾
(4) ⅛ of ¼ = ⅛
(5) ⅛ + ¾ + ½ = 1⅛, or 1c.

OPERATION 2

(1) 357×2.18
(2) ¼ of 357 = 267¾
(3) ⅛ of 218 = 145⅓
(4) ⅓ of ⅜ = 6/12
(5) 6/12 + ⅓ + ¾ = 19/12 or 2c.

2

357⅔ 1496
2.18¾ 11071
2856
357 4/12 = 6
714
267¾ = 9
145⅓ = 4/12 = 1⅙

| | | |
|---|---|---|
| | 2 | |
| 25 | 782.40 | |
| | 195.60 | |
| 12² | 586.80 | |
| | 73.35 | |
| 6' | 513.45 | 4112836 |
| | 32.09 | |
| | 481.36 net | |

NOTE.—Less than ½ cent in the gross result is rejected; more than ½ cent is counted a cent.

## EXERCISE IN INVOICE EXTENSION AND DISCOUNTING

**561.**

| | Items | Prices | Discounts | | Items | Prices | Discounts |
|---|---|---|---|---|---|---|---|
| *1.* | 297⅞ yds. | 1.37½ | 37½—18¾—10 | *41.* | 303¾ yds. | 3.37½ | 11/16—37½—50 |
| *2.* | 368½ " | 1.50 | 22⅛—11/16—5 | *42.* | 368½ " | 2.16⅔ | 25—15—5 |
| *3.* | 183⅜ " | 1.62½ | 62½—37½—12½ | *43.* | 393¾ " | 5.12½ | 5—2½—2½ |
| *4.* | 486¾ " | 2.16⅔ | 44⅛—22⅛—11/16 | *44.* | 263⅛ " | 4.18¾ | 6—3—2 |
| *5.* | 393⅞ " | 2.33⅓ | 37½—18¾—14⅞ | *45.* | 567⅜ " | 2.66⅔ | 3½—5—10 |
| *6.* | 218¾ " | 1.73¾ | 10—5—2½ | *46.* | 389¾ " | 1.33⅓ | 13½—7½—12½ |
| *7.* | 382½ " | 1.25 | 10—3½—2½ | *47.* | 373½ " | 1.95⅜ | 50—14⅞—7 |
| *8.* | 238⅞ " | 5.16⅔ | 6¼—5—3½ | *48.* | 737⅞ " | 4.22½ | 44⅛—12½—5 |
| *9.* | 196¼ " | 1.20 | 6¼—3½—2½ | *49.* | 149½ " | 1.58⅓ | 55⅜—13½—3½ |
| *10.* | 188½ " | 1.87½ | 5—3½—2½ | *50.* | 281⅞ " | 1.83⅓ | 66⅔—28⅛—6⅔ |
| *11.* | 303¾ " | 2.81¾ | 15—7½—2½ | *51.* | 193½ " | 2.43⅛ | 77/16—11/16—4 |
| *12.* | 361¼ " | 1.32½ | 13½—6⅔—3½ | *52.* | 387⅔ " | 1.93⅞ | 83⅓—16⅔—10 |
| *13.* | 245½ " | 2.12½ | 35—15—7½ | *53.* | 376¾ " | 2.11⅛ | 22⅞—18¾—6¼ |
| *14.* | 243⅛ " | 2.66⅔ | 15—7½—6⅔ | *54.* | 152⅛ " | 2.43⅞ | 75—50—20 |
| *15.* | 753⅛ " | 1.75 | 13⅓—6⅔—5 | *55.* | 235⅝ " | 4.33⅓ | 90—20—25 |
| *16.* | 615¾ " | 3.33⅓ | 75—25—10 | *56.* | 627⅔ " | 1.97⅞ | 40—10—10 |
| *17.* | 179¾ " | 2.37½ | 87½—12½—6¼ | *57.* | 184⅞ " | 5.87½ | 45 5/11—9 1/11—9 |
| *18.* | 432⅔ " | 4.18¾ | 90—10—5 | *58.* | 846¾ " | 2.58⅓ | 55—12—6 |
| *19.* | 357⅛ " | 2.50 | 66⅔—33⅓—3⅓ | *59.* | 277⅜ " | 1.52½ | 57⅛—14⅞—7 |
| *20.* | 247½ " | 5.66⅔ | 80—50—20 | *60.* | 384¾ " | 3.45⅜ | 80—25—33⅓ |
| *21.* | 595¾ " | 1.17½ | 40—30—10 | *61.* | 585⅜ " | 6.93¾ | 85—33⅓—50 |
| *22.* | 125 " | 7.56½ | 70—60—30 | *62.* | 468¾ " | 2.82½ | 20—32½—37½ |
| *23.* | 312½ " | 3.75 | 60—40—20 | *63.* | 567¼ " | 4.62½ | 25—23⅓—17½ |
| *24.* | 975 " | 1.27½ | 30—10—3 | *64.* | 462½ " | 3.65⅝ | 16⅔—26⅔—13⅓ |
| *25.* | 477¾ " | 1.55 | 70—40—30 | *65.* | 387¼ " | 2.87½ | 18—42½—18¾ |
| *26.* | 133⅓ " | 4.63½ | 33⅓—25—20 | *66.* | 383⅞ " | 2.66⅔ | 12—18¾—35 |
| *27.* | 250 " | 2.95⅝ | 25—20—16⅔ | *67.* | 387½ " | 3.81½ | 45—27½—37½ |
| *28.* | 428⅛ " | 2.62½ | 40—33⅓—25 | *68.* | 768½ " | 2.38½ | 7½—27½—4½ |
| *29.* | 333¾ " | 3.33⅓ | 50—20—12½ | *69.* | 468¾ " | 2.38½ | 5½—18¾—6¼ |
| *30.* | 750 " | 1.47⅜ | 60—33⅓—25 | *70.* | 492⅞ " | 4.62½ | 32½—18¾—37½ |
| *31.* | 163⅜ " | 1.62½ | 33⅓—16⅔—8⅓ | *71.* | 513⅓ " | 2.88½ | 50—45—15 |
| *32.* | 177⅛ " | 1.87½ | 15—7½—5 | *72.* | 875½ " | 4.87½ | 13⅓—7½—37½ |
| *33.* | 191⅛ " | 1.66⅔ | 10—3⅓—2 | *73.* | 862½ " | 5.66⅔ | 6⅔ 36⅔—26⅔ |
| *34.* | 221¼ " | 2.06¼ | 5—10—20 | *74.* | 787⅛ " | 2.88⅛ | 6¼—18¾—23⅓ |
| *35.* | 252½ " | 2.12½ | 6¼—12½—25 | *75.* | 694⅜ " | 6.47⅜ | 2½—4½—5½ |
| *36.* | 263¾ " | 2.18¾ | 15—7½—2½ | *76.* | 438¾ " | 4.83⅜ | 12½—37½—27½ |
| *37.* | 271⅛ " | 2.33⅓ | 66⅔—33⅓—16⅔ | *77.* | 268⅜ " | 4.75 | 87½—5—2½ |
| *38.* | 283⅜ " | 2.62½ | 30—15—5 | *78.* | 457⅝ " | 4.66⅔ | 75—15—7½ |
| *39.* | 295⅝ " | 2.66⅔ | 10—13⅓—3⅓ | *79.* | 638⅗ " | 3.62½ | 45—13⅓—3⅓ |
| *40.* | 297⅛ " | 2.87½ | 14⅞—16⅔—20 | *80.* | 437¾ " | 5.57¼ | 18¾—37½—11/16 |

### WRITTEN PROBLEMS

**562.** *1.* A bill of goods amounting to $347.60 is sold May 10, subject to discounts of 30%, 10% and 5%, terms, net 60 days, or 2% off if paid in ten days. How much will pay the bill May 18?

*2.* How much must be the list price of goods subject to discounts of 20% and 10% in order to net $288?

*3.* What is the difference on a bill of $582.50 between a discount of 45% and discounts of 25%, 10% and 10%?

*4.* Find the net cost of 40 doz. locks at $3.10, less 20% and 5%; 56 doz. hinges at $2.20, less 33⅓%; 90 doz. bolts at $2.55, less 60%, 10% and 10%; with an allowance of 2% for cash payment.

*5.* What must be the list price of goods that cost $97.80 in order to gain 20%, if they are sold at 25%, 10% and 2% off?

*6.* What is the net cost of 1286 yards of cloth at 87½ cents per yard, less 10% and 5%, with a discount of 2% for cash; case and packing $2.25?

*7.* If goods are bought at discounts of 25%, 20% and 5% from list price, and sold at 20% and 10% from list, what is the gain per cent.?

*8.* How much per dozen is gained by buying brushes at $5 per dozen, less 30%, 10% and 10%, and selling them at 35 cents each?

*9.* What advance on cost is necessary in order to give a discount of 25% and still make a profit of 33⅓%?

*10.* What must be the list price of goods that cost $48 per dozen in order to gain 25%; if sold at 20%, 16⅔% and 5% off?

*11.* A bill amounting to $896.50 subject to discounts of 40%, 37½% and 60%, and an additional cash discount of 2%. What amount of cash will pay the bill?

*12.* Bought goods at $6 per dozen, less 20%, 16⅔% and 25%, and sold at $8 per dozen, less 25%, 20% and 10%. Find my gain on 96 dozen.

*13.* An agent offers goods at list price less 40%, 30% and 14⅐%. Another agent offers goods at the same list price less 60%, 10% and 11⅑%. Which agent would you purchase from, and how much would you save on a bill listed at $387.40?

*14.* Which is the better—goods bought at 30%, 40% and 37½%, or goods bought at 70% and 40%? What amount would be saved on a bill of $382.65?

*15.* A jobber buys 5 doz. lawn-mowers at $50 per doz., less 40%, 12½% and 3%, and sells them to a retailer at a price that gives him a net profit of $52.65 on the lot. What price per dozen does the retailer pay, and for how much apiece must he sell the mowers to gain 25%?

# COMMISSION AND BROKERAGE

**563. Commission** is the compensation received by a *commission merchant* for buying or selling goods for another. Brokerage is the commission received by a broker.

**564.** Commissions are usually reckoned at a certain rate per cent. of the amount sold or purchased.

NOTE.—An agent receives his commission on the *gross proceeds* of a sale, or on the *prime cost* of a purchase.

**565.** An **account sales** is a detailed statement rendered by a commission merchant or other *consignee* to his principal, the *consignor*, showing the sales of the *consignment*, the incidental charges and expenses and the *net proceeds*.

The *net proceeds* are the sum left after the commission and other charges have been deducted.

**566.** An **account purchase** is a detailed statement rendered by an agent to his principal, showing the goods bought, prices paid, incidental charges and expenses, and the *gross cost*.

**567. To find the commission when the amount of purchase or sale and rate of commission are given.**

EXAMPLE.—At 3%, what does a commission merchant receive for selling $3275 worth of goods?

$$\begin{array}{r} 3275 \\ .03 \\ \hline 98.25 \end{array} \quad \text{or} \quad 3\% \,\big|\, \begin{array}{r} 3275 \\ \hline 98.25 \end{array}$$

**568.** RULE.—*Multiply the gross proceeds of the sale, or the prime cost of the purchase, by the rate per cent. of commission.*

### MENTAL PROBLEMS

**569.** *1.* An agent receives 12½ per cent. commission for selling sewing machines; what amount would he receive for selling one for $80?

SOLUTION.—12½ per cent. commission is ⅛ of the amount sold; ⅛ of $80 is $10, the amount of commission.

*2.* What amount of commission would be received for selling bicycles for $75 at a rate of 33⅓ per cent.?

*3.* What should I receive at a rate of 5 per cent. for collecting 25 per cent. of a claim of $800?

*4.* If I collect 66⅔ per cent. of a claim of $450 on a commission of 16⅔ per cent., what amount should I turn over?

*5.* What are the net proceeds of a sale of $300 worth of goods, rate of commission 5 per cent., other expenses $10?

*6.* What should I receive for buying $500 worth of produce on a commission of 2½ per cent.? How much should my principal send me to cover purchase?

*7.* What should my principal send me to cover a purchase of $600, commission 3 per cent. and 2 per cent. for guaranty of quality?

*8.* I gave a collector bills to the amount of $250 to collect; at 5 per cent. what must I pay him for collecting 80 per cent. of this amount?

## WRITTEN PROBLEMS

**570.** *1.* A real estate broker sold a house and lot for $7245; what was his commission at 2%?

*2.* At $\frac{1}{8}$% brokerage, how much must I pay my broker for purchasing $10000 worth of stock?

*3.* Find the net proceeds of the following

**Account Sales**

Of Merchandise received per P. R. R., July 2, 1915, from SANDUSKY & Co., St. Louis, Mo., to be sold on their account and risk.

| | | | | | |
|---|---|---|---|---|---|
| July | 7 | 500 lbs. Cheese | @ $0.10[2] | | |
| | 8 | 250 " Butter | .25[2] | | |
| | 10 | 75 doz. Eggs | .14[2] | | |
| | 13 | 125 lbs. Butter | .28 | | |
| | 14 | 475 " Cheese | .11[2] | | |
| | | *Charges* | | | |
| | | Freight 5.25 and Drayage 3.50 | | 8 | 75 |
| | | Commission on Sales, 2½% | | | |
| | | Your net proceeds | | | |
| | | WOODSIDE & Co. | | | |
| | | E. and O. E. | | | |
| | | Philadelphia, July 15, 1915 | | | |

*4.* Find the net proceeds of the following

**Account Sales**

PHILADELPHIA, Aug. 15, 1916.

*Sold for account of* J. D. TUCKEY & Co.,

*By* SNODGRASS, MURRAY & Co.

| | | | | | |
|---|---|---|---|---|---|
| July | 2 | 642 yds. English Tweed | @ $2.35 | | |
| | 3 | 879¾ " Broadcloth | 5.62[2] | | |
| | 9 | 48½ " Cheviot | 5.25 | | |
| | 15 | 2133½ " Satinet | .75 | | |
| | | *Charges* | | | |
| | | Freight 6.25 and Cartage 2.25 | | 8 | 50 |
| | | Commission 3% | | | |
| | | Net proceeds | | | |

*5.* Find the total cost of the following

**Account Purchase**

CHICAGO, Sept. 1, 1917.

*Bought for account of* WILSON, BROWN & Co.,

*By* WILLIAM R. ADAMS.

| | | | | |
|---|---|---|---|---|
| 200 | bbls. XX Flour | @ $2.25 | | |
| 325 | " Corn Meal | 3.12½ | | |
| 150 | " XXX Flour | 6.75 | | |
| 45 | " Flour | 6.87½ | | |
| 25 | tons Bran | 22.25 | | |
| | *Charges* | | | |
| | Cartage | | 16 | 75 |
| | Commission 3% | | | |
| | Total cost | | | |

**571. To find the gross proceeds or prime cost when the net proceeds or gross cost and rate of commission are given.**

EXAMPLE 1.—My agent remitted $2388.73, proceeds of goods sold for me at 1½% commission; how much did he receive for the goods?

100 % = Gross Sales .985)2388.730

1½% = Commission $2425.106

98½% = $2388.73 or Proceeds Result, 2425.11

**572.** RULE.—*Divide the net proceeds by 1 (100%) minus the rate expressed decimally, or the gross cost by 1 (100%) plus the rate.*

## MENTAL PROBLEMS

**573.** *1.* The gross cost of a desk is $80 and the agent's commission is 33⅓ per cent.; what is the prime cost?

SOLUTION.—The prime cost plus 33⅓ per cent. of the prime cost is ⅓ of the prime cost or the gross cost of $80; ⅓ of the prime cost is ¼ of $80 or $20, ⅓ or the prime cost is $60.

*2.* The cost of an organ was $100, which included the agent's commission of 25 per cent.; what was the price to the agent?

*3.* A manufacturer furnishes bicycles to an agent who returns him $60 each after taking out his commission of 16⅔ per cent. and deducting $5 charges; what is the gross cost to the purchaser?

*4.* A landlord received from his agent a net monthly rental of $45 after $3 was deducted for repairs and a commission of 4 per cent.; what did the tenant pay per month?

*5.* How much do I realize on a house and lot sold on commission of 3 per cent., the total charges, including $25 for advertising, being $130?

## WRITTEN PROBLEMS

**574.** *1.* An agent retained \$11.25 as his commission at 2½%; what amount did he send his principal?

*2.* What amount of goods must a salesman sell in a year at 3% to yield him an income of \$5000?

*3.* A collector received \$7500 of doubtful debts to collect and obtained 75% of the amount; if he charges 7½% for collecting, what amount shall he hand his employer?

*4.* A broker charged \$152.40 at 3½% for selling goods; what was the amount of sales?

*5.* If the brokerage for buying is \$166, rate 2½%, what is the value of the purchase? What is the cost to the principal?

*6.* How many pounds of tea at 42 cents did an agent sell if he remitted his principal \$953.30, after deducting his commission at 3½%?

*7.* How many dollars' worth of goods can an agent buy with \$14000, after deducting his commission of 3½%?

*8.* The net proceeds of an account sales were \$3060.89; what were the gross proceeds, if the commission was 5%, cartage \$13.60, storage \$8.30 and insurance \$3?

*9.* How many barrels of flour, at \$5.12½, can my agent buy for me with \$6340.96, if his commission is 2%?

*10.* A consigned to a commission merchant 346 barrels of potatoes and 218 barrels of onions, with instructions to sell and invest the proceeds in hay. The commission merchant paid \$14.60 freight and \$6.20 cartage, and sold the potatoes at \$1.40 per barrel and the onions at \$3.20 per barrel, charging 5% commission for selling. He then purchased the hay at \$12.25 per ton, charging 2% for buying. How much hay did A receive?

**575. To find the rate per cent. of commission when the commission and the gross proceeds or the prime cost are given.**

EXAMPLE.—Received from my agent an account sales, showing my net proceeds to be \$2649.89 and his commission \$96.11; what rate did he charge me?

\$2649.89 net proceeds
96.11 commission
\$2746.00 gross proceeds

$$2746\overline{)96.11}\quad .035 \text{ or } 3\tfrac{1}{2}\%$$

82 38
13730
13730

**576.** RULE.—*Divide the commission by the gross proceeds or by the prime cost.*

### MENTAL PROBLEMS

**577.** *1.* What rate per cent. of commission would I receive if I earned \$5 by selling a wheel for \$50?

SOLUTION.—\$5 is $\frac{1}{10}$ of the selling price, which is equivalent to 10 per cent.

*2.* I am offered \$3 to guarantee a debt of \$100; what is the rate of guaranty?

*3.* An agent returned \$45 on a sale of \$50; what rate of commission did he charge?

*4.* An agent sold a book for \$10 and returned \$6 to the publisher; what was the agent's rate of commission?

*5.* A consignor's net proceeds of a sale were \$380; the commission was \$10 and other charges \$10; what was the rate of commission?

*6.* What rate of commission does an agent retain who remits his principal \$350 from a sale of \$400?

### WRITTEN PROBLEMS

**578.** *1.* A lawyer took a fee of \$35 for collecting a claim of \$700; what was his rate for collecting?

*2.* A collector turned over to his employer \$2443.59, retaining a commission of \$128.61; what was his rate for collecting? What was the amount collected?

*3.* If a merchant receives \$62.50 for selling goods to the amount of \$2500, what amount of commission should he receive when he sells to the amount of \$3300?

*4.* A collector receives \$16 as his commission for collecting \$320, and, later \$28 for collecting \$560; what is his rate for collecting?

*5.* On an estate of \$32500 an inheritance tax of \$162.50 was paid; what was the tax rate?

*6.* What per cent. commission does an agent receive if he renders an account purchase showing total cost \$1428.91, his commission \$34.37, and other charges \$19.74?

*7.* A commission merchant rendered an account sales showing \$334.08 net proceeds and his commission \$13.92. What rate did he charge?

*8.* An agent received a consignment of goods which he sold for \$32414.61, charging $1\frac{1}{2}\%$ commission. He invested the proceeds in other goods, buying \$31312 worth. What per cent. did he receive for buying?

*9.* Jones consigned to a commission merchant 563 bus. of wheat and 1224 bus. of corn. The merchant sold the wheat at $82\frac{1}{2}$ cents a bushel and the corn at $67\frac{1}{2}$ cents a bushel. His charges were \$90.72, including his commission of \$51.66. What was the rate of his commission, and what were the net proceeds?

*10.* I purchase through an agent in London 1244 yds. of cloth at 14*s.* 3*d.* a yard, and remit to him in payment a draft which costs me \$4548.10 at the rate of \$4.88 for £1. What is the rate of his commission, if the other charges amount to £26 8*s.* 6*d.*?

## REVIEW PROBLEMS

**579.** *1.* A lawyer received a claim of $624.50 to collect. He succeeded in collecting $447.25. What per cent. of the claim did the holder receive, if the lawyer's commission for collecting was 5%?

*2.* What is the commission, at 2½%, on a sale of 268 bus. of oats at 38² cents a bushel?

*3.* What will 7200 doz. eggs cost at 18¾ cents a dozen, commission 3%?

*4.* An agent remits $319.60 proceeds of a sale amounting to $338.20; what rate of commission does he receive?

*5.* A consignor's net proceeds are $2314.80, the charges being $67.12 and 5% commission. Find amount of sale.

*6.* A broker charges $72 for selling 80 bales of cotton, averaging 480 lbs. each, at 7½ cents a pound. What is the rate of his commission?

*7.* Sold 750 bus. corn at 56 cents per bushel, and 460 bus. rye at 82 cents per bushel. Commission 3%, freight $264, storage $86. Find net proceeds.

*8.* How much must I remit to my agent for a purchase of $378.90 worth of goods, at 2% commission?

*9.* Received $598.55 as net proceeds of a consignment on which the commission amounted to $20.37 and the expenses $60. Find rate of commission.

*10.* A real estate broker charged me $382.50 as his commission at 5% for selling a house. How much did I receive?

*11.* Remitted my agent $690.45 to invest in cotton at 8⅞ cents per pound; how many pounds did he buy, his commission being 2½%?

*12.* What are the net proceeds of a consignment of 174 barrels of potatoes, of which 67 barrels are sold at $3.50 per barrel, 24 barrels at $3.40 per barrel, and 83 barrels at $3.20 per barrel, the charges being $37.40, and commission 3%?

*13.* What is the rate of commission on a consignment which yields $339.68 net proceeds, the commission being $12.62 and the other charges $26.30?

*14.* A commission merchant remitted $408.98 as net proceeds of a consignment on which the charges were $8.90 and his commission 2½%. How much was his commission?

*15.* If I remit to my agent $4000 to invest in flour, how many barrels can he buy at $4.90, and what is the balance left over, if his commission is 2¾%?

SUGGESTION.—Find the commission on one barrel and add to the prime cost to find the total cost of one barrel. Divide the sum remitted by the total cost of one barrel. The last remainder after finding the whole number of barrels will be the sum left over.

*16.* What is a stock broker's commission, at ⅛%, for purchasing $10000 worth of bonds?

*17.* Machines that cost $123.25 are listed at $200. If discounts of 12½%, 10% and 5% are allowed, and a commission of 10% paid the salesman, what per cent. is gained?

*18.* Find the net proceeds of the following

**Account Sales**

PHILADELPHIA, July 15, 1918.

*Sold for account of* E. N. WELSH,

*By* J. T. RODMAN & Co.

| | | | |
|---|---|---|---|
| 350 bbls. | Greening Apples | @ $3.70 | |
| 175 " | Russet " | 3.85 | |
| 412 " | Baldwin " | 3.65 | |
| | *Charges* | | |
| Freight | | | 595 60 |
| Cartage | | | 58 50 |
| Commission and guaranty, 5% | | | |
| | Net proceeds | | |

*19.* An attorney collected part of a claim, charging $12.30 for his commission of 5%. If the part collected was but 35% of the entire claim, what was the amount of the loss?

*20.* A Minneapolis merchant shipped to his agent in New York 700 barrels of wheat flour and 2000 barrels of rye flour. The agent sold the wheat flour at $3.50 per barrel and the rye flour at $1.75 per barrel. After taking out his commission he remitted $5801.25. Find per cent. of commission.

*21.* An agent receives $7500 to invest in wheat at 72⅞ cents a bushel, allowing 3% commission. What amount of wheat can he buy?

*22.* An importer buys through his agent in Paris 1217 meters of silk at 17.28 francs per meter, paying the agent 5% commission and expenses 213 francs. If the duty is $1.70 per yard, at what price per yard must the silk be sold in Philadelphia to gain 20%?

*23.* A broker charged $17 as his commission at ⅛% for buying U. S. government bonds. What amount of bonds did he purchase?

*24.* I send an agent $2472.80 to invest in cotton at $5.23 per bale, allowing 5% commission. He pays 14 cents a bale freight, 2 cents a bale storage and 3½ cents a bale drayage. How many bales does he purchase, and what is the unexpended balance?

*25.* An agent sells a consignment of 12000 bushels of wheat at 70¼ cents a bushel, deducts his commission of 2%, and invests the net proceeds in flour at $3.72 per barrel, charging 2% commission for buying. How many barrels does he buy, and what is the balance unspent?

*26.* Find the net proceeds of the following:

Marshall Brothers, Philadelphia, received a consignment of broadcloth from Brown & Sharp on which they paid $27.12 freight, and $6.50 drayage; they charge 5% commission for selling. Jan. 29, they sold 387 yards at $4.63 per yard; Feb. 13, they sold 249 yards at $3.78 per yard; March 10, sold 426 yards at $6.20 per yard; April 29, sold 512 yards at $5.50 per yard. Render an account sales showing the net proceeds due Brown & Sharp.

*27.* A commission agent in Chicago received $28674.75 with instructions to invest $\frac{1}{3}$ of it in corn at 85 cents per bu.; $\frac{3}{5}$ of the remainder is to be invested in rye at 60 cents per bu.; the remainder, after taking his commission of $2\frac{1}{2}\%$ for buying corn, and 2% for buying the rye, was to be invested in oats at 42 cents per bu., charging $1\frac{3}{4}\%$ for buying. What was the agent's total commission?

*28.* A commission house whose charge was 5%, rendered an account purchase to a customer, the total of which was $38356.84; another customer was rendered an account sales showing net proceeds of $35684.92; another customer forwarded $35000 with instructions to invest after retaining commission. What total commission was received from the three customers?

*29.* A. B. Carver & Co., Baltimore, Md., bought for Snoke & Co., Wilkes-Barre, Pa., 125 gallons oysters at 75 cents per gallon; 245 doz. clams at 8 cents per dozen; 575 lbs. blue fish at 5 cents per pound; 835 lbs. white fish at 8 cents per pound; if the commission charge is 3%, and drayage in Baltimore is $4.75, render an account purchase to Snoke & Co.

*30.* A commission merchant sold for me 1500 bales of cloth, each bale containing 42 pieces of $32\frac{3}{4}$ yds. at $9\frac{1}{4}$c. per yd., and invested the proceeds in cotton at $23\frac{3}{8}$c. per lb. His commission on the sale was 2%, for purchasing $1\frac{1}{4}\%$, for guaranteeing payment 3%. How many pounds of cotton should he send me?

*31.* I received from my agent $4957.50 as the net proceeds of a consignment of flour which he sold at $7.25 a bbl. His commission was $217.50, freight charges $212.50, storage $50. What was the rate of commission? How many bbls. did he sell?

*32.* An agent sold wheat at 5% commission, and invested the proceeds in barley at 75 cents per bushel, charging $5\frac{5}{8}\%$ commission. If the agent's commission was $1200, how many bushels of barley did he buy?

# INSURANCE

**580. Insurance** is "a contract by which one party, for an agreed consideration (which is proportioned to the risk involved), undertakes to compensate the other for loss on a specified thing from specified causes. The party agreeing to make the compensation is usually called the *insurer* or *underwriter;* the other, the *insured* or *assured;* the agreed consideration, the *premium;* the written contract, a *policy;* the events insured against, *risks* or *perils;* and the subject, right, or interest to be protected, the *insurable interest.*"—*Bouvier.*

**581.** The principal kinds of insurance are: *fire insurance, marine insurance, accident insurance,* and *life insurance.*

**582. Fire insurance** provides indemnity for loss of or damage to property by fire or the efforts to extinguish fire.

**583. Marine insurance** provides indemnity for loss by shipwreck or disaster at sea.

**584. Accident insurance** provides indemnity for loss occasioned by explosion, breakage, or other accidents to property, or loss of future earnings through personal disablement.

**585. Life insurance** is a contract by which a company, in consideration of certain premiums, agrees to pay to the heirs of a person or, under certain forms, to himself at a specified time, a certain sum of money. The rate of the premium is based on mortality tables and the kind of policy.

**586. Adjustment of losses.** Fire insurance companies pay the full amount of the loss, up to the limit of the policy, unless the contract contains the "average clause," which provides that the company shall pay only such proportion of the loss as the sum insured bears to the full value of the property. Marine insurance policies usually contain the "average clause."

NOTE.—Under the "average clause," if property worth $8000 is insured for ¾ of its value, or $6000, the company, in case of loss, will pay ¾ of the loss; that is, in case of a loss of $4000, the company would pay $3000; in case of a loss of $3000, it would pay $2250, etc. If the property were insured for ½ of its value, the company would pay but ½ of the loss, etc.

**587. To find the premium.**

EXAMPLE.—What is the premium on $12000 insurance at 1½%?

$12000
.01½
12000
6000
$180.00

The other cases under insurance are more theoretical than practical, and are simply applications of percentage principles already explained.

**588.** RULE.—*Multiply amount of insurance named in policy by the rate per cent. of premium.*

## MENTAL PROBLEMS

**589.** *1.* What is the premium on an insurance policy of \$2000 at 2 per cent.?

SOLUTION.—The premium is $\frac{2}{100}$ or $\frac{1}{50}$ of \$2000, which is \$40.

*2.* What is the premium on a policy of \$1200 at $2\frac{1}{2}$ per cent.?

*3.* What is the premium on a policy of \$5000 at $1\frac{1}{2}$ per cent.?

*4.* What premium must I pay for insuring a vessel worth \$8000 for $\frac{3}{4}$ of its value at 2 per cent.?

*5.* Which is the better and how much—an insurance on \$1200 at $2\frac{1}{2}$ per cent. for three years, or \$1200 at 1 per cent. per annum?

*6.* What is the amount of premium on furniture worth \$4200 at $1\frac{1}{2}$ per cent.?

*7.* At 2 per cent., what will be the face of a policy whose premium is \$30?

*8.* If I pay \$50 premium quarterly on insurance at $1\frac{1}{4}$ per cent., how much insurance do I carry?

*9.* What is the face of a policy whose premium is \$30 a quarter, at $1\frac{1}{2}$ per cent.?

*10.* \$23 are charged upon my obtaining an insurance policy at $\frac{3}{4}$ per cent. premium; what is the face of my policy?

*11.* If an agent's commissions for writing insurance during a week amount to \$100 at $\frac{1}{2}$ per cent. commission, what amount has he written?

*12.* If a company has received premiums during a week amounting to \$200, written at 1 per cent., what amount of insurance has it issued?

*13.* I write insurance for a company which charges 2 per cent. and allows me $\frac{1}{4}$ per cent. If I earn \$60 a week, what amount must the company issue?

## WRITTEN PROBLEMS

**590.** *1.* What is the premium at $1\frac{1}{8}\%$ on \$6250 insurance?

*2.* If \$105 are paid as premiums for five years on a policy of \$3500, what is the rate per annum?

*3.* At the rate of $2\frac{1}{8}\%$, \$264.69 were paid as premium; what was the amount of the risk?

*4.* What is the cost of insuring a cargo valued at \$5674.50 for 75% of its value, at the rate of $1\frac{3}{4}\%$?

*5.* If the cargo mentioned in the preceding problem were damaged to the extent of \$820, how much would the policy holder receive from the insurance company?

*6.* A insured his house on March 1, 1914, for \$7500 for five years, at $3\frac{1}{2}\%$. What was the amount of "unearned premium" on Sept. 1, 1916?

7. A cargo worth $22,567.30 was insured for 80% of its value at 1½%, premium paid. The ship was lost at sea. How much did the insurance company lose?

8. My store, worth $8000, is insured for ¾ of its value at 1½ per cent and its contents, valued at $6000, for ⅔ of their value at 2 per cent. What is the entire premium?

9. My charge for insuring a house worth $6000 for ⅞ of its value is 1½ per cent. What is the amount of my bill?

10. My property, worth $9000, is insured for ⅔ of its value in a company at 1½ per cent and for ⅓ of its value in a company at 2 per cent. What amount of insurance do I carry, and what does it cost me?

11. A merchant has a stock valued at $12,000 which one company offers to insure for ⅔ of its value at 1½ per cent, and another for ¾ of its value at 1¼ per cent. What will be the difference in cost between the premiums?

12. A merchant insured his stock of goods with one company for $3000 with another for $2000, and with a third for $2500. If the goods should be damaged by fire to the extent of $4000, how much should each company pay?

13. What is an insurance broker's commission at 3%, for placing $40,000 insurance at ½%?

14. What per cent of loss would a marine insurance company pay on goods valued at $46,574.18 and insured at $35,000?

15. A man had his life insured for $5000, paying an annual premium of $102.85. After paying premiums for 12 years he died. What per cent more did the company pay the beneficiary than had been received in premiums?

16. B insured his house in one company at 1½% premium and in another at ¾% premium. His total premium was $105. If the amount of the second policy was half that of the first, what was the amount of each?

17. A cargo was insured for equal amounts in three different companies at a premium of 1½%, in the first, 1%, in the second and 1¼%, in the third, the total insurance being ⅘ of the value of the cargo. The cargo was lost by shipwreck and the owner thereby lost $7145, including the premiums he had paid. For what amount were the policies drawn?

18. What is merchandise worth that is insured for ⅞ of its value at a premium of ¾%, if the premium amounts to $17.06?

19. A building and its contents are separately insured, the building for ⅔ of its value at ¾%, premium, and the contents for ⅘ of their value at 1% premium. The premium for insuring both is $1491.67. If the value of the contents is $8000 less than that of the building, what is the building worth?

20. A manufacturer had his factory insured at 1½% premium, and while the building was erected, thus increasing the risk, the rate was raised to 2½%, and $2000 additional insurance taken. The additional premium was $52.50. What was the original premium?

*21.* Certain insurance policies contain the following co-insurance clause:

"If at the time of fire the whole amount of insurance on the property covered by each separate item of this policy on property as described in such item shall be less than 80% of the actual cash value thereof, this company shall in case of loss or damage be liable for only such proportion of such loss or damage as the amount insured under said item shall bear to the said 80% of the actual cash value of the property covered by such item." The co-insurance clause is an average clause in which the per cent. named (often 80%) of the actual value is the basis of payment in case of loss.

Under the provisions of this clause, how much would each company pay in case of damage amounting to $420 on property worth $12000, insured for $3000 in each of three companies? How much would each pay if the property had been insured for $3500 in each company?

*22.* Under the 80% co-insurance clause, as above, how much would be recovered, in case of total loss, on property valued at $60000 and insured to the extent of $48000? How much on property valued at $60000 and insured for $40000?

*23.* A factory (worth $3000) and its contents were insured for $10000, as follows: $2000 on building, $3000 on machinery (worth $5000), and $5000 on stock (worth $8000). The building was damaged by fire to the amount of $1000, the machinery $4000, and the stock was a total loss. How much was the claim against the insurance company? What was the premium at 1½%? What was the owner's loss? What was the company's loss, if the risk was covered (1) by an "ordinary policy"; (2) if the policy contained the "average clause"; (3) if the policy contained the "co-insurance clause"?

*24.* A building and its contents were separately insured; the building for $\frac{3}{4}$ of its value; the contents for $\frac{7}{8}$ of its value; fire damaged the building to the amount of $3500, and destroyed stock valued at $15000. What should be recovered from the insurance company?

*25.* A property was insured for 80% of its value at 1⅛% premium; fire damaged the property to the amount of $8000; after the company had paid the claim, its loss was found to be $6184. What did the owner lose?

*26.* An insurance company having taken a risk on a vessel of $185000 at 1¾%, reinsured 20% of its risk at 1⅛% premium; 25% of the remainder was re-insured at 1½% premium; 10% of the remainder was re-insured at 1% premium. For what net amount is the first company liable? What would be their loss if the vessel should be totally destroyed?

## REVIEW PROBLEMS

**591.** *1.* A railway company's earnings for the years 1912 and 1913 were $3648729.13. The earnings for 1913 were 14% more than for 1912. Find earnings for each year.

*2.* What per cent. above cost must goods be marked to allow a discount of 20% and 5% and still yield a profit of 16⅔%?

*3.* One-fourth of a consignment of goods is sold at a loss of 18%, and one-third at a gain of 6½%. For how much above cost must the balance be sold to net a profit of 20% on the whole consignment.

*4.* In a certain compound the weights of the ingredients are, respectively, 4 oz.; 2 oz. 13 pwt.; 20 gr.; and 1 oz. 4 gr. Find the per cent. of each ingredient.

*5.* Sold a bicycle for 10% less than it cost me. If I had sold it for $60, my gain would have been 20%. How much did I lose?

*6.* If A sells goods at list price less 40%, 25% and 10%, and B sells at same list less 30%, 10% and 10%, what per cent. of A's net selling price is B's?

*7.* How much is Jones's investment if his income at 8% is $1893.76?

*8.* Consigned merchandise to an agent, which he sold for $3211.22, charging 2½% commission. According to instructions, he invested the proceeds in wheat after taking out his commission of 2% for purchasing. How much did he have to invest?

*9.* A sold B a horse for $114, which is 5% less than A paid for him. B sells the horse for 15% more than it cost A. How much did B receive for the horse?

*10.* What will be the cost of insuring a house for $7500 at ⅞%, and the furniture, etc., for $3000 at ⅝%?

*11.* After retaining 3½% commission for selling a consignment of cotton, my agent paid me $4174.59. How much was his commission?

*12.* Sold a farm for $3904.32, thereby losing 17%. The sum I paid for the farm was what I received as my one-third share of a legacy, after the executor had received 2% for settling the estate. What was the value of the entire estate?

*13.* A property being sold at 23⅞% above cost, the gain amounted to $1741.92; what was the cost?

*14.* In a school examination, James obtained the correct results to 7 problems out of 10 in arithmetic, spelled correctly 34 out of 40 words, answered correctly 14 out of 15 questions in geography, 19 out of 23 in grammar, and 13 out of 18 in history; what per cent. general average did he make?

*15.* If the above examination had been averaged according to the U. S. civil service method, and arithmetic had been given a value of 6, spelling 5, geography 3, grammar 4, and history 2, what would have been his general average?

*16.* What per cent. profit is made by buying wine at 2 francs a liter and selling it at $2 a gallon?

*17.* Bought goods for $382.75; what price must I ask for them so as to be able to allow a discount of 25%, 10% and 5%, and net a profit of 20%?

*18.* If the par value of the stock of a certain railway is \$50 per share, and it is selling for $97\frac{3}{8}\%$ of its par value, what will 150 shares cost if purchased through a broker who receives \$9.38 commission?

*19.* A grocer bought a quantity of sugar at 22 pounds for a dollar, and sold half of it at 18 pounds for a dollar, and half of it at 19 pounds for a dollar; what per cent. did he gain on the whole?

*20.* On June 8, 1915, E. S. Grayson & Bro., commission merchants, of Philadelphia, received from L. N. Clark, of Chester, Pa., a consignment of 1437 lbs. of butter, to be sold for his account and risk. They paid \$6.70 charges, and sold the butter as follows: On June 10, 329 lbs. @ 35 cents; June 12, 412 lbs. @ $41\frac{1}{4}$ cents; June 14, 224 lbs. @ $37\frac{7}{8}$ cents; June 15, 118 lbs. @ $36\frac{1}{2}$ cents; June 17, 272 lbs. @ 39 cents; June 18, 82 lbs. @ $38\frac{1}{4}$ cents. On June 21 they rendered an account sales, charging 5% commission. Make out the account sales in proper form, showing Clark's net proceeds.

*21.* A man having a certain sum of money, invested $\frac{1}{2}$ of it in wheat, $\frac{1}{3}$ of it in oil, and the remainder in cotton. He sold the wheat at a profit of 6%, the oil at a loss of 2%, and the cotton at a profit of 9%. If the total amount received from the sales was \$6230, how much had he at first?

*22.* If a grocer pays \$57.60 for 180 lbs. of Mocha coffee, how many pounds of Java coffee costing 26 cents per pound must he mix with it in order to gain 20% by selling the mixture at 36 cents per pound?

*23.* What is 6% of £43 13s. 8d.?

*24.* A sold goods to B for \$967.20 and gained 3%. B sold them to C and lost 3%. How much more or less did C pay than A?

*25.* What per cent. larger than a gallon is a half peck?

*26.* If I lose 10% by selling cloth at 72 cents per yard, at what price must I sell it to gain 25%?

*27.* The net amount of a bill of goods is \$3783.18, the discounts allowed being 35%, 15% and 10%, and 2% for cash. What is the gross amount of the bill?

*28.* The capital stock of a railroad is \$28500000. Its gross earnings for the year 1913 were \$3642819.76, and its operating expenses \$1938743.29. After paying \$825000 interest on its bonded debt, what per cent. of its earnings remains?

*29.* If an agent's commission for selling \$3264 worth of goods is \$96.92, how much would he receive, at the same rate, for selling \$7492.85?

*30.* Bought two houses, 28% of the cost of the first being equal to 20% of the cost of the second. If the second cost \$2000 more than the first, what was the cost of each?

*31.* By selling 40% of a purchase at 20% profit, 65% of the remainder at 15% profit, and what then remained at 10% loss, a net gain of \$468.69 was made. Find amount of purchase.

*32.* A, B and C are partners. 30% of A's share in the business equals 18% of B's share, and 12% of B's share equals 40% of C's share. C's share is \$5400. Find A's and B's shares.

12. Robertson withdrew his money from a business that was yielding him an annual income of 8½%, and invested in another business that turned out less profitable than he expected, yielding only 7½% on his investment. His annual income was reduced $100.25 by the change. How much capital had he invested?

13. An attorney succeeded in collecting 70% of a claim placed in his hands, and received $55.20 as his commission at 2%. What was the whole amount of the claim?

14. Half of a cargo that cost $32465.18 was sold at 12% profit, and 30% of it at a loss of 7½%. For how much must the remainder be sold to yield a net gain of 10%, on the whole?

15. If a dealer sells ⅞ of a pound for what ⅞ of a pound cost, what per cent does he gain?

16. What will it cost me in U. S. money to buy, through my agent at Brussels, 500 meters of lace at 6.15 francs per meter, if the agent's commission is 3%?

17. What per cent profit would I make on the lace in the above problem if, after paying freight and other expenses amounting to $60.20, and $250 duty, I sell the lace at $3 per yard?

18. What per cent of 10 francs is ⅞ of a pound sterling?

19. How must I mark goods that cost $4.17 a yard in order to be able to allow a discount of 20%, and 5%, and still gain 25%?

20. A real estate broker sold two houses for $1200 and $4500 respectively. The first house brought 25½% more than cost, and the second 10% less than cost. If the broker's commission for selling was 3%, how much did his principal gain or lose on both houses?

21. My agent in Pittsburgh purchased for me 1,200 bbls. of oil at $3 per barrel. He sold 500 bbls. of it at $3.75 per barrel, and the remainder at $2.60. His commission was 2% for buying and 3% for selling. How much should he remit in settlement of my account?

22. X bought a house for 15% more than its assessed value. He made improvements costing $800, and then sold the house at a net gain of 20%, receiving $11,280 for it. Find how much the taxes on the house amount to at $1.40 on $100.

23. After paying an agent's commission of 2½%, $1,247.40 were received as proceeds of a sale. For how much were the goods sold?

24. If goods are bought at a discount of 25%, 10%, and 5%, off list and sold at same net price, less 15%, and 10%, what is the gain per cent?

25. A grocer bought five dozen eggs at the rate of 4 for 5 cents and five dozen at the rate of 2 for 5 cents. He sold them at the rate of 5 for 10 cents under the impression that he was selling them at cost. What per cent did he lose?

26. What per cent of £5 17s. 6d. is 93.75 francs if £1 = 25.25 francs?

27. From a cask containing 217.56 liters of wine 5% was lost by leakage. How many quarts remained?

28. How much will it cost to excavate a cellar 60 ft. x 40 ft. and 8 ft. deep, ½ of it being rock at 90 cents a cubic yard and the remainder clay at 60 cents a cubic yard?

# INTEREST

**592. Interest** is a charge made for the use of money.

**593.** The **principal** is the sum for the use of which interest is charged.

**594.** The **rate** is the per cent. of the principal charged for its use for one year.

**595.** The **amount** is the sum of the principal and interest.

**596. Simple interest** is interest on the principal only.

**597. Compound interest** is interest on the sum of the principal and unpaid interest.

**598. Common interest** is interest computed on the basis of 12 months of 30 days each, or 360 days to the year.

**599. Accurate** or **exact interest** is interest computed on the basis of 365 days to a year.

**600. Legal interest** is interest at the rate per cent. established by law for cases in which the rate is not specified.

**601. Accrued interest** is the unpaid interest on an obligation, usually not yet due.

**602. Usury** is the crime of charging a higher rate of interest than is allowed by law.

**603. To find the interest on any principal.**

## GENERAL FORMULAS

**For one year.** Principal × Rate (expressed decimally) = Interest

**For part of a year.**

Principal × Rate (expressed decimally) × $\frac{\text{No. of days}}{360}$ = Interest

EXAMPLE 1.—Find the interest of $360 for 3 mo. 13 da. at 6% per annum.

EXPLANATION.—The interest for 1 year is .06 times the principal, or $21.60. 3 mos. are equal to 90 days; adding the 13 days makes 103 days, or $\frac{103}{360}$ of a year. The interest for $\frac{103}{360}$ of a year is $\frac{103}{360}$ of $21.60, or $6.18.

Or, by cancelation

$$\frac{360 \times .06 \times 103}{360} = 6.18$$

$360 Prin
.06 Rate
21.60 Int. for 1 yr
103
6480
2160
360)2224.80($6.18 Int. for 103 da
2160
648
360
2880
2880

EXAMPLE 2.—What is the interest of \$360 for 2 yr. 3 mo. 13 da. at 6 % per annum?

| | | |
|---|---|---|
| \$360 principal | \$21.60×2 | \$43.20 int. 2 yr. |
| .06 rate | 1.80×3 | 5.40 int. 3 mo. |
| 12)\$21.60 interest 1 yr. | .06×13 | .78 int. 13 da. |
| 30) 1.80 int. 1 mo. | | \$49.38 int. 2 yr. 3 mo. 13 da. |
| .06 int. 1 da. | | |

EXPLANATION.—The interest for one year is .06 of the principal, or \$21.60; the interest for one month is $\frac{1}{12}$ of a year's interest, or \$1.80; the interest for one day is $\frac{1}{30}$ of a month's interest, or \$.06. The interest for two years is twice one year's interest or \$43.20; for 3 mos. it is three times one month's interest, or \$5.40; for 13 das. it is thirteen times one day's interest, or \$.78. Adding, we have \$49.38, the interest for 2 yr. 3 mo. 13 da.

REMARK.—There is no one application of the general interest formulas that is best for all cases. The student who has mastered the general formulas should be led to see clearly the special applications and to discriminate in their use.

**604.** RULE.—*To find one year's interest, multiply the principal by the rate expressed decimally.*

*To find the interest for part of a year, multiply one year's interest by as many 360ths as there are days.* (*Or, for each month, take* $\frac{1}{12}$ *of a year's interest; and for each day,* $\frac{1}{30}$ *of a month's interest.*)

### MENTAL PROBLEMS

**605.** *1.* I lend Jones \$100 for one year, interest at 6 per cent.; what sum should he pay me for the use of the money?

SOLUTION.—At 6 per cent. for 1 year, the interest would equal .06 of the principal, which is \$6; hence he should pay me \$6 interest.

*2.* What should I receive for lending Borrower \$200 for 6 months, at 6 per cent.?

*3.* What should I return to Lender, principal and interest, for the use of \$200 for 3 months at 6 per cent.?

*4.* What would be the yearly interest on a mortgage of \$400 at 5 per cent.?

*5.* The holder of a note for \$400, bearing interest at 6 per cent., receives the interest in quarterly payments. What sum does he receive at each payment?

*6.* For what amount should I draw a check in full payment of a loan of \$200 obtained 90 days ago, interest at 6 per cent.?

*7.* Find the interest on \$360 for 4 months at 4 per cent.

*8.* What amount of income shall I receive semi-annually on a \$500 bond bearing 4 per cent. interest?

*9.* What amount must I return in 6 months for a loan of \$350 at 4 per cent.?

## ORAL EXERCISE

**606.** What is the interest on

*1.* $100 for 1 yr. at 6%?
*2.* $200 for 1 yr. at 6%?
*3.* $100 for 2 yrs. at 6%?
*4.* $200 for 2 yrs. at 6%?
*5.* $100 for 2 yrs. at 5%?
*6.* $200 for 2 yrs. at 5%?
*7.* $200 for 2 yrs. at 4%?
*8.* $200 for 1 yr. 6 mo. at 6%?
*9.* $300 for 1 yr. 3 mo. at 4%?
*10.* $100 for 6 mos. at 6%?
*11.* $200 for 3 mos. at 4%?
*12.* $100 for 60 das. at 6%?
*13.* $200 for 60 das. at 5%?
*14.* $300 for 90 das. at 6%?
*15.* $100 for 120 das. at 6%?
*16.* $200 for 2 yrs. at 5%?
*17.* $150 for 1 yr. 6 mo. at 4%?
*18.* $50 for 2 yr. 4 mo. at 6%?
*19.* $3600 for 20 das. at 2%?
*20.* $1000 for 36 das. at 8%?

## WRITTEN EXERCISE

**607.** *1.* Find the interest on $1530 for 2 yr. 6 mo. at 6%.

*2.* Find the interest on $890 for 3 yr. 3 mo. at 6%.

*3.* What is the amount of $364 for 2 yr. 9 mo. at 6%?

NOTE.—Principal + Interest = Amount

*4.* What is the interest on $1000 from Jan. 1, 1914, to April 1, 1915?

NOTE.—When the time extends beyond a year, find time by compound subtraction.

*5.* Find the amount of $2520 from May 1, 1914, to November 1, 1915, int at 5%.

*6.* Find the interest of $378 for 1 yr. 4 mo. at 6%.

*7.* Of $756 for 2 yr. 2 mo. 15 da. at 4%.

*8.* Of $2425.60 for 3 yr. 5 mo. 18 da. at 5%.

*9.* Of $589.70 for 84 days at 7%.

*10.* Of $642.50 for 7 mo. 19 da. at 3%.

*11.* Find the amount of $342.42 from Feb. 5, 1914, to March 15, 1915, at 7%

*12.* I borrowed $360.50 Aug. 1, 1914, and returned it March 5, 1915, interest at 6½%. How much was paid?

*13.* If $150 was loaned Aug. 5, 1913, and returned with interest at 7% on Mar 1915, how much was paid?

*14.* A merchant sells an article for $35 cash. How much more should he ch a customer who wants 5 months' credit, if money is worth 6 per cent.?

*15.* A speculator bought $25000 U. S. Bonds. He held them 38 days. How m interest at 2% did they earn while he owned them?

*16.* A money lender who charges 6% on his money called in a loan of $185370 having it out 28 days. How much interest shall he receive?

## SIX PER CENT. METHOD

**608.** At six per cent the interest for one year is .06 of the principal. For one month, or $\frac{1}{12}$ of a year, it is $\frac{1}{12}$ of .06, or .005 of the principal. For one day, or $\frac{1}{30}$ of a month it is $\frac{1}{30}$ of .005 or .000$\frac{1}{6}$ of the principal.

EXAMPLE. What is the interest of $1348 at 6% for 3 yr. 6 mo. 18 da.?

| | | |
|---|---|---|
| 3 × .06 = | .18 | $1348 |
| 6 × .005 = | .030 | 213 |
| 18 × .000$\frac{1}{6}$ = | .003 | 4 044 |
| | .213 | 13 48 |
| | | 269 6 |
| | | $287.124 |

Or

| | |
|---|---|
| $1348 | $1348 |
| .213 | .06 |
| 4 044 | 80.88 |
| 40 44 | .3 |
| $84.484 interest for 6 mo. 18 da. | $242.64 interest for 3 yr. |
| | 44.48 |
| | $287.12 interest for given time. |

**609.** RULE. *Find what per cent of the principal the interest will be for the whole time required as follows:*

*Take as many times .06 as there are years, as many times .005 as there are months, and as many times .000$\frac{1}{6}$ as there are days; add these rates, and multiply the principal by their sum.*

NOTE. Another way to state this rule is: *Find first the interest of one dollar for the given time, and then take as many times this amount as there are dollars in the principal.* Or take $\frac{1}{2}$ of the months and call the result cents, and take $\frac{1}{6}$ of the days and call the result mills.

**610.** The product by the above rule is the interest at 6%. To find the interest at any other rate, increase or diminish this amount proportionally. Thus, to find the interest at 7%, add $\frac{1}{6}$; to find the interest at 5%, subtract $\frac{1}{6}$; at 8%, add $\frac{1}{3}$; at 4%, subtract $\frac{1}{3}$, etc.

### WRITTEN EXERCISE

**611.** What is the interest of

1. $[illegible] for 2 yr. 7 mo. 12 da. at 6%?
2. $[illegible] for 1 yr. 5 mo. 6 da. at 6%?
3. $[illegible] for 4 yr. 9 mo. at 6%?
4. $[illegible] for 1 yr. 4 mo. 24 da. at 6%?
5. $[illegible] for 5 yr. 9 mo. 18 da. at 6%?
6. $[illegible] for 10 mo. 15 da. at 5%?
7. $[illegible] for 3 yr. 3 mo. 3 da. at 7%?
8. $[illegible] for 2 yr. 11 mo. 21 da. at 8%?
9. $[illegible] for 6 yr. 8 mo. 27 da. at 4%?
10. $5645.20 for 1 yr. 2 mo. 2 da. at 3%?

## DAY METHOD

**612. To find the interest of any principal for any number of days at 6%.**

EXAMPLE.—Find the interest of $2560 at 6%, from June 16 to September 7.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| June | 14 days | $2560 | or, | $25.60 | int. for | 60 | da. |
| July | 31 " | 83 | | 8.53 | " | 20 | " |
| Aug. | 31 " | 7680 | | 1.28 | " | 3 | " |
| Sept. | 7 " | 20480 | | $35.41 | " | 83 | " |
| | 83 days | 6)212.480 | | | | | |
| | | $35.413\frac{2}{3}$ | | | | | |

June 16 to Sept. 7 = 83 days — $35.41, interest for 83 days

**613.** RULE.—*Multiply the principal by the number of days, and divide by six, pointing off three places if the principal be dollars, or five, if the principal be dollars and cents.*

**614.** In Pennsylvania, the legal rate of interest being 6% and money loans being usually for terms not exceeding four months, this method is the basis for nearly all interest calculations. The formula is usually modified as below and the operation shortened by cancelation.

Multiply the principal by the days, divide by 60 and point off two places if the principal be dollars only, four places if there be cents in the principal.

EXAMPLE.—Find the interest on $360 for 61 days at 6%.

$$\frac{\overset{6}{\cancel{\$360}} \times 61}{\underset{}{\cancel{60}}} = \$3.66$$

NOTE.—This may be modified for different rates by changing the divisor. For 5% the divisor is 72; for 9%, 40; for 4%, 90; for 4½%, 80; and in general, the divisor is the number obtained by dividing 360 by the rate per cent.

### WRITTEN EXERCISE

**615.** Find the interest of

1. $892 for 79 days at 6%
2. $1763 for 125 days at 6%
3. $4367 for 217 days at 6%
4. $2250 for 87 days at 4%
5. $3975 for 225 days at 3%
6. $1223 for 167 days at 5%
7. $3271.38 for 1 yr. 4 mo. 17 da. at 6%
8. $2794.72 for 2 yr. 5 mo. 23 da. at 5%
9. $745 from March 16 to June 19 at 6%
10. $987 from May 3 to October 14 at 6%
11. $723.93 from August 3, 1914, to June 17, 1915, at 5%
12. $3242.25 from July 2, 1914, to June 7, 1915, at 4%

## 60-DAY METHOD

**616.** The interest on any principal for 60 days at 6% is one per cent of the principal.

EXAMPLE. Find the interest of $1296 for 45 days at 6%.

| | |
|---|---|
| 1% of $1296 is | $12.96 = 60 days' int |
| ½ of 60 da. | 6.48 = 30 " " |
| ½ of 30 da. | 3.24 = 15 " " |
| | $22.68 = 45 days' int |

**617.** RULE. *Find one per cent of the principal by pointing off two decimal places. This gives the interest for 60 days.*

*Then find the interest for the required time by taking aliquot parts of 60 days.*

### WRITTEN EXERCISE

**618.** Find the interest of

1. $1296 for 30 days at 6%.
2. $240 for 60 days at 6%.
3. $7.20 for 45 days at 6%.
4. $5000 for 90 days at 3%.
5. $1200 for 30 days at 6%.
6. $1200 for 60 days at 6%.
7. $1700 for 60 days at 5%.
8. $600 for 70 days at 7%.
9. $300 for 90 days at 8%.
10. $1700 for 45 days at 6½%.
11. $4.17 for 37 days at 6%.
12. $1.25 for 60 days at 6%.
13. $720.32 for 9 mo. 30 da. at 6%.
14. $1250 for 1 yr. 2 mo. 15 da. at 6%.
15. $493.72 for 2 yr. 7 mo. 10 da. at 6%.
16. $1267.50 for 183 days at 6%.
17. $27.84 for 9 mo. 9 da. at 7%.
18. $521.56 for 1 yr. 5 mo. 21 da. at 8%.
19. $672.83 for 231 days at 5%.
20. $1250 for 7 mo. 20 da. at 6%.

## $6000 RULE

**619.** The interest of $6000 at 6% is $1 a day.

EXAMPLE. What is the interest of $6000 at 6%, for 210 days?

| | |
|---|---|
| Interest of $6000 for 210 days | = $210 |
| " $3000 ½ of $6000 | = 70 |
| " $6000 for 210 days | = $280 |

### WRITTEN EXERCISE

**620.** Find the interest of

1. $2000 for 2 yr. 3 mo. 15 da. at 6%.
2. $2400 for 1 yr. 9 mo. 10 da. at 6%.
3. $4500 for 3 yr. 4 mo. 22 da. at 6%.
4. $1200 for 1 yr. 1 mo. 3 da. at 3%.
5. $12500 for 2 yr. 7 mo. 9 da. at 5%.
6. $1000 for 172 days at 6%.
7. $1200 for 312 days at 6%.
8. $3000 for 1 yr. 5% da. at 6%.
9. $4000 for 270 days at 7%.
10. $6000 for 75 days at 6%.

### Common Year Method

**621.** The interest on $1 for 360 days at 1%, or on $360 for 1 day at 1% is one cent. In proportion as the rate is increased, the time or principal to yield one cent of interest is decreased. Thus $1 at 3%, 4%, or 6%, will yield one cent of interest in respectively 120, 90, 60 days, or $120, $90, or $60 respectively for one day will yield one cent. This applies to any rate divisible evenly into 360.

Note.—In the illustrative table it will be seen that pointing off two places in the principal gives the interest for as many days as the rate is contained times in 360; or, conversely, pointing off two places in the days gives the interest on a like number of dollars. Also, that pointing off one place gives the interest for ten times as many days or on ten times as many dollars; while three places pointed off gives the interest for one-tenth as many days or on one-tenth as many dollars.

### Illustrative Table

**622.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $1 at 2 % | for | 180 | days, | or | $180 at 2 % | for | 1 day | $.01 int. |
| $1 at 3 % | " | 120 | " | or | $120 at 3 % | " | " | .01 " |
| $1 at 6 % | " | 60 | " | or | $60 at 6 % | " | " | .01 " |
| $1 at 4 % | " | 90 | " | or | $90 at 4 % | " | " | .01 " |
| $1 at 4½% | " | 80 | " | or | $80 at 4½% | " | " | .01 " |
| $1 at 5 % | " | 72 | " | or | $72 at 5 % | " | " | .01 " |
| $1 at 6 % | " | 6 | " | or | $6 at 6 % | " | " | .001 " |
| $1 at 6 % | " | 600 | " | or | $600 at 6 % | " | " | .10 " |

Example 1.—Find interest on $720 at 4% for 135 days.

| (1) | | | | (2) | |
|---|---|---|---|---|---|
| $7.20 | int. for | 90 days | | $1.35 int. on | $90 |
| 3.60 | " " | 45 " | (½ of 90) | 8 | 8 |
| $10.80 | " " | 135 " | | $10.80 " " | $720 (90×8) |

Example 2.—Find interest on $960 at 4½% for 64 days.

| (1) | | | | (2) | |
|---|---|---|---|---|---|
| $9.60 | int. for | 80 days | | $.64 int. on | $80 |
| 1.92 | " " | 16 " | (⅕ of 80 days) | 12 | 12 |
| $7.68 | " " | 64 " | | $7.68 | $960 (80×12) |

Example 3.—Find the interest on $780 for 19 days at 6%.

| (1) | | | | (2) | |
|---|---|---|---|---|---|
| $.780 | int. | 6 days | | $.019 int. on | $6 |
| 3 | | 3 | | 130 | 130 |
| 2.340 | " | 18 " | (6×3) | $2.470 | $780 (6×130) |
| 130 | " | 1 da. | | | |
| $2.470 | " | 19 days | | | |

### WRITTEN EXERCISE

**623.** *1.* Find the interest on \$475.84 for 96 days at 5%.

*2.* What is the interest on \$286.80 for 126 days at 6%?

*3.* Find the interest on \$876.92 for 69 days at 8%.

*4.* What is the interest on \$345.75 for 136 days at 4½%?

*5.* Find the interest on \$595 for 288 days at 4%.

*6.* What is the interest on \$1350 for 298 days at 4%?

*7.* Find the interest on \$330 for 129 days at 6%.

*8.* What is the interest on \$1320 for 217 days at 5%?

### REVIEW PROBLEMS

**624.** *1.* Find the interest on \$4500 for 1 yr. 1 mo. 1 da. at 6%.

*2.* What is the amount of \$2250 for 3 mo. 3 da. at 6%?

*3.* Find the interest on \$405.50 for 66 days at 7%.

*4.* What is the interest on \$1606 from March 2, 1914, to May 17, 1915, at 4%?

*5.* John Hamaker took up to-day, January 27, 1915, his note for \$350 which was dated April 1, 1914, and bore interest at 5%; what amount did he pay?

*6.* What is the interest on \$778.75 at 6% from September 1, 1914, to January 1, 1915?

*7.* Find the amount of \$500 for 5 mo. 5 da. at 5%.

*8.* What is the interest on \$1755 for 1 yr. 9 mo. 9 da. at 6%?

*9.* What is the interest on \$299.30 for 192 days at 7%?

*10.* What is the interest on \$1666 for 6 mo. 6 da. at 6%?

*11.* What is the interest on \$777 for 77 days at 7%?

*12.* Find the amount of \$2200 from January 1, 1915, to September 1, 1915, at 4½%.

*13.* What is the interest on \$920 for 8 mo. 8 da. at 8%?

*14.* What is the amount of \$444 for 44 days at 4%?

*15.* What is the interest on \$333 for 3 yr. 3 mo. 3 da. at 3½%?

*16.* Find the interest on \$508.50 for 11 mo. 15 da. at 9%.

*17.* Find the interest on \$1532.75 from February 1, 1915, to October 1, 1915, at 10%.

*18.* What is the amount of \$1410.18 for 10 mo. 26 da. at 12%?

*19.* What amount will take up January 1, 1915, a note for \$2000 given July 1, 1914, bearing interest at 6%?

*20.* I buy a house subject to a 5% mortgage of \$2500, payable half-yearly. What is the amount of my semi-annual payment?

*21.* What semi-annual income do I receive from a \$1200 bond bearing interest at 3½%?

*22.* A ground rent of $1500 bears interest at 6%, payable quarterly; what is the amount of the quarterly payment?

*23.* My property has against it a 5% mortgage of $1200 and a 6% ground rent of $800. What is the amount of my yearly interest payments?

*24.* Borrowed $750 on January 18, 1915, at 4 per cent. What amount must I pay September 6, 1915?

*25.* Bought a house on August 20, 1913, for $8000, paying $2500 cash and giving a mortgage at 5% for the balance. Paid for taxes, repairs, etc., $218.30. Collected rents amounting to $780. Sold the house September 13, 1915, for $8600. How much did I gain?

*26.* I have a lot of ground which I offer for sale at $1300. In order to get 6% on my money how much should I charge a man for it, who offers to pay $300 down and settle the balance by paying $200 every three months?

*27.* On March 13, 1915, at Philadelphia, Pa., A sells B a bill of goods amounting to $492.83, on three months' credit. How much should B pay A on October 26, 1915?

*28.* I borrowed $327 on May 24, 1915, interest at 4½%, and $725.75 on January 15, 1916, interest at 5%. What was due on the two debts September 1, 1916?

*29.* On May 4, 1914, a merchant bought goods for $780.16 on 90 days' credit, but did not pay for them until April 7, 1915. What amount did he pay, interest being reckoned at 6% from the expiration of the 90 days?

*30.* John Smith borrowed $752, June 23, 1913, interest at 5%, and $634.40 on December 23, 1913, interest at 5½%. What total amount did he owe on October 1, 1914?

*31.* Sept. 4, 1914, I bought a quantity of grain for $840.70; Nov. 7, 1914, I sold part of it for $580.90; Jan. 27, 1915, the remainder for $325.40. Money being worth 6%, how much did I gain by the transaction?

*32.* May 9, 1914, a dealer borrowed sufficient money at 5% to pay for 280 bushels corn, at 65c. a bushel and for 300 bushels oats at 40c. a bushel. Corn advanced 8c. a bushel, and oats 4c. a bushel. He sold at these advanced prices on July 27, 1914. After returning the money borrowed and the accrued interest, what was his gain?

*33.* I have an account with a firm of bankers and brokers, and have an understanding with them that they are to allow me 4 per cent. interest on the money I have on deposit with them, but, whenever my account is overdrawn, I am to pay them 6 per cent. interest on the debit balance. On February 21, 1914, I deposit $3216.36; on April 13, 1914, they purchased stock on my account, amounting to $9218.75; on July 7, 1914, they sell the stock for me for $9462.25. What is the balance to my credit on November 10, 1914?

## INTEREST PROBLEMS

**625. To find the rate when the principal, interest and time are given.**

EXAMPLE.—At what rate will \$480 produce \$4.50 interest in 75 days?

| | | | | | |
|---|---|---|---|---|---|
| \$480 | principal. | | | | |
| \$4.80 | int. for | 60 | days | at 6% | |
| 1.20 | " | 15 | " | " | ($\frac{1}{4}$ of 60 days) |
| \$6.00 | " | 75 | " | " | |
| \$1.00 | " | 75 | " | 1% | ($\frac{1}{6}$ of 6%) |
| \$4.50 ÷ \$1 = 4½ | | Result, 4½% | | | |

**626.** RULE.—*Divide the given interest by the interest of the given principal at 1% the given time.*

### MENTAL PROBLEMS

**627.** *1.* At what rate per cent. will \$100 produce \$6 interest in 1 year?

SOLUTION.—As 1 per cent., which is $\frac{1}{100}$ of the principal, will produce \$1 in one year, to produce will require as many times 1 per cent. as \$1 is contained times in \$6, or 6 per cent.

*2.* At what rate per cent. will \$200 produce \$12 in 1 year?

*3.* At what rate per cent. will \$300 produce \$20 in 2 years?

*4.* Find what rate per cent. will produce \$7 interest on \$200 in 6 months.

*5.* I received \$3 interest by investing \$180 for 60 days; what rate per cent. was aid?

*6.* Having \$200 on interest for 3 years, I received principal and interest, \$236; hat rate per cent. did I get for my money?

*7.* A loan of \$300, which had been out for 4 years, was returned with \$60 nterest; what was the rate per cent.?

*8.* At what rate per cent. will \$160 in 5 years give \$64 interest?

*9.* Find at what rate per cent. \$100 will in 4 years amount to \$128.

*10.* At what rate per cent. will \$500 amount to \$510 in 90 days?

### WRITTEN EXERCISE

**628.** At what rate will

*1.* \$725 produce \$8.82 interest in 73 days?

*2.* \$2430 produce \$25.92 interest in 6 mo. 12 da.?

*3.* \$718.75 produce \$39.53 interest in 1 yr. 2 mo. 20 da.?

*4.* \$522.50 produce \$96.41 interest in 2 yr. 7 mo. 19 da.?

*5.* \$358.60 produce \$9.40 interest in 5 mo. 27 da.?

*6.* \$2332 produce \$282.43 interest in 3 yr. 10 da.?

*7.* \$2765.10 produce \$230.23 interest in 1 yr. 6 mo. 5 da.?

*8.* \$563.25 produce \$23.31 interest in 213 days?

*9.* \$917 amount to \$989.81 in 2 yr. 5 mo. 17 da.?

*10.* \$3223.96 amount to \$3250.60 in 10 mo. 25 da.?

**629. To find the time when the principal, interest and rate are given.**

EXAMPLE.—In what time will $375 amount to $412.38, at 5% per annum?

$412.38 amt.
375.00 prin.
37.38 int.

$375
.05
$18.75 int. for 1 yr.

```
        1|9936
18.75)37.38|0000
   18 75|
   18 630
   16 875
    1 7550
    1 6875
       6750
       5625
       11250
```

```
1|9936 yrs.
 |  12
11|9232 mo.
 |   30
27.6960 days
```

Result, 1 yr. 11 mo. 28 da.

**630.** RULE.—*Divide the given interest by the interest of the given principal for one year at the given rate. The result will be years; reduce the fraction, if any, to months and days.*

### MENTAL EXERCISE

**631.** *1.* In what time will $100 at 6 per cent. earn $18 interest?

SOLUTION.—In 1 year the interest will equal $\frac{6}{100}$ or $\frac{3}{50}$ of $100, which is $6, and to earn $18 will require as many years as $6 is contained times in $18, or 3 years.

*2.* In what time will $200 at 5 per cent. earn $50 interest?
*3.* In what time will $200 at 7 per cent. earn $28 interest?
*4.* In what time will $200 at 6 per cent. earn $2 interest?
*5.* In what time will $300 at 10 per cent. earn $90 interest?
*6.* In what time will $80 at 5 per cent. earn $10 interest?
*7.* In what time will $300 at 6 per cent. amount to $309?
*8.* In what time will $50 at 6 per cent. earn $1½ interest?

### WRITTEN EXERCISE

**632.** *1.* In what time will $5000 produce $112.70 interest at 6%?
*2.* $2230 produce $64.27 interest at 6%?
*3.* $3917.25 produce $910 interest at 5%?
*4.* $325 produce $72.18 interest at 4%?
*5.* $1897.10 produce $321.37 interest at 3%?
*6.* $376.75 produce $184.12 interest at 8%?
*7.* $632.80 amount to $711.13 interest at 6%?
*8.* $421.89 amount to $500 interest at 5½%?
*9.* $1124 produce $321.17 interest at 2½%?
*10.* $789.73 amount to $821.50 interest at 7%?
*11.* In what time will $4263.13 produce $289.10 interest at 5½% per annum?
*12.* In what time will $750 produce $150 interest at 5½%?

**633. To find the principal when the interest, time and rate are given.**

EXAMPLE.—What principal will produce $42.99 interest in 2 yr. 3 mo. 15 da. at 5%?

| | |
|---|---|
| $1 | 375 186 |
| .05 | .1145⅝)42.9900 |
| $.05 int. for 1 yr. | .6875)257.9400 000 |
| .10 int. for 2 yr. | |
| .0125 int. for 3 mo. (¼ of 1 yr.) | |
| .0020⅝ int. for 15 da. (⅛ of 3 mo.) | |
| $.1145⅝ int. for 2 yr. 3 mo. 15 da. | Result, $375.19 |

**634.** RULE.—*Divide the given interest by the interest of $1 at the given rate for the given time.*

**635. To find the principal when the amount, time and rate are given.**

EXAMPLE.—What principal will amount to $594.04 in 2 yr. 3 mo. 18 da. at 6%?

| | |
|---|---|
| $1 | 522 000 |
| .06 | 1.138)594.040 000 |
| $.06 int. for 1 yr. | 569 0 |
| .12 int. for 2 yr. | 25 04 |
| .015 int. for 3 mo. | 22 76 |
| .003 int. for 18 da. | 2 280 |
| $.138 int. for 2 yr. 3 mo. 18 da. | 2 276 |
| $1.138 amt. for 2 yr. 3 mo. 18 da. | 4 Result, $522 |

**636.** RULE.—*Divide the given amount by the amount of $1 at the given rate for the given time.*

## MENTAL EXERCISE

**637.** *1.* What principal will in two years, at 6 per cent., amount to $112?

SOLUTION.—At 6 per cent. for 2 years $\frac{12}{100}$ of the principal equals the interest. $\frac{100}{100}$ plus $\frac{12}{100}$ or $\frac{112}{100}$ equals the amount or $112, and $\frac{1}{100}$ equals $\frac{1}{112}$ of the amount or $1; hence the principal is $100.

*2.* What principal will in 7 years, at 5 per cent., amount to $270?

*3.* What principal will in 4 years, at 10 per cent., amount to $140?

*4.* What principal will in 3½ years, at 6 per cent., amount to $242?

*5.* What principal will in 2 years, at 4 per cent., produce $120 interest?

*6.* What principal will in 1 year 6 months, at 7 per cent., amount to $221?

*7.* What principal will in 6 months, at 6 per cent., amount to $412?

*8.* What principal will in 60 days, at 6 per cent., produce $6 interest?

*9.* What principal will in 3 months, at 12 per cent., amount to $206?

*10.* What sum of money was put at interest 3 years ago at 5 per cent., if it now amounts to $575?

*11.* What principal, placed at interest at 8 per cent. for 2 years and 6 months, will produce $60 interest?

*12.* What principal, at interest for 120 days at 6 per cent., will produce $16 interest?

*13.* What principal, at interest for 3 years at 5 per cent., will amount to $805?

*14.* What principal, put at interest for 30 days at 6 per cent., will produce $6 interest?

*15.* I received to-day $1080, which is the amount of a sum of money placed on interest for 2 years at 4 per cent.; what was the sum?

### WRITTEN EXERCISE

**638.** What principal will produce

*1.* $37.27 interest in 179 days at 6%?

SUGGESTION.—Multiply the interest by 60 and divide by the number of days (considered as cents) for which interest is computed; the result is the principal. If the rate is 5%, multiply by 72; if 4%, 90; if 4½%, 60, etc.

*2.* $184.29 interest in 1 yr. 3 mo. 3 da. at 4%?

*3.* $274.50 interest in 8 mo. 21 da. at 6%?

*4.* $381.75 interest in 175 days at 5%?

*5.* $92.17 interest in 2 yr. 7 mo. 11 da. at 6%?

*6.* $188.77 interest in 5 mo. 12 da. at 7%?

*7.* $941.15 interest in 11 mo. 18 da. at 3½%?

*8.* $62.75 interest in 3 yr. 1 mo. 27 da. at 6%?

*9.* $713.41 interest in 4 mo. 18 da. at 2%?

*10.* $825 interest in 1 yr. 10 mo. 2 da. at 5½%?

What principal will amount to

*11.* $4692.10 in 7 mo. 8 da. at 6%?

SUGGESTION.—Multiply the required amount by 60 and divide the result by $60 plus the number of days (considered as cents) for which interest is to be reckoned; at 5% use 72 in place of 60; at 40%, 90, etc.

*12.* $378.12 in 2 yr. 1 mo. 5 da. at 4%?

*13.* $927.38 in 1 yr. 7 mo. 4 da. at 7%?

*14.* $1250 in 5 mo. 23 da. at 6%?

*15.* $3565.15 in 11 mo. 14 da. at 8%?

*16.* $2732.44 in 3 yr. 8 mo. at 5%?

*17.* $5723.92 in 4 yr. 2 mo. 7 da. at 4½%?

*18* $125.78 in 10 mo. 28 da. at 3%?

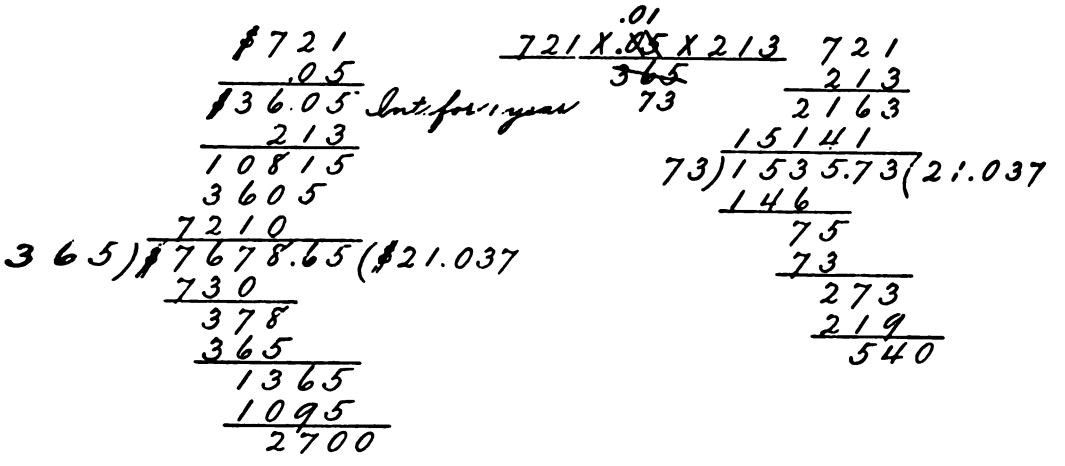
*19.* $3716.17 in 1 yr. 3 mo. 13 da. at 2%?

*20.* $2244.39 in 2 yr. 4 mo. 24 da. at 6%?

## ACCURATE INTEREST

**639. Accurate Interest** is found by taking the exact number of days, and reckoning them as 365ths of a year.

EXAMPLE.—What is the accurate interest of $721 at 5% for 213 days?




NOTE.—In ordinary interest a year consists of 360 days; in accurate interest it consists of 365 days. Since 360 days is $\frac{5}{73}$ or $\frac{1}{73}$ less than 365 days, it follows that the interest for a year of 360 days is $\frac{1}{73}$ more than the interest for a year of 365 days; hence accurate interest may be reckoned by first finding the ordinary interest and then deducting $\frac{1}{73}$ of this amount.

**640.** RULE.—*Find the interest for one year by multiplying the principal by the rate expressed decimally.*

*Multiply the interest for one year by the number of days, and divide the product by 365.*

### WRITTEN EXERCISE

**641.** Find the accurate interest of

1. $623 for 26 days at 6%
2. $937.25 for 123 days at 5%
3. $685 for 73 days at 4%
4. $427.60 for 92 days at 6%
5. $1272.50 for 34 days at 3%
6. $561 from Jan. 13, 1914, to June 3, 1914, at 7%
7. $743.25 from April 17, 1914, to Aug. 13, 1914, at 5%
8. $1864.73 from Feb. 1, 1915, to Dec. 21, 1915, at 6%
9. $324 from April 20, 1914, to Oct. 5, 1914, at 2½%
10. $911.75 from June 27, 1914, to Nov. 5, 1914, at 4½%

## COMPOUND INTEREST

**642. Compound Interest** is interest on the principal, and on the unpaid interest after it becomes due.

NOTE.—Compound interest will not be allowed legally unless there has been an account settled between the parties, or a judgment has been obtained, thus producing a new principal, aggregating principal and interest which has fallen due, or where there is a special agreement in valid form to pay compound interest.

It will also be allowed upon coupons which are overdue, or upon interest wrongfully withheld by a trustee from a beneficiary.

EXAMPLE.—Find the compound interest of $650 for 3 yr. 7 mo. 12 days at 6%.

| | |
|---|---|
| $650 | prin. |
| 39. | int. for 1st yr. |
| 689. | amt. at end of 1st yr. |
| 41.34 | int. for 2d yr. |
| 730.34 | amt. at end of 2d yr. |
| 43.82 | int. for 3d yr. |
| 774.16 | amt. at end of 3d yr. |
| 28.64 | int. for 7 mo. 12 days |
| 802.80 | amt. for full time |
| 650.00 | prin. |
| $152.80 | compound int. |

EXPLANATION.—Since the interest is to be compounded annually, the amount due at the end of the first year, which is $689, will be the basis of interest for the second year; and the amount due at end of second year, $730.34, will be the basis of interest for the third year; the amount due at the end of 3d year, $774.16, will be the basis of the interest for the remaining 7 mo. 12 days of the time; and since the compound amount thus found, $802.80, is made up of the compound interest and the principal, if from this amount the principal be subtracted, the remainder, $152.80, will be the compound interest.

**643.** RULE.—*Find the amount of the principal for the first period of time for which interest is to be reckoned, and make this the principal for the second. Find the amount of this principal for the second period, and thus continue to the end of the given time. The last amount will be the required amount. To find the compound interest, subtract the given principal from the last amount.*

NOTE.—If the time contains fractional parts of a period, as months and days, find the amount due for the full periods, and to this add its interest for the months and days.

REMARKS.—1. For interest to be compounded semi-annually, take one-half the rate for twice the time.

2. For interest to be compounded quarterly, take one-fourth the rate for four times the time.

3. For interest to be compounded bi-monthly, take one-sixth the rate for six times the time.

4. For periods beyond the scope of the table, multiply together the amounts shown for periods, the sum of which will equal the time required. Thus, the amount of $1 at compound interest for 65 years at 4%, is equal to the product of $1 at 4% for 30 years, and the amount of $1 at 4% for 35 years, that is, 3.24339751 × 3.94608899, or 12.79873520.

### WRITTEN PROBLEMS

**644.** *1.* What is the compound interest of $340 for 3 years at 5%?

*2.* What is the compound interest of $750 for 2 years 6 months at 6%, payable semi-annually?

*3.* I lent a contractor $5400 for 7 years at 10% per annum, interest payable quarterly, and took a bond and mortgage to secure the debt and its interest. Not having been paid until the end of the 7 years, how much was required in full settlement?

4 Aug. 4, 1914, I paid in full a note for $1275, dated April 21, 1909, bearing 10% interest. If the interest was compounded annually, what was the amount due at settlement?

5 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 5 years hence at 6% compounded annually?

6 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 6 years hence at 6% compounded semi-annually?

7 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded annually?

8 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 5 years hence at 6% compounded annually?

9 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 6 years hence at 6% compounded annually?

10 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded semi-annually?

11 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded annually?

12 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 5 years hence at 6% compounded annually?

13 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded semi-annually?

14 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded annually?

15 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 5 years hence at 6% compounded annually?

16 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded semi-annually?

17 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded annually?

18 Which is worth more, $1000 due 5 years hence at 6% compounded semi-annually, or $1000 due 5 years hence at 6% compounded annually?

19 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded semi-annually?

20 Which is worth more, $1000 due 5 years hence at 6% compounded annually, or $1000 due 6 years hence at 6% compounded annually?

**645.** The labor of computing compound interest may be greatly sho the use of the following

## COMPOUND INTEREST TABLE

Showing the amount of $1 at compound interest at various rates pe any number of years, from 1 year to 50 years inclusive

| Yrs. | 1 per ct. | 1 1-2 per ct. | 2 per ct. | 2 1-2 per ct. | 3 per ct. | 3 1-2 per ct. | |
|---|---|---|---|---|---|---|---|
| 1 | 1.0100 000 | 1.0150 000 | 1.0200 0000 | 1.0250 0000 | 1.0300 0000 | 1.0350 0000 | 1 |
| 2 | 1.0201 000 | 1.0302 250 | 1.0404 0000 | 1.0506 2500 | 1.0609 0000 | 1.0712 2500 | 1 |
| 3 | 1.0303 010 | 1.0456 784 | 1.0612 0800 | 1.0768 9062 | 1.0927 2700 | 1.1087 1787 | 1 |
| 4 | 1.0406 040 | 1.0613 636 | 1.0824 3216 | 1.1038 1289 | 1.1255 0881 | 1.1475 2300 | 1 |
| 5 | 1.0510 101 | 1.0772 840 | 1.1040 8080 | 1.1314 0821 | 1.1592 7407 | 1.1876 8631 | 1 |
| 6 | 1.0615 202 | 1.0934 433 | 1.1261 6242 | 1.1596 9342 | 1.1940 5230 | 1.2292 5533 | 1. |
| 7 | 1.0721 354 | 1.1098 450 | 1.1486 8567 | 1.1886 8575 | 1.2298 7387 | 1.2722 7926 | 1 |
| 8 | 1.0828 567 | 1.1264 926 | 1.1716 5938 | 1.2184 0290 | 1.2667 7008 | 1.3168 0904 | 1 |
| 9 | 1.0936 853 | 1.1433 900 | 1.1950 9257 | 1.2488 6297 | 1.3047 7318 | 1.3628 9735 | 1. |
| 10 | 1.1046 221 | 1.1605 408 | 1.2189 9442 | 1.2800 8454 | 1.3439 1638 | 1.4105 9876 | 1. |
| 11 | 1.1156 683 | 1.1779 489 | 1.2433 7431 | 1.3120 8666 | 1.3842 3387 | 1.4599 6972 | 1. |
| 12 | 1.1268 250 | 1.1956 182 | 1.2682 4179 | 1.3448 8882 | 1.4257 6089 | 1.5110 6866 | 1. |
| 13 | 1.1380 933 | 1.2135 524 | 1.2936 0663 | 1.3785 1104 | 1.4685 3371 | 1.5639 5606 | 1. |
| 14 | 1.1494 742 | 1.2317 557 | 1.3194 7876 | 1.4129 7382 | 1.5125 8972 | 1.6186 9452 | 1. |
| 15 | 1.1609 690 | 1.2502 321 | 1.3458 6834 | 1.4482 9817 | 1.5579 6742 | 1.6753 4883 | 1. |
| 16 | 1.1725 786 | 1.2689 855 | 1.3727 8570 | 1.4845 0562 | 1.6047 0644 | 1.7339 8601 | 1. |
| 17 | 1.1843 044 | 1.2880 203 | 1.4002 4142 | 1.5216 1826 | 1.6528 4763 | 1.7946 7555 | 1. |
| 18 | 1.1961 475 | 1.3073 406 | 1.4282 4625 | 1.5596 5872 | 1.7024 3306 | 1.8574 8920 | 2. |
| 19 | 1.2081 090 | 1.3269 507 | 1.4568 1117 | 1.5986 5019 | 1.7535 0605 | 1.9225 0132 | 2. |
| 20 | 1.2201 900 | 1.3468 550 | 1.4859 4740 | 1.6386 1644 | 1.8061 1123 | 1.9897 8886 | 2. |
| 21 | 1.2323 919 | 1.3670 578 | 1.5156 6634 | 1.6795 8185 | 1.8602 9457 | 2.0594 3147 | 2. |
| 22 | 1.2447 159 | 1.3875 637 | 1.5459 7967 | 1.7215 7140 | 1.9161 0341 | 2.1315 1158 | 2. |
| 23 | 1.2571 630 | 1.4083 772 | 1.5768 9926 | 1.7646 1068 | 1.9735 8651 | 2.2061 1448 | 2. |
| 24 | 1.2697 346 | 1.4295 028 | 1.6084 3725 | 1.8087 2595 | 2.0327 9411 | 2.2833 2849 | 2. |
| 25 | 1.2824 320 | 1.4509 454 | 1.6406 0599 | 1.8539 4410 | 2.0937 7793 | 2.3632 4498 | 2. |
| 26 | 1.2952 563 | 1.4727 095 | 1.6734 1811 | 1.9002 9270 | 2.1565 9127 | 2.4459 5856 | 2. |
| 27 | 1.3082 089 | 1.4948 002 | 1.7068 8648 | 1.9478 0002 | 2.2212 8901 | 2.5315 6711 | 2. |
| 28 | 1.3212 910 | 1.5172 222 | 1.7410 2421 | 1.9964 9502 | 2.2879 2768 | 2.6201 7196 | 2. |
| 29 | 1.3345 039 | 1.5399 805 | 1.7758 4469 | 2.0464 0739 | 2.3565 6551 | 2.7118 7798 | 3. |
| 30 | 1.3478 490 | 1.5630 802 | 1.8113 6158 | 2.0975 6758 | 2.4272 6247 | 2.8067 9370 | 3. |
| 31 | 1.3613 274 | 1.5865 264 | 1.8475 8882 | 2.1500 0677 | 2.5000 8035 | 2.9050 3148 | 3. |
| 32 | 1.3749 407 | 1.6103 243 | 1.8845 4059 | 2.2037 5694 | 2.5750 8276 | 3.0067 0759 | 3. |
| 33 | 1.3886 901 | 1.6344 792 | 1.9222 3140 | 2.2588 5086 | 2.6523 3524 | 3.1119 4235 | 3. |
| 34 | 1.4025 770 | 1.6589 964 | 1.9606 7603 | 2.3153 2213 | 2.7319 0530 | 3.2208 6033 | 3. |
| 35 | 1.4166 028 | 1.6838 813 | 1.9998 8955 | 2.3732 0519 | 2.8138 6245 | 3.3335 9045 | 3. |
| 36 | 1.4307 688 | 1.7091 395 | 2.0398 8734 | 2.4325 3532 | 2.8982 7833 | 3.4502 6611 | 4. |
| 37 | 1.4450 765 | 1.7347 766 | 2.0806 8509 | 2.4933 4870 | 2.9852 2668 | 3.5710 2543 | 4. |
| 38 | 1.4595 272 | 1.7607 983 | 2.1222 9879 | 2.5556 8242 | 3.0747 8348 | 3.6960 1132 | 4. |
| 39 | 1.4741 225 | 1.7872 103 | 2.1647 4477 | 2.6195 7448 | 3.1670 2698 | 3.8253 7171 | 4. |
| 40 | 1.4888 637 | 1.8140 184 | 2.2080 3966 | 2.6850 6384 | 3.2620 3779 | 3.9592 5972 | 4. |
| 41 | 1.5037 524 | 1.8412 287 | 2.2522 0046 | 2.7521 9043 | 3.3598 9893 | 4.0978 3381 | 4. |
| 42 | 1.5187 899 | 1.8688 471 | 2.2972 4447 | 2.8209 9520 | 3.4606 9589 | 4.2412 5799 | 5. |
| 43 | 1.5339 778 | 1.8968 798 | 2.3431 8936 | 2.8915 2008 | 3.5645 1677 | 4.3897 0202 | 5. |
| 44 | 1.5493 176 | 1.9253 330 | 2.3900 5314 | 2.9638 0808 | 3.6714 5227 | 4.5433 4160 | 5. |
| 45 | 1.5648 107 | 1.9542 130 | 2.4378 5421 | 3.0379 0328 | 3.7815 9584 | 4.7023 5855 | 5. |
| 46 | 1.5804 589 | 1.9835 262 | 2.4866 1129 | 3.1138 5086 | 3.8950 4372 | 4.8669 4110 | 6. |
| 47 | 1.5962 634 | 2.0132 791 | 2.5363 4351 | 3.1916 9713 | 4.0118 9503 | 5.0372 8404 | 6. |
| 48 | 1.6122 261 | 2.0434 783 | 2.5870 7039 | 3.2714 8956 | 4.1322 5188 | 5.2135 8898 | 6. |
| 49 | 1.6283 483 | 2.0741 305 | 2.6388 1179 | 3.3532 7680 | 4.2562 1944 | 5.3960 6459 | 6. |
| 50 | 1.6446 318 | 2.1052 424 | 2.6915 8803 | 3.4371 0872 | 4.3839 0602 | 5.5849 2686 | 7. |

## COMPOUND INTEREST TABLE

Showing the amount of $1 at compound interest at various rates per cent. for any number of years, from 1 year to 50 years inclusive

| Yrs. | 4 1-2 per ct. | 5 per ct. | 6 per ct. | 7 per ct. | 8 per ct. | 9 per ct. | 10 per ct. |
|---|---|---|---|---|---|---|---|
| 1 | 1.0450 0000 | 1.0500 000 | 1.0600 000 | 1.0700 000 | 1.0800 000 | 1.0900 000 | 1.1000 000 |
| 2 | 1.0920 2500 | 1.1025 000 | 1.1236 000 | 1.1449 000 | 1.1664 000 | 1.1881 000 | 1.2100 000 |
| 3 | 1.1411 6612 | 1.1576 250 | 1.1910 160 | 1.2250 430 | 1.2597 120 | 1.2950 290 | 1.3310 000 |
| 4 | 1.1925 1860 | 1.2155 063 | 1.2624 770 | 1.3107 960 | 1.3604 890 | 1.4115 816 | 1.4641 000 |
| 5 | 1.2461 8194 | 1.2762 816 | 1.3382 256 | 1.4025 517 | 1.4693 281 | 1.5386 240 | 1.6105 100 |
| 6 | 1.3022 6012 | 1.3400 956 | 1.4185 191 | 1.5007 304 | 1.5668 743 | 1.6771 001 | 1.7715 610 |
| 7 | 1.3608 6183 | 1.4071 004 | 1.5036 303 | 1.6057 815 | 1.7138 243 | 1.8280 391 | 1.9487 171 |
| 8 | 1.4221 0061 | 1.4774 554 | 1.5938 481 | 1.7181 862 | 1.8509 302 | 1.9925 626 | 2.1435 888 |
| 9 | 1.4860 9514 | 1.5513 282 | 1.6894 790 | 1.8384 592 | 1.9990 046 | 2.1718 933 | 2.3579 477 |
| 10 | 1.5529 6942 | 1.6288 946 | 1.7908 477 | 1.9671 514 | 2.1589 250 | 2.3673 637 | 2.5937 425 |
| 11 | 1.6228 5305 | 1.7103 394 | 1.8982 986 | 2.1048 520 | 2.3316 390 | 2.5804 264 | 2.8531 167 |
| 12 | 1.6958 8143 | 1.7958 563 | 2.0121 965 | 2.2521 916 | 2.5181 701 | 2.8126 648 | 3.1384 284 |
| 13 | 1.7721 9610 | 1.8856 491 | 2.1329 283 | 2.4098 450 | 2.7196 237 | 3.0658 046 | 3.4522 712 |
| 14 | 1.8519 4492 | 1.9799 316 | 2.2609 040 | 2.5785 342 | 2.9371 936 | 3.3417 270 | 3.7974 983 |
| 15 | 1.9352 8244 | 2.0789 282 | 2.3965 582 | 2.7590 315 | 3.1721 691 | 3.6424 825 | 4.1772 482 |
| 16 | 2.0223 7015 | 2.1828 746 | 2.5403 517 | 2.9521 638 | 3.4259 426 | 3.9703 059 | 4.5949 730 |
| 17 | 2.1133 7681 | 2.2920 183 | 2.6927 728 | 3.1588 152 | 3.7000 181 | 4.3276 334 | 5.0544 703 |
| 18 | 2.2084 7877 | 2.4066 192 | 2.8543 392 | 3.3799 323 | 3.9960 195 | 4.7171 204 | 5.5599 173 |
| 19 | 2.3078 6031 | 2.5269 502 | 3.0255 995 | 3.6165 275 | 4.3157 011 | 5.1416 613 | 6.1159 390 |
| 20 | 2.4117 1402 | 2.6532 977 | 3.2071 355 | 3.8696 845 | 4.6609 571 | 5.6044 108 | 6.7275 900 |
| 21 | 2.5202 4116 | 2.7859 626 | 3.3995 636 | 4.1405 624 | 5.0338 337 | 6.1088 077 | 7.4002 499 |
| 22 | 2.6336 5201 | 2.9252 607 | 3.6035 374 | 4.4304 017 | 5.4365 404 | 6.6586 004 | 8.1402 749 |
| 23 | 2.7521 6635 | 3.0715 238 | 3.8197 497 | 4.7405 299 | 5.8714 637 | 7.2578 745 | 8.9543 024 |
| 24 | 2.8760 1383 | 3.2250 999 | 4.0489 346 | 5.0723 670 | 6.3411 807 | 7.9110 832 | 9.8497 327 |
| 25 | 3.0054 3446 | 3.3863 549 | 4.2918 707 | 5.4274 326 | 6.8484 752 | 8.6230 807 | 10.8347 059 |
| 26 | 3.1406 7901 | 3.5556 727 | 4.5493 830 | 5.8073 529 | 7.3963 532 | 9.3991 579 | 11.9181 765 |
| 27 | 3.2820 0956 | 3.7334 563 | 4.8223 459 | 6.2138 676 | 7.9880 615 | 10.2450 821 | 13.1099 942 |
| 28 | 3.4296 9999 | 3.9201 291 | 5.1116 867 | 6.6488 384 | 8.6271 064 | 11.1671 395 | 14.4209 936 |
| 29 | 3.5840 3649 | 4.1161 356 | 5.4183 879 | 7.1142 571 | 9.3172 749 | 12.1721 821 | 15.8630 930 |
| 30 | 3.7453 1813 | 4.3219 424 | 5.7434 912 | 7.6122 550 | 10.0626 569 | 13.2676 785 | 17.4494 023 |
| 31 | 3.9138 5745 | 4.5380 395 | 6.0881 006 | 8.1451 129 | 10.8676 694 | 14.4617 695 | 19.1943 425 |
| 32 | 4.0899 8104 | 4.7649 415 | 6.4533 867 | 8.7152 708 | 11.7370 830 | 15.7633 288 | 21.1137 768 |
| 33 | 4.2740 3018 | 5.0031 885 | 6.8405 899 | 9.3253 398 | 12.6760 496 | 17.1820 284 | 23.2251 544 |
| 34 | 4.4663 6154 | 5.2533 480 | 7.2510 253 | 9.9781 135 | 13.6901 336 | 18.7284 109 | 25.5476 699 |
| 35 | 4.6673 4781 | 5.5160 154 | 7.6860 868 | 10.6765 815 | 14.7853 443 | 20.4139 679 | 28.1024 369 |
| 36 | 4.8773 7846 | 5.7918 161 | 8.1472 520 | 11.4239 422 | 15.9681 718 | 22.2512 250 | 30.9126 805 |
| 37 | 5.0968 6049 | 6.0814 069 | 8.6360 871 | 12.2236 181 | 17.2456 256 | 24.2538 353 | 34.0039 486 |
| 38 | 5.3262 1921 | 6.3854 773 | 9.1542 524 | 13.0792 714 | 18.6252 756 | 26.4366 805 | 37.4043 434 |
| 39 | 5.5658 9908 | 6.7047 512 | 9.7035 075 | 13.9948 204 | 20.1152 977 | 28.8159 817 | 41.1447 778 |
| 40 | 5.8163 6454 | 7.0399 887 | 10.2857 179 | 14.9744 578 | 21.7245 215 | 31.4094 200 | 45.2592 556 |
| 41 | 6.0781 0094 | 7.3919 882 | 10.9028 610 | 16.0226 699 | 23.4624 832 | 34.2362 679 | 49.7851 811 |
| 42 | 6.3516 1548 | 7.7615 876 | 11.5570 327 | 17.1442 568 | 25.3394 819 | 37.3175 320 | 54.7636 992 |
| 43 | 6.6374 3818 | 8.1496 669 | 12.2504 546 | 18.3443 548 | 27.3666 404 | 40.6761 098 | 60.2400 692 |
| 44 | 6.9361 2290 | 8.5571 503 | 12.9854 819 | 19.6284 596 | 29.5559 717 | 44.3369 597 | 66.2640 761 |
| 45 | 7.2482 4843 | 8.9850 078 | 13.7646 108 | 21.0024 518 | 31.9204 494 | 48.3272 861 | 72.8904 837 |
| 46 | 7.5744 1961 | 9.4342 582 | 14.5904 875 | 22.4726 234 | 34.4740 853 | 52.6767 419 | 80.1795 321 |
| 47 | 7.9152 6849 | 9.9059 711 | 15.4659 167 | 24.0457 070 | 37.2320 122 | 57.4176 486 | 88.1974 853 |
| 48 | 8.2714 5557 | 10.4012 697 | 16.3938 717 | 25.7289 065 | 40.2105 731 | 62.5852 370 | 97.0172 338 |
| 49 | 8.6436 7107 | 10.9213 331 | 17.3775 040 | 27.5299 300 | 43.4274 190 | 68.2179 083 | 106.7189 572 |
| 50 | 9.0326 3627 | 11.4673 998 | 18.4201 543 | 29.4570 251 | 46.9016 125 | 74.3575 201 | 117.3908 529 |

## REVIEW PROBLEMS

**646.** *1.* Find the interest of \$4723.69 for 2 yr. 8 mo. 14 da. at 6%.

*2.* What will \$528.50 amount to in 1 yr. 4 mo. 22 da. at 4%?

*3.* Loaned \$4500 on February 16, 1914, at 5%. Received the amount due me on May 4, 1915; how much did I receive?

*4.* What sum must be invested at 4% for a child 10 years old, that he may receive \$5000 when he is 21 years of age?

*5.* What principal will amount to \$3273.40, at 6%, if loaned January 15, 1913, and paid August 3, 1915?

*6.* What sum of money will amount to \$3630 in 3 years at 7%?

*7.* What principal will produce \$7 interest, at 5%, in 90 days?

*8.* What sum must be invested in a property that pays 7½% per annum, to produce an income of \$300 a year?

*9.* What sum of money will amount to \$2562 in 9 months at 9%?

*10.* At what rate will \$120 gain \$21, if placed on interest for 3 yr. 6 mo.?

*11.* If \$800 amounts to \$832 in 180 days, what is the rate per annum?

*12.* In what time will \$900 amount to \$1005 at 5 per cent. per annum?

*13.* In what time will \$880 produce \$55 interest at 5%?

*14.* In what time will \$500, placed on interest at 4%, double itself?

*15.* Jan. 1, 1915, I borrowed \$1500 at 6%; on what date will I owe \$1770?

*16.* If a man buys a bill of goods amounting to \$2762.48, terms 60 days or 3% off for cash, how much does he save by borrowing the money at 6% and paying cash?

*17.* A buys a house for \$6000, paying \$2000 cash and giving a mortgage at 5% for the balance. At the end of two years he finds he has paid \$224 in taxes and \$97.50 for repairs, and has received \$800 for rent. What rate of interest does he receive on his investment of \$2000 (reckoning the house to be still worth \$6000)?

*18.* At what rate will \$2475, loaned April 6, 1914, amount to \$2559.94 on March 25, 1915?

*19.* A man buys a piano on the instalment plan. The price is \$275, payable \$35 cash and \$10 monthly until paid. The piano could be bought for \$250 cash. What rate of interest is paid by the instalment plan? What rate if based on the cash price?

*20.* On Jan. 13, 1914, a man borrowed \$5500 at 5%, with which he purchased a piece of land. He afterward sold the land for \$5950, paid the loan with interest and found that he had gained \$172.44. On what date did he sell? Jan 16

*21.* April 1, 1911, I borrowed \$10500 at 4½% interest, and invested it in a farm at \$75 an acre. Aug. 15, 1913, my agent sold 50 acres at \$105 an acre, charging me 3%. The money was deposited in a bank paying 2½% on deposits. April 1, 1915, I sold the remainder of the farm at \$95 an acre. After paying the interest, what was my gain? 1457.74

*22.* How much money invested at 4% will amount to \$10000 in 8 yr. 5 mo. 29 da., compounded semi-annually?

*23.* A firm bought goods on credit and agreed to pay 7% interest on each purchase from its date. Oct. 16, 1913, goods were bought to the amount of $268; Dec. 31, 1913, to the amount of $765.80; Feb. 26, 1914, to the amount of $600; Apr. 1, 1914, to the amount of $325.25. If full settlement was made Aug. 25, 1914, how much cash was paid? 2040.85

*24.* On August 27, 1913, Smith sold his farm for $16000; the terms were, $4000 cash on delivery, $5000 on May 27, 1914, $3000 on Feb. 26, 1915, and the remainder in two years from date of purchase, with 6% interest on all deferred payments. What was the total amount paid?

*25.* Oct. 16, 1915, goods were bought to the amount of $268; Dec. 31, 1915, to the amount of $567.90; on Feb. 26, 1916, to the amount of $575; on May 1, 1916, to the amount of $235.75. The firm agreed to pay 7% interest on each purchase from its date. If full settlement was made Sept. 5, 1916, how much cash was paid?

*26.* How much money must I deposit in a savings bank during each of four years, in order to be able to draw out $440, if the bank pays 4% compound interest on its deposits?

*27.* I bought a house and lot on speculation for $14325; 4 months 23 days from date of purchase, I sold the property for $15840. If money was worth 7½% per annum, how much more did the transaction yield me than if I had lent the purchase money at interest?

*28.* An attorney collects a claim of $875 with ordinary interest thereon from August 29, 1914, to December 14, 1914, at 7%. If the attorney's rate for collecting is 10%, what net proceeds should be paid to the creditor?

*29.* Find the difference between the accurate interest and the common interest on $8750 from May 12, 1911, to October 16, 1911, at 6¾% per annum.

*30.* A contractor borrowed a sum of money for 3 months 24 days at 6%. Not having sufficient funds to meet this obligation when due, he paid 60% of the debt and entire accrued interest by giving his check for $5054, and agreed to continue the balance at 8% per annum until paid. The balance was paid 3 months 24 days later, with interest. What was the amount of the second payment?

*31.* A man engaged in business was making 12½% annually on his capital of $16840. He quit his business and loaned his money at 7½%. What did he lose in 2 yr. 3 mo. 18 da. by the change?

*32.* A tract of land containing 516 acres was bought at $36 an acre, the money being loaned at 5½%. At the end of 2 yr. 9 mo. 15 da., ⅔ of the land was sold at $42 an acre and the money loaned at 5%; 1 yr. 2 mo. 18 da. later the remainder was sold at $38½ an acre and the money loaned at 6%. Five years after borrowing the original sum, the loans were collected and the original sum returned with interest. What was gained or lost?

## BANK DISCOUNT

**647. Bank Discount** is a deduction made from the face of a promissory note or draft for cashing such negotiable paper before maturity. This deduction is the interest of the sum due at the maturity of the note or draft for the number of days for which it is discounted.

**648.** It is called bank discount because one of the chief functions of a bank is the cashing, or buying, of such commercial paper. When such notes are cashed by individuals, however, the same rules are observed.

**649.** The methods of bank discount depend upon State laws and in some cases upon local customs. The methods which follow comply with the statute laws of Pennsylvania and the practise in its chief money center, Philadelphia, and banking circles contributory to it. The legal and usual rate of discount in Pennsylvania is 6%.

**650.** The **due date** of a note is the date upon which the maker is legally bound to make payment at some designated bank or place of business, and if not named in the note, is found by adding to the date of the note the time expressed in it.

**651.** When the time is expressed in months, the due date is determined by calendar months, a calendar month being from a certain date in a month to the same date in the succeeding month; should this month have fewer than 29, 30 or 31 days, the note is due on the last day of the month. A note dated December 31, 1915, for two months, falls due February 28, 1916.

When the **time** is expressed in days, the due date is determined by counting forward the actual number of days. A note dated December 31, 1914, for 60 days, falls due March 1, 1915.

It is a growing custom to write the date of payment in a note instead of the time; as, "On March 1, 1915, I promise to pay, etc."

**652.** Should the due date of a note fall upon a Sunday, a Saturday, or any legal holiday, the note is due and payable on the next business day.

**653.** The **legal holidays** in the State of Pennsylvania are: New Year's Day (Jan. 1), Lincoln's Birthday (Feb. 12), Washington's Birthday (Feb. 22), Good Friday, Memorial Day (May 30), Independence Day (July 4), Labor Day (first Monday in September), Columbus Day (Oct. 12), General Election Day (*Tuesday after the first* Monday in November), Thanksgiving Day (by custom the last Thursday in November), Christmas Day (Dec. 25). Should any of these legal holidays fall on Sunday, they are observed on the Monday following. Saturday is a legal holiday in Pennsylvania, so far as commercial paper is concerned, and paper nominally due on that day is legally due on the following business day.

**654. Days of grace** are abolished in many States, but where recognized by statute the date of maturity falls upon the last day of grace.

**655.** It is not common in Philadelphia to offer interest-bearing notes for discount, but if the banks accept such notes for discount, the discount is reckoned on the face of the note.

**656. To determine the due date of a note.**

EXAMPLE.—(1) What is the maturity date of a note dated Thursday, September 3, 1914, drawn for 3 months? (2) If drawn for 90 days?

(1) September 3 + 3 months = December 3
From September 3 to October 3 = 30 days
" October 3 to November 3 = 31 "
" November 3 to December 3 = 30 "
Total, 91 "
91 days or 13 weeks from September 3 (Thurs.) is December 3 (Thurs.)

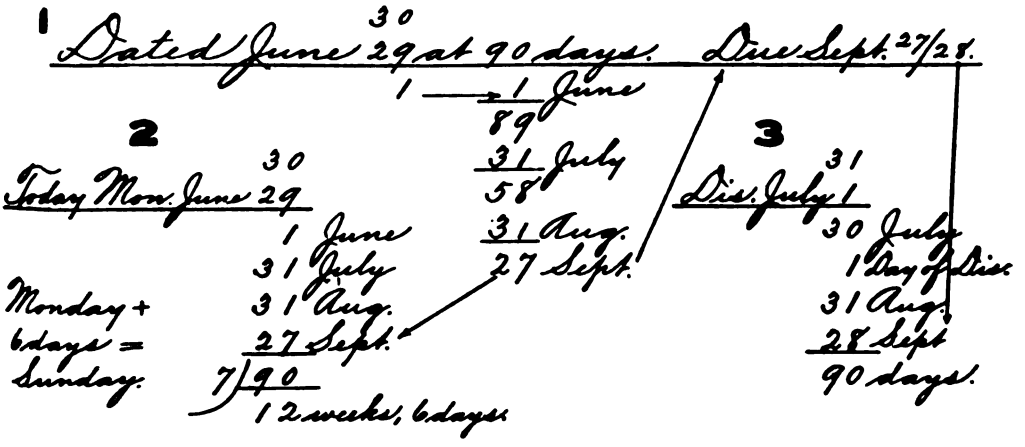
(2) September 3 plus 90 days
September 3 + 30 days = October 3
October 3 + 30 " = November 2
November 2 + 30 " = December 2
90 days = 12 weeks, 6 days
12 weeks and 6 days from Thursday, September 3, is Wednesday, December 2

**657** RULE.—*Divide the number of days between date and due date by 7, and from the day of the week on which dated count forward as many days as there are in the remainder; if Saturday, Sunday or a legal holiday, the date will be the next business day.*

**658.** The face of a note, less the interest for the term of discount, is called the **proceeds.**

**659. To find the proceeds of a note.**

EXAMPLE.—A note dated Monday, June 29, 1914, at 90 days, for $1450 is discounted July 1, 1914. What are the proceeds?




$$\frac{1450 \times .06 \times 90}{360} = \begin{array}{r} 1450.00 \text{ Face of note.} \\ 21.75 \text{ Discount} \\ \hline 1428.25 \text{ Proceeds.} \end{array}$$

EXPLANATION.—OPERATION 1.—To find on what date the 90 days will expire, or the nominal due date.

If the note was dated June 29, one of the 90 days will be in June, leaving 89 days; 31 days will be in July, leaving 58 days; 31 days will be in August, leaving 27 days to be in September. Hence, the 90 days will end September 27, which is the nominal due date.

OPERATION 2.—To find on what day of the week the nominal due date will fall, or to find the legal due date.

Using Monday, June 29, as the day of working the example, we find 1 day left in June; 31 days in July, 31 days in August and 27 days in September, or 90 days from the date of working the example to the due date. Dividing 90 by 7 we get 12 weeks and 6 days; 6 days from Monday is Sunday. Hence, September 27 is Sunday, and the legal due date is Monday, September 28.

*Operation 3.*—To find the term of discount.

This note was discounted July 1, hence the bank would have it 29 days in July plus 1 day for the day of discount, 31 days in August and 29 days in September, or 90 days, which is the term of discount.

*Operation 4.*—Find the interest on $1450 at 6% for 90 days, which is $21.75, or the bank discount, and the proceeds are the difference between the face of the note $1450 and $21.75 the discount, or $1428.25.

or. Date of note, June 29, 1914
Due date, September 27, Sunday; 28 Monday
Discount term, 90 days

| | | |
|---|---|---|
| $1450 face of note | | |
| $14.50 int. for 60 days | | $1450.00 face |
| 7.25 " 30 " | | 21.75 int. |
| $21.75 " 90 " | | $1428.25 proceeds |

**660. Rule.**—*First find the due date of the note by adding to the date of the note the time expressed in it; if the resulting date falls on Saturday, Sunday, or a legal holiday, extend the time to the next business day. When the legal due date has been determined, find the interest on the face of the note for the number of days from the date of discount to the legal due date, including both.*

*Note.*—The usual custom observed in Philadelphia and some other cities, of including both the day of discount and the day of maturity in the term of discount, gives one day more than the difference found by subtracting the dates. As, January 5 to January 15 = 10 days; but January 5 to January 15, inclusive = 11 days.

## MENTAL PROBLEMS

**661.** *1.* What is the bank discount of a note of $60, discounted for 90 days, at 6 per cent.?

Solution.—For 60 days the discount is 60 cents, and for 30 days, which is a half more than 60 days, the discount is a half more than 60 cents, or 90 cents.

2. What is the bank discount of $300 for 60 days?
3. What is the bank discount of $80 for 60 days?
4. What is the bank discount of $120 for 30 days?
5. What is the bank discount of $300 for 120 days?
6. What is the bank discount of $80 for 30 days?
7. What is the bank discount of $150 for 60 days?
8. What is the bank discount of $240 for 63 days?
9. What is the bank discount of $600 for 120 days?
10. What is the bank discount of $300 for 20 days?
11. What are the proceeds of a note for $800 discounted for 30 days?
12. What are the proceeds of a note for $240 discounted for 30 days?
13. What are the proceeds of a note for $400 discounted for 90 days?
14. What are the proceeds of a note for $200 discounted for 60 days?
15. What are the proceeds of a note for $600 discounted for 15 days?
16. What are the proceeds of a note for $600 discounted for 20 days?
17. What are the proceeds of a note for $120 discounted for 12 days?
18. What are the proceeds of a note for $200 discounted for 15 days?

**662.** Find when due, bank discount and proceeds

| | Face | Date of Note | Time | When Due | When Discounted | Discount | Proceeds |
|---|---|---|---|---|---|---|---|
| *1.* | $400.00 | Jan. 4 | 90 da. | | Jan. 11 | | |
| *2.* | $500.00 | Jan. 12 | 60 da. | | Jan. 13 | | |
| *3.* | $1000.00 | Jan. 5 | 3 mo. | | Jan. 7 | | |
| *4.* | $387.25 | Jan. 28 | 30 da. | | Jan. 28 | | |
| *5.* | $487.37 | Jan. 11 | 90 da. | | Jan. 14 | | |
| *6.* | $587.00 | Feb. 1 | 1 mo. | | Feb. 2 | | |
| *7.* | $1250.00 | Feb. 8 | 90 da. | | Feb. 8 | | |
| *8.* | $1187.50 | Feb. 9 | 3 mo. | | Feb. 10 | | |
| *9.* | $988.75 | Feb. 15 | 4 mo. | | Feb. 23 | | |
| *10.* | $546.27 | Mar. 1 | 90 da. | | Mar. 1 | | |
| *11.* | $787.37 | Mar. 9 | 2 mo. | | Mar. 10 | | |
| *12.* | $1358.68 | Mar. 23 | 3 mo. | | Mar. 23 | | |
| *13.* | $186.75 | Mar. 29 | 90 da. | | Mar. 30 | | |
| *14.* | $580.72 | Apr. 1 | 2 mo. | | Apr. 5 | | |
| *15.* | $758.60 | Apr. 12 | 4 mo. | | Apr. 12 | | |
| *16.* | $789.70 | Apr. 26 | 120 da. | | Apr. 27 | | |
| *17.* | $486.92 | Apr. 27 | 60 da. | | Apr. 28 | | |
| *18.* | $767.75 | May 3 | 3 mo. | | May 3 | | |
| *19.* | $463.87 | May 17 | 20 da. | | May 20 | | |
| *20.* | $50.27 | May 26 | 10 da. | | May 26 | | |
| *21.* | $408.75 | May 31 | 90 da. | | May 31 | | |
| *22.* | $568.70 | June 1 | 60 da. | | June 1 | | |
| *23.* | $750.00 | June 3 | 30 da. | | June 3 | | |
| *24.* | $950.00 | June 15 | 60 da. | | June 16 | | |
| *25.* | $567.27 | June 30 | 3 mo. | | June 30 | | |
| *26.* | $868.00 | July 5 | 10 da. | | July 5 | | |
| *27.* | $487.70 | July 6 | 20 da. | | July 7 | | |
| *28.* | $460.75 | July 31 | 2 mo. | | July 31 | | |
| *29.* | $467.50 | Aug. 2 | 60 da. | | Aug. 3 | | |
| *30.* | $1275.00 | Aug. 10 | 120 da. | | Aug. 12 | | |
| *31.* | $1100.00 | Aug. 19 | 30 da. | | Aug. 20 | | |
| *32.* | $1375.87 | Aug. 31 | 1 mo. | | Aug. 31 | | |
| *33.* | $987.70 | Sept. 1 | 3 mo. | | Sept. 2 | | |
| *34.* | $778.70 | Sept. 13 | 90 da. | | Sept. 15 | | |
| *35.* | $1500.00 | Sept. 28 | 60 da. | | Sept. 30 | | |
| *36.* | $1476.80 | Oct. 4 | 15 da. | | Oct. 4 | | |
| *37.* | $1156.76 | Oct. 11 | 3 mo. | | Oct. 13 | | |
| *38.* | $568.95 | Nov. 1 | 90 da. | | Nov. 3 | | |
| *39.* | $759.63 | Nov. 16 | 90 da. | | Nov. 16 | | |
| *40.* | $963.42 | Nov. 30 | 90 da. | | Dec. 1 | | |
| *41.* | $867.54 | Dec. 8 | 60 da. | | Dec. 9 | | |
| *42.* | $568.95 | Dec. 21 | 4 mo. | | Dec. 22 | | |
| *43.* | $1156.76 | Dec. 31 | 60 da. | | Dec. 31 | | |

NOTE.—If date of note or discount date falls on Sunday or legal holiday, consider next business day as date.

**663. To find the face of a note.**

EXAMPLE.—What is the face of a note at 90 days, dated June 29, 1914, discounted on that date, that will yield $1200 proceeds?

June 29+90 days = Sunday, September 27 = Monday, September 28

Fom June 29 to September 28, inclusive = 92 days

$$\begin{array}{rrrrrrr} \$1 & .984\tfrac{2}{3} & ) & 1200 & & \text{Or,} & \\ 92 & 3 & & 3 & & \$60. & \$1200 \text{ Proc} \\ \hline 6\not{0}\not{0}\not{0} ) & 92 & 2.954 & ) & 3600.0000 & ( 1218.68 \text{ face of note} & .92 \quad 60 \\ \$.015\tfrac{1}{3} & & & & & & \$59.08 ) \ 72000 \\ & & & & & & \$1218.68 \text{ fac} \end{array}$$

$\$1.000$

$.015\frac{1}{3}$

$\$.984\frac{2}{3}$ = Proceeds of $1

**664.** *Divide the given proceeds by the proceeds of $1; or, multiply the desired p ceeds by 60; subtract the number of days for which discounted (considered as cents) fr $60 and use this as a divisor. The quotient will be the face.*

NOTE.—If the discount rate is 5%, use 72 in place of 60; if $4\frac{1}{2}$%, use 80; if 4%, use 90, etc.

**665.** Find when due and the face of note.

| | Date of Note | Time | When Due | Date of Discount | Proceeds | Face |
|---|---|---|---|---|---|---|
| *1.* | Jan. 11 | 60 da. | | Jan. 11 | $450.00 | |
| *2.* | Feb. 15 | 30 da. | | Feb. 15 | $500.00 | |
| *3.* | Feb. 23 | 90 da. | | Feb. 23 | $650.00 | |
| *4.* | Mar. 10 | 2 mo. | | Mar. 10 | $450.25 | |
| *5.* | Apr. 12 | 3 mo. | | Apr. 12 | $375.60 | |
| *6.* | June 7 | 4 mo. | | June 7 | $2000.00 | |
| *7.* | Jan. 27 | 1 mo. | | Jan. 27 | $3000.00 | |
| *8.* | Dec. 30 | 2 mo. | | Dec. 31 | $400.75 | |
| *9.* | Dec. 21 | 2 mo. | | Dec. 29 | $786.50 | |
| *10.* | Nov. 29 | 90 da. | | Nov. 30 | $10000.00 | |
| *11.* | July 7 | 4 mo. | | July 7 | $100.00 | |
| *12.* | Aug. 17 | 60 da. | | Aug. 18 | $50.00 | |
| *13.* | July 19 | 90 da. | | July 20 | $300.00 | |
| *14.* | Nov. 17 | 4 mo. | | Nov. 17 | $487.75 | |
| *15.* | May 19 | 30 da. | | May 20 | $568.46 | |
| *16.* | Dec. 3 | 4 mo. | | Dec. 6 | $787.80 | |
| *17.* | Oct. 11 | 30 da. | | Oct. 12 | $1160.75 | |
| *18.* | Jan. 11 | 4 mo. | | Jan. 14 | $1230.50 | |
| *19.* | Aug. 31 | 30 da. | | Aug. 31 | $520.00 | |
| *20.* | July 20 | 60 da. | | July 20 | $1250.00 | |
| *21.* | Feb. 23 | 90 da. | | Feb. 24 | $1178.80 | |
| *22.* | Jan. 21 | 120 da. | | Jan. 22 | $125.75 | |
| *23.* | Oct. 25 | 3 mo. | | Oct. 29 | $499.60 | |
| *24.* | Jan. 4 | 90 da. | | Jan. 5 | $568.90 | |
| *25.* | June 21 | 30 da. | | June 30 | $5000.00 | |

## WRITTEN PROBLEMS

**668.** *1* Find the proceeds of a note for $1190, at 90 days, dated and discounted to-day

*2* I send to bank two notes the first dated to-day, at three months, for $2300, the second dated to-day, at sixty days, for $1190.25 If both these notes are discounted and the proceeds placed to my credit, what is my bank balance?

*3* Find the proceeds of a note dated and discounted to-day, at three months, the face of which is $975.62

*4* What is the bank discount of a note at five months, dated four weeks ago and discounted to-day, if the face of the note is $1500?

*5* What were the proceeds of a note for $1000, dated January 22, at 60 days, which was discounted February 17?

*6* Sent to bank John Alpane's 60-day note, dated 7 days ago, for $1500, and had it discounted to-day Find the proceeds

*7* Sold John Halter 925 yards broadcloth at $2.16, on his ninety-day note which I immediately had discounted Find the discount

*8* Hiram Hillegas to-day discounted his note in my favor, given 14 days ago, for $1140 at four months Find the proceeds

*9* Fourteen days ago I sold Abram Lewis 1200 yards of linen at 18 cents on a sixty-day note and to-day had it discounted Find the proceeds

*10* A merchant sends to bank $320 cash, John Evans's note dated seven days ago, at 90 days, for $350, Abram Kinkler's note for $1250 dated seven days ago, at two months, William Witherspoon's note for $1500, dated to-day, at 90 days and John Larkin's note for $300 at 2 months, dated to-day If all these notes are discounted, find the total amount placed to the merchant's credit

*11* I drew a draft on A Beaver of New York City, dated to-day, at four months, for $3000, and offered it for discount at my bank, and it was accepted What amount should be placed to my credit?

*12* What is the net amount of a note dated December 30, at four months for $1200 discounted February 1?

*13* What amount will a sixty-day sight draft for $5000, dated June 5, yield if discounted June 10?

*14* I hold A Buyer's 90-day note, for $1875, dated one week ago If my bank discounts it for me to-day, what sum will be placed to my credit?

*15* I want to secure $1000 cash to-day, and to do so I have discounted my note at 90 days, dated to-day Find the face of the note

*16* I have in bank a balance of $98.70 Wishing to give a check for $700 I have discounted my note dated to-day, at three months for a sum sufficient to secure the balance Find the face of my note

*17* John Hamerlin has in bank a balance of $19.30 He sends to bank $250 cash, and for discount Warren Bates's note dated to-day at three months, for $1130, and his own note at ninety days, dated to-day, for a sum sufficient to enable him to write a check for $4500 and have $1500 remaining in bank Find the face of his own note

*18.* For what amount must I write a note at four months that will yield me $2211 if dated and discounted to-day?

*19.* For what amount must I write a note, dated to-day, which I intend to have discounted the next business day, the time of the note being three months, to yield $900?

*20.* I have in bank to-day a balance of $750.22. I owe a debt of $1200, and in order to secure the balance I give a note with collateral, at three months, dated to-day. What must be the face of the note?

*21.* Find the face of a note that will yield $1720, if the time of the note is five months and it is dated and discounted to-day.

*22.* The proceeds of a 90-day note, dated and discounted to-day, are $1115.42. Find its face.

*23.* I have in bank to-day a balance of $950. I send to bank for discount four notes: The first for $1125, dated to-day, at 90 days; the second for $1050, dated to-day, at 2 months; the third for $975, dated 7 days ago, at 60 days; and the fourth is my own note at 30 days, dated to-day, for such an amount as will make my bank balance $5000. Find the face of my own note.

*24.* Raise $1000 to-day, January 5, Tuesday, by having Merchant's note, dated December 21, for $375, at 4 months, discounted and by giving your own 90-day note, dated to-day, for such an amount that its proceeds, with the proceeds of Merchant's note, will make up the sum needed.

*25.* Borrow at bank $1800 on three notes at one, two and three months respectively. The notes are all to be dated and discounted to-day. For what sum shall they be drawn so that the proceeds of the notes shall be the same?

*26.* My bank discounts for me to-day an interest-bearing, 4-month note, dated a week ago. The proceeds are $540; what is the face of the note?

*27.* For what amount must a 4-month note, dated to-day, bearing interest at 5%, be drawn so that discounted to-day at 6%, the proceeds will be $900?

*28.* If I buy goods for $1200 cash, and immediately dispose of them for $1300, receiving a 4-month note, which I have discounted at once at bank, receiving the proceeds, what is my per cent. of gain on the sale?

*29.* A man has a 60-day note discounted, term of discount 61 days, and receives $12.20 less than the face of the note. What sum does he receive?

*30.* To-day I desire to pay for an invoice of silks; 178.5 meters at 7.25 francs per meter; 325.25 meters at 5.85 francs per meter; 875.8 meters silk ribbon at 2.35 francs per meter; commission on purchase 2½%. What should be the face of a 90-day note, dated and discounted to-day, that would yield the required amount?

*31.* A contractor contracted to dig the cellar of a house 74 ft. long, 41 ft. 3 in. wide and 6 ft. 9 in. deep @ 35 cents a *load;* also to build the cellar walls 8 ft. 3 in. high and 18 in. thick @ $2.16 per cu. yd. for the material and $1.45 a perch for the labor. Find the face of a 90-day note dated and discounted to-day that will pay the contractor's bill.

*32.* A cistern 8 ft. 3 in. long, 5 ft. 4 in. wide and 9 ft. 6 in. high is lined with zinc costing 12 cents a lb., allowing 15 lbs. for every 6 sq. ft. Find the face of a 60-day note, dated to-day, which when discounted to-day will pay for lining the cistern.

*33.* The walls of a room 16 ft. 8 in. long, 10 ft. 4 in. wide are tiled 4 ft. high, with tile 2½ in. by 5 in., and the floor of the same room is tiled with tile 6 in. by 3½ in. The tile for the walls cost $4.50 per C; the tile for the floor cost $5.40 per C; and the cost of laying them was 34 cents a sq. yd. To-day a 3-months note to pay for the labor and tiling was received and discounted. What were the proceeds?

*34.* Three bills for $2896.50, $4867.92 and $7864.95, are respectively 45, 80 and 115 days overdue; if interest is charged on these at 3%, 4% and 4½% respectively, what must be the face of a 60-day note, dated and discounted to-day, at 5%, that would yield sufficient to cancel the indebtedness?

*35.* Three creditors have claims against a firm totaling $32875; A's claim is 20% more than C's, and B's is 40% of the total claim. Settlement is made February 15th by giving notes for 15% less than the amount of each claim, A's note maturing April 1st; B's note maturing June 1st; and C's note maturing August 2d. A has his note discounted March 15th, B has his discounted May 10th, and C has his discounted June 30th. What is each man's loss?

*36.* An agent received a 60-day note payable April 25th; March 3d he had it discounted at bank, and used the proceeds to purchase equal quantities of hay at $18.75 per ton, and feed at $22.50 per ton; his commission was 1½%; if the discount on the note was $63.54, how many tons of each were purchased and what was the unspent balance? What was the face of the note?

*37.* Purchased 3 car-loads shelled corn, 86437 lbs. each, at 74c. per bu.; 2 car-loads oats, 78465 lbs. each, at 54½c per bu.; 5 car-loads baled hay, 52795 lbs. each, at $26.50 per gross ton, and 7 car-loads baled straw, 45370 lbs. each, at $11.75 per net ton, terms 5 per cent. discount within 20 days, 5% interest after 30 days. I discounted the bill on the last day, securing the money by discounting a 60-day note with collateral. How much less did it cost me at the maturity of the note than if I had let the bill draw interest until that time?

*38.* Three notes for 30, 60 and 90 days respectively were dated to-day. The first note was for $350.85; the second for $867.45, and the third for $948.75. I had them all discounted to-day and loaned the entire sum at 6% for 90 days. How much more or less would I have had if I had collected each note at maturity and loaned the money at 6% until maturity of the 90-day note.

*39.* April 19th I wrote a note at 90 days for a sum such that, when discounted, the proceeds would pay for 45 horses at $135 each, the freight, $23.85, other charges of $65, and a commission of 3%. At the end of the 90 days I sold the horses at $188.25 each, took a 60-day note for the sale, had the note discounted at bank immediately, paid out of the proceeds 75c. each for delivering the horses, keeping $8.85 per head, and 1½% commission for selling. What was my gain per cent.?

## PARTIAL PAYMENTS

**867.** A **partial payment** is a payment of a part of a debt and its accrued interest.

**868.** It is customary to acknowledge the partial payment of a note by writing on its back the amount of payment and the date. These payments are sometimes called indorsements.

**869.** There are a number of rules for determining the amount due at any time on a debt on which partial payments have been made. The most important are the *United States Rule* and the *Mercantile Rule*.

## UNITED STATES RULE

**870.** The **United States Rule** is the method adopted by the United States Supreme Court, and is used by courts in general.

EXAMPLE.—On a note of $2800 bearing interest at 5%, dated July 7, 1914, the following payments were made: August 5, 1914, $25; October 23, 1914, $15; Dec. 17, 1914, $10; Feb. 6, 1915, $500; April 5, 1915, $100; June 2, 1915, $300. What is due July 7, 1915?

**Operation**

| | | | |
|---|---|---|---|
| Face of note July 7, 1914 | | | $2800.00 |
| First payment Aug. 5, 1914 | | $25.00 | |
| Interest on face of note to first payment (29 da. | | 12.94 | 12.94 |
| New principal Aug. 5, 1914 | | | 2787.06 |
| Second payment Oct. 23, 1914 | | 15.00 | |
| Interest on new principal to Oct. 23, 1914 (79 da. | $29.05 | | |
| Third payment December 17, 1914 | | 10.00 | |
| Interest on same principal to Dec. 17, 1914 (54 da. | 19.86 | | |
| Fourth payment Feb. 6, 1915 | | 500.00 | |
| | | 525.86 | |
| Interest on same principal to Feb. 6, 1915 (48 da. | 19.67 | 19.67 | 458.10 |
| New principal Feb. 6, 1915 | | | 2329.96 |
| Fifth payment April 5, 1915 | | 100.00 | |
| Interest on new principal to April 5, 1915 (57 da. | | 19.48 | 951.54 |
| New principal April 5, 1915 | | | 1378.42 |
| Sixth payment June 2, 1915 | | 300.00 | |
| Interest on new principal to June 2, 1915 (58 da. | | 7.16 | 288.94 |
| New principal June 2, 1915 | | | 1089.48 |
| Interest to July 7, 1915 (35 da. | | | 5.16 |
| Amount due July 7, 1915 | | | $1094.64 |

**671.** UNITED STATES RULE.—*Find the interest on the face of the note from the date of the note to the time of the first payment (time by compound subtraction). From the first payment subtract this interest and reduce the face of the note by the balance. Find the interest on this balance for the time from the first payment to the second payment. From the second payment subtract this interest and reduce the debt by the balance. Proceed in this way until all the payments have been disposed of. Then add the interest from the last payment to the date of settlement.*

NOTE.—Should any interest item exceed a payment, simply make a note of the payment and accumulated interest, and find interest on same amount to next payment. If the sum of both payments exceeds both interest items, reduce the face of the note by the difference; if not, continue in this way until the sum of the payments does exceed the sum of the interest items, and then reduce the debt by the difference.

*Interest must never be taken on a sum larger than that on which the preceding interest was taken.*

## MERCANTILE RULE

**672.** The **Mercantile Rule** is the method commonly used by merchants.

EXAMPLE.—On a note of $2800 bearing interest at 5%, dated July 7, 1914, the following payments were made: Aug. 8, 1914, $25; Oct. 23, 1914, $15; Dec. 17, 1914, $10; Feb. 6, 1915, $500; Apr. 3, 1915, $1000; June 2, 1915, $300. What is due July 7, 1915?

| Operation | | |
|---|---|---|
| Face of note July 7, 1914 | | $2800.00 |
| Interest to July 7, 1915 | | 140.00 |
| Amount due July 7, 1915 | | $2940.00 |
| First payment Aug. 8, 1914 | $25.00 | |
| Interest to July 7, 1915 (333 da.) | 1.16 | |
| Second payment Oct. 23, 1914 | 15.00 | |
| Interest to July 7, 1915 (257 da.) | .54 | |
| Third payment Dec. 17, 1914 | 10.00 | |
| Interest to July 7, 1915 (202 da.) | .28 | |
| Fourth payment Feb. 6, 1915 | 500.00 | |
| Interest to July 7, 1915 (151 da.) | 10.49 | |
| Fifth payment April 3, 1915 | 1000.00 | |
| Interest to July 7, 1915 (95 da.) | 13.19 | |
| Sixth payment June 2, 1915 | 300.00 | |
| Interest to July 7, 1915 (35 da.) | 1.46 | 1877.12 |
| Amount due July 7, 1915 | | $1062.88 |

**673.** MERCANTILE RULE.—*Find the interest on the face of the note to the date of settlement, if the date of settlement is not later than one year from date. Add this interest to the face of the note. Find the interest on each payment from the date on which it was made to the date of settlement (exact days). Add the several payments and interest items, and subtract the sum from the amount of the note.*

NOTE.—Should the date of settlement extend beyond a year from the date of the note, settlement of the note must be made at the end of every year from date, then for the fraction of the year, if any.

We present a problem worked out in both methods with the object of showing that for large amounts there is a considerable difference between the results obtained by the United States and the Mercantile Rules. When worked by the mercantile Rule, the difference is in favor of the payer of the note. It will be seen that the Mercantile Rule Method is used for small amounts and short periods of time.

[illegible] $3000. Philadelphia, January 2, 1914.

On demand I promise to pay John Doe or order

Three thousand [illegible] Dollars,

with interest at 6%, without defalcation for value received.

Richard Roe.

Payments indorsed. March 2, 1914, $1000; May 2, 1914, $1000; July 2, 1914, $1000; Sept. 2, 1914, $1000; Nov. 2, 1914, $1000. What amount will take up the note January 2, 1915?

**United States Rule**

| | | |
|---|---|---|
| Face of note January 2, 1914 | | $3000.00 |
| Payment March 2, 1914 | $1000.00 | |
| Interest on face, Jan. 2, 1914, to March 2, 1914 | 30.00 | 1030.00 |
| Balance March 2, 1914 | | 1970.00 |
| Payment May 2, 1914 | 1000.00 | |
| Interest on balance, March 2, 1914, to May 2, 1914 | 19.70 | 1019.70 |
| Balance May 2, 1914 | | 970.30 |
| Payment July 2, 1914 | 1000.00 | |
| Interest on balance, May 2, 1914, to July 2, 1914 | 9.71 | 19.71 |
| Balance July 2, 1914 | | 990.41 |
| Payment Sept. 2, 1914 | 1000.00 | |
| Interest on balance, July 2, 1914, to Sept. 2, 1914 | 9.93 | 1009.93 |
| Balance Sept. 2, 1914 | | 990.44 |
| Payment November 2, 1914 | 1000.00 | |
| Interest on balance, Sept. 2, 1914, to Nov. 2, 1914 | 9.90 | 1009.90 |
| Balance November 2, 1914 | | 9.90 |
| Interest on balance, Nov. 2, 1914, to Jan. 2, 1915 | | .20 |
| Amount to take up note, January 2, 1915 | | $10.10 |

**Mercantile Rule**

| | | |
|---|---|---|
| Face of note January 2, 1914 | | $3000.00 |
| Interest for one year | | 180.00 |
| Amount due Jan. 2, 1915 | | 3180.00 |
| Payment March 2, 1914 | $1000.00 | |
| Interest on payment to January 2, 1915 306 da. | 30.60 | |
| Payment May 2, 1914 | 1000.00 | |
| Interest on payment to Jan. 2, 1915 245 da. | 34.05 | |
| Payment July 2, 1914 | 1000.00 | |
| Interest on payment to Jan. 2, 1915 184 da. | 18.40 | |
| Payment Sept. 2, 1914 | 1000.00 | |
| Interest on payment to Jan. 2, 1915 122 da. | 12.20 | |
| Payment November 2, 1914 | 1000.00 | |
| Interest on payment to Jan. 2, 1915 61 da. | 6.10 | 3106.00 |
| | | $74.00 |
| Amounts needed to take up note | | |
| By United States Rule | $10.10 | |
| By Mercantile Rule | 74.00 | |
| Difference in favor of payer by Mercantile Rule | $63.90 | |

## WRITTEN PROBLEMS

**675.** *1.* A note of $1460, dated June 3, 1914, with interest at 4½%, has the following indorsements: Aug. 6, 1914, $150; Nov. 19, 1914, $250; Jan. 25, 1915, $210; March 19, 1915, $290; May 20, 1915, $100. What was due June 3, 1915, by United States Rule?

*2.* On a mortgage of $5000, dated March 6, 1914, with interest at 5%, the following payments have been made: May 23, 1914, $950; July 20, 1914, $290; Oct. 17, 1914, $450; Dec. 18, 1914, $1000; Feb. 12, 1915, $1500. What is due March 6, 1915, by Mercantile Rule? 917.25

*3.* A demand note dated February 5, 1914, bearing interest at 5½%, for $2200, has the following indorsements: May 15, 1914, $25; June 11, 1914, $200; Aug. 20, 1914, $10; Nov. 5, 1914, $250; Jan. 9, 1915, $300. What is due Feb. 5, 1915, by United States and Mercantile Rules?

*4.* On a note dated August 6, 1914, for $5200, with interest at 6%, the following payments have been made: Nov. 19, 1914, $550; Jan. 2, 1915, $30; March 27, 1915, $33; May 22, 1915, $38; July 23, 1915, $42. What is due Aug. 6, 1915, by United States Rule?

*5.*

$5000. Philadelphia, June 15, 1914.

On demand, I promise to pay to the order of Morris and Lewis, Five thousand Dollars with interest at 5%, without defalcation, value received.

H. H. ETTER,
104 S. Eighteenth St.

On the above note the following indorsements have been made: July 30, 1914, $100; Sept. 16, 1914, $10; Nov. 18, 1914, $25; Feb. 11, 1915, $1000; March 18, 1915, $90; May 13, 1915, $25. What is due one year from date by United States and Mercantile Rules?

*6.* A note of $2500, dated Aug. 12, 1914, with interest at 6%, has the following indorsements: Oct. 26, 1914, $400; Dec. 9, 1914, $40; Jan. 25, 1915, $350; May 25, 1915, $25; July 15, 1915, $250; Sept. 16, 1915, $500; Nov. 5, 1915, $200. How much is due Jan. 1, 1916, by United States Rule?

*7.* What is due Mar. 8, 1915, on a note for $1878.90, with interest at 6%, dated Jan. 12, 1914, on which the following payments have been made: Mar. 13, 1914, $100; May 18, 1914, $200; June 10, 1914, $75; Aug. 19, 1914, $150; Oct. 14, 1914, $250? (Mercantile Rule.)

*8.* What sum will take up January 1, 1916, a note for $10000, interest at 4½%, bearing date of September 5, 1914, upon which the following indorsements have been made: January 2, 1915, $2000; April 1, 1915, $2000; July 1, 1915, $2000; Oct. 2, 1915, $2000?

# STOCKS AND BONDS

## STOCKS

**676. Stock** is the share capital of a corporation or commercial company; that is, the fund employed in conducting the business.

**677.** The capital stock of a company is divided into **shares** of equal amount, which are owned by the individuals who jointly form the corporation.

**678. Certificates of stock** are papers issued by a company, signed by the proper officers, indicating the number of shares each stockholder is entitled to, and are an evidence of ownership; they are transferable by assignment and may be bought or sold like any other property.

**679.** The **par value** of a share of stock is the amount named in the certificate—usually $100, though it may be any amount, as $50, $25, $10, etc. Shares of $50 are sometimes called *half-stock*, and those of $25, *quarter-stock*.

**680.** The **market value** of a share of stock is the price at which it is sold.

**681.** When the market value of shares is greater than the par value, they are said to be *above par* or *at a premium*, and when the market value is less than the par value, they are said to be *below par* or *at a discount*.

**682. Quotations** of the market value are generally a percentage of the par value, though they sometimes indicate the number of dollars per share.

**683.** The **gross earnings** of a company are the total receipts before expenses have been paid.

**684.** The **net earnings** are what is left of the gross earnings after paying expenses.

**685.** The accumulated profits which are distributed among the shareholders at stated periods are called **dividends**, and are "declared" at a certain per cent. of the par value of their shares.

**686.** An **assessment** is a sum levied upon stockholders to make up losses or to provide for extensive improvements.

**687. Preferred stocks** are shares entitled to a dividend of a certain per cent. before the common stock can receive any dividend. Preferred stock is sometimes issued as a special inducement to raise money during financial embarrassment.

**688.** The term **watered stock** is applied to stock which has been increased by shares issued in excess of the capital stock subscribed for and actually paid in.

**689.** The **Stock Exchange** is a place where stock brokers meet daily to execute their customers' orders for the purchase and sale of railroad stocks and bonds, Government securities, and such other stocks and bonds of various corporations as may be listed at the Exchange.

**690.** The regular **commission** for buying or selling at the New York Stock Exchange is *one-eighth of one per cent.* on the par value of all stocks and bonds; that is, $12.50 on 100 shares of stock of the par value of $100 each, or $1.25 on a $1000 bond.

The rate of commission on other exchanges may vary somewhat on local shares or shares of stock not quoted on the New York Exchange ½%; is to be considered in all problems unless otherwise specified

**691.** A **margin** is a sum of money, or its value in securities, deposited with a broker to protect him against loss in buying or selling for a customer's account It is 10%, 20%, or more of the par value of the stocks bought or sold Many brokers base the amount of margin on the marketable value and fluctuations of stocks

NOTE Brokers allow their customers interest on money deposited as margin and when a customer buys stock on margin since the broker has to pay the whole cost of the stock in cash he charges his customer interest on the amount so expended for his account

**692.** Stocks are usually bought or sold either "regular," "cash," "seller three" days or "buyer three" days Stock sold in the regular way is to be delivered the next day, if sold "cash" it is to be delivered on the day sold When stock is bought "seller three" it means that the seller can deliver it on any one of three days at his option the day sold, the next day, or the day after next "Buyer three" means that the buyer may demand delivery of the stock at any time within three days, but must take and pay for it before the end of the third day

When stock is sold at either buyer's or seller's option for more than three days as 5-10, 5-20, 5-30, 5-60, 5-90, 5-10, 5-20, 5-30, 5-60, the buyer pays six per cent interest, unless "flat" is specified or other rate designated

**693.** **Selling short** means selling stock which one does not own either by seller's option in which case the seller buys and delivers the stock before the expiration of the time specified or by borrowing the stock in order to make delivery, and afterwards buying to repay the loan In either case the person making the "short" sale expects to be able to buy at a lower price to make a profit

**694.** In stock-exchange slang a "bull" is one who endeavors to effect a rise in the price of stock, and a "bear" is one who endeavors to bring down prices A "bull" buys stock, expecting to sell it at a higher price, while a "bear" sells stock "short" expecting to buy it at a lower price

**695.** A **corner** is a combination to buy all the available supply of a stock, so that those who have sold short may be unable to fulfil their contracts except by buying from the combination at an exorbitant price

**696.** To **hypothecate** stocks or bonds is to deposit them as collateral security for money borrowed

**697. To find cost when buying or proceeds when selling.**

EXAMPLE 1 What must I pay for 150 shares of B & Q R R stock, at 102½ usual brokerage?

Quotation 102½ plus brokerage ¼ equals 102¾

102½
.75
1.12
153.93 Ans.

EXAMPLE 2.—What do I obtain from the sale of 150 shares of N. Y. C. R. R. stock, at 118¾, usual brokerage?

Quotation 118¾ less brokerage ⅛ equals 118⅝.

$118\frac{3}{4}$
150
5900
118
93.75
$17793.75 proceeds

**698.** RULE.—*Add the brokerage to the quotation and multiply this sum (expressed as dollars) by the number of shares to find the cost of purchase; subtract the brokerage from the quotation and multiply this difference (expressed as dollars) by the number of shares to find the proceeds of sale.*

NOTE.—A list of the stocks named in the problems of this book whose par value is other than $100 per share follows:

*Catawissa R. R., $50; Lehigh Valley R. R., $50; Lehigh Navigation, $50; North Penn. R. R., $50; Pennsylvania R. R., $50; Reading R. R., $50; P., Wil. & Balto., $50; United Gas Improvement, $50.*

## WRITTEN PROBLEMS

**699.** *1.* What must I pay for 750 shares Catawissa preferred at 53?

NOTE.—When no brokerage is given, the usual brokerage of ⅛%, or 12½ cents per share, par $100, is understood.

*2.* Through my broker I sold 1250 shares L. V. R. R. at 24½; what should I receive from him?

*3.* Bought 600 shares Lehigh Navigation at 40, and sold them at 42, usual brokerage; what amount of money do I gain?

*4.* My broker buys for me 800 shares P. R. R. at 59, and 350 shares North Penn. at 91½; what is the full cost to me?

*5.* I bought 500 shares Un. Gas. Imp. at 87⅛, and to-day it is quoted at 107¼; what would I gain in net money by selling at once?

*6.* Bought Pacific Mail at 57⅛, and sold it during a decline at 41¾; what did I lose per share?

*7.* How many shares of Chi. R. I. & P. R. R. at 92⅛ can I buy with $10000, and what surplus will remain?

*8.* I sold 350 shares Lehigh Nav. at 47⅛, and invested in P. R. R. at 59½; how many shares did I buy, and what is the amount of surplus?

*9.* How many shares of Reading stock at $11\frac{13}{16}$ were bought through a broker, for $2375?

*10.* What is the cost of 100 Reading 1st preferred at $23\frac{1}{16}$, 200 N. Pacific preferred at 41¼, 300 St. Paul at 87⅜, and 200 Atchison preferred at 25⅞?

*11.* Purchased, through a New York broker, 100 Cent. of N. J. at 88½, 500 Del. & Hudson at 112½, 300 Illinois Cent. at 99¾, and 200 Lake Shore at 171⅛. Find cost.

*12.* Sold, through a broker, 200 St. Paul at 86¾, 150 Lehigh Val. at 30½, 425 Reading at 11¾, and 300 West N. Y. & Pa. at 2¼. Find proceeds.

*13.* Find the proceeds of the following stocks if sold through a New York broker: 100 Balt. and Ohio at 11½, 250 Wabash preferred at 15⅛, 200 Chi. R. I. & P. R. R. at 76⅝, 150 West. U. Tel. at 84½, 25 Del., Lack. & W. at 158, and 300 Southern Pac. Co. at 16½.

**700. Buying and selling on margin.**

EXAMPLE.—On June 4, 1914 A. Lamb instructed his broker to buy for him 200 shares of a certain stock and deposited with him $2000 margin. On June 8 he bought at 89⅜. On June 24 he sold the stock at 87½. What is the balance of Lamb's account July 2? What is his loss?

*Dr.* A. LAMB *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| June | 8 | Bot. 200 shares Chi. R. I. & P.R.R. at 89½ | 17900 00 | June | 4 | Cash deposit | 2000 00 |
| " | 24 | Int. on bal. | 42 40 | " | 8 | Int. on bal. | 1 33 |
| *July* | *2* | *Balance* | *1536 03* | " | 24 | Sold 200 shares Chi. R. I. & P.R.R. at 87⅜ | 17475 00 |
| | | | | July | 2 | Int. on bal. | 2 10 |
| | | | 19478 43 | | | | 19478 43 |

| | |
|---|---|
| Margin deposited | $2000.00 |
| Balance received | 1536.03 |
| Loss | $463.97 |

NOTE.—The brokerage is added to purchase and deducted from sale, and interest is calculated on daily balances. There is a credit balance of $2000 from June 4 to June 8, on which the interest is $1.33; a debit balance of $15900 from June 8 to June 24, on which the interest is $42.40, and a credit balance of $1575 from June 24 to July 2, on which the interest is $2.10.

**701. Buying and selling short.**

EXAMPLE.—Aaron Bear sold "short" December 1, 1914, through his broker, 250 shares Northern Pacific R. R. pref. at 60⅜, and "covered" his "short" December 15, 1914, at 56. Allowing ⅛% brokerage for buying and selling, what was his net profit?

*Dr.* A. BEAR *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| Dec. | 15 | Bot. 250 N. P. pref. at 56⅛ | 14031 25 | Dec. | 1 | Cash deposit | 2500 00 |
| " | 15 | *Balance* | *3537 08* | " | 1 | Sold 250 N. P. pref. at 60¼ | 15062 50 |
| | | | | " | 15 | Int. on cash | 5 83 |
| | | | 17568 33 | | | | 17568 33 |

| | |
|---|---|
| Balance | $3537.08 |
| Margin | 2500.00 |
| Gain | $1037.08 |

NOTE.—On "short" sales the stock is borrowed for delivery and replaced when purchase is made. No interest is charged for this. The margin is allowed interest as in any other stock transaction.

## WRITTEN PROBLEMS

**702.** *1.* A speculator directed his broker to purchase 500 shares of stock (par $50), and deposited 10% of par value as margin April 6, 1914. On April 8 the stock was purchased at 52½. On May 1 a dividend of 3% was collected. On May 12 the stock was sold at 54. How much did the speculator gain, and what was the balance due him by the broker on May 14?

*2.* October 5, 1914, my broker buys for me 500 shares of L. V. R. R. at 29⅛, brokerage ¼%, and holds it till February 25, 1915, then sells it at 34½, brokerage as before. There were no dividends on the stock. I put up $2500 as a margin in the hands of the broker at the time of purchase. What does he owe me at the time of sale?

*3.* A broker sold "short" for I. Flyer, February 1, 1915, 700 shares Illinois Steel at 55 and "covered" sale February 14, 1915, at 52⅜. What was Flyer's net profit?

*4.* Bought 150 shares of P. R. R. at 55¼, received a semi-annual dividend of 2½ per cent, and then sold the stock for 55½. How much did I gain?

*5.* On April 3, 1914, a speculator deposited $2000 with his broker, who purchased for him 100 shares of stock at 89, charging ¼% brokerage. On May 1, 1914, he received a dividend of $175. On October 14, 1914, he sold the stock at 92, charging ¼% brokerage, and made settlement, charging 6% interest. How much did the speculator receive?

*6.* March 15th, this year, I ordered my broker to purchase for me 18 Municipal 4½% bonds, interest payable April 30th and October 30th, and deposited a 15% of par as margin. He purchased April 5th at 107¾ and interest; April 30th he collected the dividend; June 5th he sold at 106⅝ and interest, and made settlement. What was my profit if he charged 6% interest and allowed me 4%?

*7.* September 10th my broker sold "short" for me 115 shares stock ($50) at 62½ and covered the sale September 24th at 60⅞; what was my profit?

*8.* March 4th, this year, my broker bought for me 75 shares Ill. Cent., at 108⅛, bal. March 12th, he called for the stock and sold 50 shares at 110⅝; March 25th he sold the remaining shares at 109⅜, and settled the account. If my margin was $1500, and I was allowed 4½% interest on my money, but was required to pay him 6% on his money, what was my gain?

## BONDS

**703.** A **bond** is the written obligation of a corporation, or of a City, County, State or Government, to pay a certain sum of money at a certain time with a fixed rate of interest, payable at certain periods

**704. Bonds** may be **classified**: 1) as to the character of the issuing corporation (government, municipal, railroad, etc.), 2) as to security (1st mortgage, debenture, guaranteed), (3) as to purpose (construction, redemption, etc.), 4) as to interest (coupon, registered, profit-sharing, etc.)

**705. Government bonds** have no security other than the faith and credit of the people, but business corporations usually secure their bonds by a mortgage on all or part of their property

**706. Debenture bonds** are issued on the credit of a corporation, and are unsecured promises to pay

**707. Coupon bonds** are those with small certificates of interest attached, which are to be cut off and presented for payment as they become due. These bonds and coupons are payable to bearer and are transferable by delivery. These bonds may be registered as to principal

**708. Registered bonds** are those payable to the registered owner or to his order, they are transferable by assignment. The interest is paid by check to the owner

**709.** Bonds are issued in various **denominations**, $1000 being the most common. They are always quoted at so much per cent of the par value

**710.** The **interest** on **bonds** is a fixed charge paid out of the net earnings. The interest is paid at regular stated periods, annually, semi-annually or quarterly. The principal is paid at the expiration of the time from the accumulated profits set aside as a redemption or sinking fund, if a business corporation, or in a sinking fund if a municipal corporation. Often a corporation reserves the right to redeem bonds at a fixed price prior to their expiration

**711. Bond brokers** usually buy and sell bonds at a fixed price and interest, or **market** price and interest

**712.** The **interest** is calculated on the basis of a 360-day year or 30-day month, regardless of the number of days in a month. The fractional part of a month is computed by exact days

**713.** A **bond** bought at a **premium** tends to decrease in value as it approaches maturity, since at maturity it will be redeemed at its face value, while a **bond** bought at a **discount** correspondingly increases in value as it approaches maturity

Note. The above statements are based on bond mathematics only. As a matter of fact, other conditions affect the prices of bonds, such as the money market, the condition of issuing company, etc.

**714. To find cost when bought, proceeds when sold, and income.**

EXAMPLE 1.—What must I pay for $12000, par value, of U. S. 4s registered, at 113⅛? 2. What sum should my broker return to me from the sale of $8500, par value, of U. S. 4s coupon, at 114¼? What is the annual income from each?

113⅛ + ⅛ = 113¼

| | | | |
|---|---|---|---|
| $12000 | $12000 par value | $8500 × 114¼% = | $9753.75 market value |
| 1.13¾ | .04 rate | $8500 × ⅛% = | 10.63 brokerage |
| 135000 | $480.00 = annual income | | $9743.12 proceeds |
| 7500 | | $8500 par value | |
| $13635.00 cost | | .04 rate | |
| | | $340.00 = annual income | |

**715.** RULE.—*Add rate of brokerage to quotation and multiply the sum by the par value of the bonds to find cost; subtract rate of brokerage from quotation and multiply the difference by the par value of the bonds to find proceeds.* OR, *Multiply the par value by the rate of brokerage to find the brokerage. Add brokerage to market value to find the cost; subtract to find proceeds. Multiply the par value by the interest of bond to find income.*

**WRITTEN PROBLEMS**

**716.** 1. What will be the cost of $12000, par value, of U. S. 4s coupon, at 128¾? The income?

2. What must I pay for $8500 U. S. 4s registered, at 127½?

3. Find the proceeds of $7000 U. S. 3s coupon, at 113⅝.

4. I sell $8500 U. S. 3s registered, at 113½ and invest proceeds in U. S. Currency 6s at 105. What is the par value of bonds bought, to the nearest $100, what the surplus, and what the change in income?

5. What will $8000 Lehigh Coal & Nav. 3s cost at 96? If I sell them at 98½, after receiving two dividends, what is my profit?

6. Bought $35000 extended 3s at 99½ and sold them at once at 102: what was my gain?

**717. To find cost or proceeds with interest adjustments.**

EXAMPLE 1. Bought January 16, 1914, five $1000 Fairmount Park Transportation Company First Mortgage 5%, Sinking Fund Gold Bonds at 96, interest payable April 1st and October 1st. What do I owe my broker?

Interest from Oct. 1st to Jan. 16 = 3 mo. 15 da.

| | | |
|---|---|---|
| $5000 × 96% | = | $4800.00 market value |
| $5000 × ¼% | = | 12.50 brokerage |
| $5000 × 5% × 3 mo. 15 da. | = | 72.92 accumulated interest |
| | | $4885.42 total cost |

NOTE. The purchaser of the bond receives the advance interest at the next regular interest payment period.

**718.** RULE.—*Add interest from last interest period to date of sale or date of delivery when sold to cost or proceeds of bonds.*

## WRITTEN PROBLEMS

**719.** *1.* Bought February 16, 1915, two ($1000) New Jersey Public Service General Mortgage 5% Gold Bonds at 90, interest payable semi-annually, April 1st and October 1st. For what amount shall I draw my check in favor of my broker to cover the cost of these bonds?

*2.* I gave my broker, April 3, 1915, $10000 to buy City of Philadelphia 3½% miscellaneous improvement bonds, interest payable January 1st and July 1st. If he purchases for me ten bonds at 92½, what amount of money will I still have remaining to my credit?

*3.* I gave my broker an order to purchase for me five ($1000) Pennsylvania Railroad Equipment Trust Series "A" 4% bonds, maturing in 1930, interest payable March 1st and September 1st, at 99½. He made the purchase to-day and sent me a statement of my account. Make out the statement.

NOTE.—When bonds are issued in series, they have generally a letter designation.

*4.* A coupon bond bearing 5% interest has the coupons dated April 15th and October 15th of each year; March 10th my broker bought eight of these bonds for me at 108½ and accrued interest. What do I pay?

*5.* If 4½% coupon bonds, coupons payable May 15th and November 15th, are bought February 10th, at 98⅜ and interest, and after cashing 9 coupons are sold June 28th at 100¼ and interest, how much more did I realize on my investment than if I had invested the same amount for the same time in a 5% mortgage?

*6.* Having $22500 to invest, how many Baltimore & Ohio 4% bonds ($500), at 96⅜ and interest, can I buy, if the purchase is made April 5th, and the interest on the bonds is payable January 1st and June 1st. How much of the money is not invested? What will be my annual income from the bonds?

*7.* April 18th my broker bought for me $12000 Lehigh and Wilkes-Barre Consolidated 4% (Serial) Gold Bonds, interest payable June 1st and December 1st. He paid 104¼ and accrued interest. June 1st he collected the dividend; July 15th I paid him the amount due on the bonds. What amount did I pay?

*8.* Needing $25000 cash, I had my broker sell for me 20 Adams Express 4s, at 90½ and interest. If the sale was made December 12th and interest on the bonds is payable March 1st and September 1st, how much additional money must I secure?

*9.* Bought July 1, 1914, ten ($1000) Chattanooga Railway & Light Company First Mortgage 6% Bonds at 97⅜, interest payable June 1st and December 1st. What do I owe my broker?

*10.* I gave my broker an order to purchase six ($1000) Tennessee Power Company First Mortgage 5% Gold Bonds, maturing 1962, interest payable May 1st and November 1st, at 95⅜. He made the purchase to-day. What amount do I owe my broker, and what is my annual income from these bonds?

### WRITTEN PROBLEMS

**720. To find rate of income.**

EXAMPLE.—Bought U. S. 4s registered, at 113. What is rate of income?

$4\% \div 113\frac{1}{8}\% = 3.54\%$ nearly

Or—

| | |
|---|---|
| \$1000 bond $\times 113\frac{1}{8}\% =$ | \$1131.25 cost of bond |
| \$1000 @ 4% | 40.00 annual income |
| \$40 ÷ 1131.25 | 3.54% nearly |

**721.** RULE.—*Diride the interest rate by the cost (quotation plus brokerage).*

**722.** *1.* What is my rate of income from U. S. new 4s registered, bought at $128\frac{1}{2}$?

*2.* What is my rate of income from City of Cincinnati $7\frac{3}{10}$s bought at $127\frac{1}{2}$?

*3.* What would be my rate of income from P. & R. Gen. Mort. 4s at $86\frac{7}{8}$?

*4.* What net rate do Catawissa R. R. 6s pay an investor if bought at $105\frac{3}{4}$?

*5.* Which is the better investment—City of Camden 7s at 114 or City of Harrisburg 6s at $112\frac{1}{2}$?

**723. To find the cost of an interest-bearing bond that will yield a given rate of income.**

EXAMPLE.—At what price must I buy 5% bonds to get 7% on my investment? No brokerage.

As quotations run in eighths, it is evident that the result obtained must be expressed in nearest eighth, which would be a business result, but not a mathematical one.

$7\overline{)5.00}$

$.71\frac{3}{7}$ cost, or $71\frac{3}{8}$ business result

**724.** RULE.—*Diride the rate expressed in bond by the rate of income to be obtained, and the result will be the cost.*

### WRITTEN PROBLEMS

**725.** *1.* What would be the cost of bank stock paying annual dividends of 10%, so that the holder would realize 6% on his investment? No brokerage.

*2.* A bank stock whose par is \$50 sells at 140; semi-annual dividends of 10% are declared. At what price should it be sold to make it an 8% investment? No brokerage.

*3.* At what price must I buy U. S. new 4s to obtain 3% on my investment? No brokerage.

*4.* Southwark Bank stock is \$50 per share at par. It pays 20% dividends. What price may be paid for it per share to realise 8% on the cost?

*5.* At what price must I buy U. S. Currency 6s to make 5% on my investment?

*6.* What must be the market price of a $5\frac{1}{2}\%$ bond to yield the same per cent. income to an investor as a 6% bond bought at $119\frac{3}{8}$, brokerage $\frac{1}{8}$?

*7.* A bond bought on a 4% basis cost $112\frac{3}{8}$ and brokerage; I desire to purchase another bond paying the same rate of interest on a $3\frac{1}{2}\%$ basis; what should I pay for it?

*8.* At what price must 4% bonds be purchased to yield $3\frac{1}{8}\%$ on the investment?

**726. To find average rate of income derived from a bond bought and held until redeemed.**

EXAMPLE.—What will be the average rate of income from a 4% bond bought at 118, maturing in 30 years? No brokerage.

| | |
|---|---|
| 4% for 30 years | 120% |
| Price rec'd on redemp'n | 100% |
| Full return | 220% |
| Cost | 118 |
| Received above cost | 102 |
| Average annual income $\frac{102}{30}$ | |
| $\frac{102}{30} \div 118 = 2.88$ | |

3540)102.00(2.88
70 80
31 200
28 320
2 8800

Or concretely,

| | |
|---|---|
| Income on $1000 bond | |
| 30 yr. at 4% | $1200 |
| Bond redeemed | 1000 |
| Full return | 2200 |
| Cost | 1180 |
| Received above cost | 1020 |
| Average annual income | 34 |

$34 \div 1180 = 2.88$, or $2\frac{8}{17}\%$ nearly

Result, 2.88% approximately

**727.** RULE.—*Add the income derived from the bond to the redemption value, subtract the cost from this sum and divide the difference by the term of years; this will give the average annual income; the average annual income divided by cost will give the average rate of income.*

## WRITTEN PROBLEMS

*1.* If I pay 124$\frac{1}{8}$ for a 4% bond maturing in 20 years, what per cent. income do I receive on my investment?

*2.* What per cent. on the investment do 6% bonds pay, maturing in 10 years, if bought at 112$\frac{1}{2}$?

*3.* What per cent. on the investment do 5 per cent. bonds pay, maturing in 20 years, if bought at 104$\frac{3}{8}$?

*4.* If I purchase 5% bonds maturing in 10 years from date of investment at 89$\frac{1}{4}$, what rate per cent. of my investment will I realize?

*5.* What rate per cent. per annum is realized from an investment in 4$\frac{1}{2}$% bonds maturing in 20 years, purchased at 95$\frac{1}{8}$%?

*6.* If a 5% bond maturing in 20 years costs 86.59, how much will it net the purchaser?

NOTE.—By bond and accrued interest tables used by bankers, the rate is easily ascertained; the arithmetical process is somewhat tedious and complicated. See remark, page 220.

**728. To find price that may be paid for a bond drawing a certain rate of interest to produce a certain rate of income.**

EXAMPLE.—What must be paid for a 4% bond maturing in 30 years, to produce 3% income?

30 yrs. @ 3% or
90% desired income
100% redemption value
190% expected return

30 yrs. @ 4% or
120% income received
100% redemption value
220% full return
220 ÷ 190 = 115.78 or price paid

Result, 115.78, approximately
or
115⅞, nearly

Or concretely,

$1000 bond at 4% in 30 years
will produce $1200
Redemption value 1000
Full return 2200

A 3% income on $1000
bond would give in
30 years $900
Redemption value 1000
Desired return 1900

1900 : 2200.00 ( 115.78, or 115⅞ nearly

REMARKS.—1. There are various methods in use for determining the average annual income in cases of this kind, one being based upon the condition that the annual income might be at once reinvested (compound interest); another, that the income is put to other uses than investment. The above method is simply a business method of getting a closely approximate result. For convenience and accuracy bond tables are used.

2. Figures in bond tables may be found as follows: Add the dividend to the maturity value and divide by 1 plus the desired rate of income; add the dividend to this result and again divide by 1 plus the rate, and so continue as many times as there are dividend payments until the maturity of the bond.

**729.** RULE.—*Divide full return received from bond, income and redemption, by full return acceptable, and the result will be the approximate cost.*

## WRITTEN PROBLEMS

1. How much must be paid for 5% bonds, maturing in 12 years, in order to realize 4½% on the investment.

2. How much must be paid for 7% bonds to realize 3% on the investment if the bonds mature in 6 years? No brokerage.

3. At what price must 4% bonds, maturing in 1920, be purchased in 1914, to yield 6% on the investment?

4. Find the quotation of 5% bonds, maturing in 12 years, to realize 4% on the investment.

5. What should be paid Jan. 1, 1915, for 6% bonds, maturing January 1, 1930, to net 5% on the investment?

## REVIEW PROBLEMS

**730.** *1.* How much must be invested in Lehigh Valley 6s at 103½ to produce a yearly income of $1200?

*2.* What quarterly income will $29345.63 yield if invested in U. S. new 4s coupon at 124¾, usual brokerage?

*3.* On July 2 I bought 300 Lehigh Valley at 28¼, b30, and called for the stock July 21. What was the cost, including interest? What was my profit if I sold on July 21 at 31⅜?

*4.* What was paid for 400 Lehigh Nav., bought July 6 at 41¾, s20, and delivered July 22?

*5.* A man having $45000 to invest, bought $10000 N. Y. P. & N. 6s at 104, $15000 Lehigh Nav. cons. 7s at 129¾, and $12000 S. H. & W. 1st 5s at 106, and deposited the balance with a trust company which paid 2% interest on deposits. Find his annual income. What rate per cent. is it on his investment? 2277.43 5.3 %.

*6.* What per cent. income on the investment do 6 per cent. bonds yield, if purchased at 104⅜ (brokerage ⅛%)?

*7.* Which will produce the greater annual income, and how much—$35000 invested in stock at 297 paying 5% quarterly dividends or the same amount invested in stock at 118 paying 3% semi-annually?

*8.* Bought through a broker sufficient 4% bonds to produce an annual income of $1000. What did they cost at 123⅞ and what is the per cent. of income on the investment?

*9.* A invested $25000 as follows: 100 shares of 5% stock at 102, 50 shares of 6% stock at 136⅜, 25 shares of 3% stock at 80, brokerage ⅛%, and loaned the balance on mortgage at 5%. What is his total income? 1172.97

*10.* How much invested in 5% bonds at 122⅛ will produce an annual income of $1600?

*11.* June 2, 1914, a broker purchased for the account of a customer 600 shares of stock at 62⅛, the customer depositing $3000 margin. July 8 the stock was sold at 71. What was the gain? (Usual commissions and interest.) 492.72

*12.* My broker sold "short" for me on July 2, 1915, 500 shares of stock at 127⅛, and "covered" on July 22 at 126. What was my gain?

*13.* Smith purchased, through a broker, 500 Reading at 12, 200 Lake Shore at 170¾, and 100 Illinois Cent. at 99¼. He sold the Reading at 12⅞, the Lake Shore at 172, and the Illinois Cent. at 98¾, investing the proceeds in U. S. 4s at 124⅛. What per cent. annual income does he realize on his original investment?

*14.* Bought May 1st, 1912, 3 ($500) Portland Railway Light and Power Company First and Refunding Mortgage 5% Sinking Fund Gold Bonds, dated Feburary 1, 1912, at 96¼, interest payable February 1st and August 1st. Sold the bonds April 21, 1916, at 98. Explain the nature of this bond, its security and its retirement. Find cost per cent. of yield, and the proceeds.

*15.* At what rate should stock paying annual dividends of 15% be bought to realize 5½% on the investment?

*16.* Chemical Bank Stock is quoted at 4000; what rate of dividend should it pay to give the investor a 6% income?

*17.* A man buys Bank of North America at 260, which pays 20% dividend (par $100). What is the rate of income to the investor? No brokerage.

*18.* Bought 150 shares of P. R. R. stock at 55¼, received a semi-annual dividend of 2½%, and then sold the stock at 55½. How much did I gain?

*19.* Which pays the better interest—railway stock at 147 (par $50), which pays 20% dividends, or bank stock at 237 (par $100), which pays 16% dividends? No brokerage.

*20.* A market company has a capital stock of $100000 divided into shares of $100 each. They have a bonded debt of $250000. Their receipts for the year are $50000. Expenses, exclusive of interest, $11000. They, of right, will pay the interest on their bonded debt and divide the balance among the stockholders. What rate of dividend will they pay the stockholders?

*21.* Which pays the better, to buy a New Jersey 7% mortgage at 5% discount or a Pennsylvania 6% mortgage at 10% discount? No brokerage.

*22.* What is my rate of income from Reading Terminal 5s bought at 118¼? Usual brokerage.

*23.* If I buy $10000 (par value) U. S. Gov. 4% bonds at 112, payable in ten years, and receive $400 each year and the principal, $10000, at the end of the tenth year, what rate of interest have I received on my investment?

*24.* I bought stock, par $50, at 6½% discount, and received two semi-annual dividends of 2½% each; I then sold the stock at 7% discount, netting a gain of $50, after paying a broker ¼% for buying, and the same rate for selling. How many shares were bought?

*25.* A owned sufficient U. S. 4s, 1907, to pay him an annual income of $300; he sold them when they were quoted at 113¼, brokerage ¼%, and invested the proceeds in 5% R. R. stock at 10% premium, brokerage ¼%; find his change of income, and the surplus from the sale of the bonds.

*26.* If stock bought at 110 pays 4 1/11% on the investment, what rate of dividend does the stock pay, and what rate of interest would it yield if the price should advance 20% (brokerage not considered)?

*27.* What rate of interest does an investor receive on his money who buys, through a broker, P. R. R. stock, ($50), at $61, if this stock pays a semi-annual dividend of 2¼% and the rate of brokerage is ¼%?

*28.* I own sufficient mining stock, par $50, paying a semi-annual dividend of 2¼% to yield me $400 annually. The rate of brokerage being ¼%, and the market value of the stock being 67⅛, how many U. S. 3% bonds could I get if I were to sell the stock and invest the proceeds in bonds at 107⅛? What would be my surplus? What change will be made in my income?

# EXCHANGE

**731. Exchange** is the method or system by which debts are settled between persons in different places without the actual transmission of the money. It consists in the giving or receiving of a sum of money in one place for a bill ordering the payment of an equivalent sum in another.

Payment may be made by checks, postal money orders, express money orders, telegraph money orders, bank drafts or commercial drafts.

**732. Bank Drafts** are the principal means employed by merchants in making remittances from one part of the country to another. Banks located in the smaller cities and towns of the country keep money deposited in the great financial centers, such as New York, Chicago, Boston, Philadelphia, St. Louis, Baltimore, New Orleans and San Francisco. The banks draw upon their accounts in distant cities and sell their drafts to their customers, making a profit on the charge for exchange. They also buy commercial drafts, drawn by one merchant upon another who owes him money, and transmit these to their correspondents for collection, thus keeping their accounts replenished.

**733.** Drafts upon **New York** are extensively used in effecting exchange between the other financial centers of the country. New York is thus the ultimate center of exchange for the whole country.

**734.** The principal financial centers of Europe are London, Paris, Amsterdam, Antwerp, Hamburg, Vienna, Geneva, Berlin, Frankfort and Bremen. **London** is the center of exchange for the commerce of the world.

**735. A draft**, or **bill of exchange**, is an order drawn by one person (*the drawer*) upon another (*the drawee*) living in a different place, directing *the drawee* to pay a sum of money to the order of the drawer or to a third person (*the payee*).

**736. Bills of exchange** are of two kinds: *foreign* and *domestic*.

**737. A foreign bill of exchange** is one payable in a country other than the one in which it is drawn.

**738. A domestic**, or **inland bill of exchange**, is one drawn and payable in the same country.

**739. The rate of domestic exchange** depends on the condition of the trade balances between different cities. If in one city, as New York, there is little demand for drafts on another city, as St. Louis, but a large demand in St. Louis for drafts on New York, the funds of St. Louis banks on deposit in New York will become exhausted. Due to this condition the price of exchange on New York rises, or is at a premium, the reverse prevails in New York, where it is discount. The rate of collection, commonly called exchange, is arbitrary, depending not on trade conditions.

**740.** With reference to time, drafts are drawn in three ways: at sight, a certain number of days after sight and a certain number of days after date. The first are called **sight drafts**; the other two are called **time drafts.**

**741. Time drafts** are discounted in the same manner as promissory notes.

**742. To find the cost of a draft, the face being given.**

EXAMPLE 1.—Find the cost of a draft for \$1200, exchange $\frac{1}{4}\%$ premium.

\$1200 Face of draft + $\frac{1}{4}\%$ of Face of draft, or \$3 = \$1203 total cost

EXAMPLE 2.—Find the cost of a draft for \$800, exchange $\frac{1}{4}\%$ discount.

\$800 Face of draft − $\frac{1}{4}\%$ of Face of draft, or \$2 = \$798 net cost

**743.** RULE.—*Multiply face by par (100%) plus the rate of premium, or by par less the discount.* Or, *Add the premium to the face or subtract the discount from the face.*

**WRITTEN EXERCISE**

**744.** Find the cost of a draft for

1. \$1000 at $\frac{1}{8}\%$ discount
2. \$2000 at $\frac{1}{4}\%$ premium
3. \$5000 at $\frac{1}{2}\%$ discount
4. \$3875.50 at $\frac{1}{16}\%$ premium
5. \$9217.84 at $\frac{3}{16}\%$ discount
6. \$7382.19 at $\frac{5}{16}\%$ premium

**745. To find the face of a draft, the cost and rate being given.**

EXAMPLE.—What is the face of a draft that cost \$6015, exchange $\frac{1}{4}\%$ premium?

\$1.00 of face + $\frac{1}{4}\%$ of Face, or .0025 = 1.0025

6015 ÷ 1.0025 = \$6000 Face

**746.** RULE.—*Divide cost by par (100%) plus the rate of premium or by par less the discount.*

**WRITTEN EXERCISE**

**747.** Find the face of the following drafts (amount given being the *cost*, and the rate expressed in dollars and cents per \$1000):

1. \$6234.18 at \$1.25 premium
2. \$3697.25 at \$2.50 discount
3. \$6800.00 at \$1.00 premium
4. \$5675.00 at 25 cents discount
5. \$8322.12 at \$2.00 premium
6. \$4725.80 at \$1.75 discount

**WRITTEN PROBLEMS**

**748.** Find the cost of the following drafts:

1. A 30-day draft for \$5000, $\frac{1}{8}\%$ premium.

EXPLANATION.—At $\frac{1}{8}\%$ the premium is \$6.25, making the cost \$5006.25; but the discount on the face for 30 days, or \$25, will reduce the cost by that much (\$5006.25 − \$25) or \$4981.25.

2. A 60-day draft for \$7500, $\frac{3}{4}\%$ discount.
3. A 20-day draft for \$9873.25, $\frac{5}{8}\%$ premium.
4. A 12-day draft for \$6589.62, $\frac{1}{8}\%$ discount.
5. A 33-day draft for \$4437.18, $\frac{3}{8}\%$ premium.
6. Find the face of a 30-day draft that cost \$7289, \$1.25 premium.

EXPLANATION.—Add the premium on \$1 to \$1, then subtract the interest for 30 days; divide this into the entire cost.

*7.* What is the face of a 36-day draft that cost $4578.27, 50 cents discount?

*8.* Find the face of a 63-day draft that cost $5539.65, $\frac{3}{8}\%$ premium.

*9.* I wish my agent in Chicago to buy 2000 bushels of wheat at $68\frac{1}{4}$ cents a bushel. If his commission is 2% and exchange $\frac{1}{8}\%$ premium, what will be the cost of the sight draft I send him?

*10.* A cotton broker in New York sells for a dealer in New Orleans 300 bales of cotton averaging 470 pounds each at $7\frac{7}{8}$ cents per pound, his commission being 5%. The dealer in New Orleans draws a 60-day draft for the amount due him; how much can he get for the draft if exchange is $2.05 premium?

*11.* A of New York owes B of San Francisco $15000. If exchange is $\frac{1}{8}\%$ premium, how much must he pay for a 30-day draft, the rate of interest being 6%?

*12.* A man in St. Louis sends to a New York broker 500 shares of stock with instructions to sell and remit proceeds by bank draft. If the broker sells the stock at $87\frac{3}{4}$ and exchange is $\frac{1}{4}\%$ premium, how much does the St. Louis man receive?

*13.* A commission merchant of Boston sold a consignment of 988200 pounds of wheat at $1.20 per bushel, charging $2\frac{1}{2}\%$ commission and $85 for other expenses. If exchange is at $\frac{1}{4}\%$ discount and interest 6% per annum, how large a draft, payable 20 days after sight, can he buy with the net proceeds, allowing 4 days for transmission of the draft?

*14.* A jobber purchased a bill of goods at $7865.95 less 5% for cash. To make immediate payment, he discounted a draft for $2836.75 due in 48 days, paying $\frac{1}{2}\%$ discount; a note for $3578.25 due in 27 days, paying $\frac{1}{4}\%$ discount; and purchased a bank draft at $\frac{1}{4}\%$ premium for the balance. What was the face of the bank draft? What will it cost? How much was saved by paying cash?

*15.* A draft drawn at 20 days after date was presented at bank for discount 12 days before maturity. If drafts are selling at $3.25 premium and $5687.45 was realized, what was the face of the draft?

*16.* My agent sold produce for $15685 and charged $2\frac{1}{4}\%$ commission. He then purchased a 60-day draft, paying $\frac{3}{8}\%$ premium, which he sent to me. Twenty-three days before maturity, I had it discounted. What did I receive?

*17.* My agent in Chicago bought $10000 Chi., R. I. & Pacific at $96\frac{3}{4}$ and 87 shares Ill. Central ($50) at $57\frac{3}{4}$. To pay for the stock and brokerage, I purchased a 30-day bank draft, exchange $\frac{1}{4}\%$ discount; what was the face of the draft, and what did it cost?

*18.* My draft on Peter Gray at 20 days was returned to me 4 days after acceptance. Eight days later I presented it to bank for discount. I received $5.48 less than face value. What amount did I receive?

*19.* Sent my agent 350 bbls. potatoes, on which he paid $18.50 charges. He sold them at $2.85 a bbl., charging $2\frac{1}{2}\%$ commission. Cash payment being made, a discount of 3% was given the purchaser. After deducting all expenses, the agent remitted by 30-day bank draft bought at a premium of $3.25. What difference would it have made to me if he had remitted by Express Money Order, paying 15 cents per $100?

*20.* I have a 20-day draft which cost $9468.75, bought at $\frac{3}{8}\%$ premium. Eight days before it was due I had it discounted and also had discounted the same day a 10-day acceptance, the face of which was $4385. What were the proceeds of the two drafts?

# FOREIGN EXCHANGE

**749. Foreign exchange** is the settlement of debts by means of bills drawn in one country and payable in another.

**750.** The sum of a **foreign bill of exchange** is expressed in the money of the country on which it is drawn.

NOTE.—Foreign bills are usually drawn *at sight* or at *60 days after sight*. No discount is reckoned on 60-day bills, as the quotation includes the allowance for time.

**751.** The **intrinsic par of exchange** is the value of the monetary unit of one country in that of another, based on the comparative weight and fineness of the coins as determined by government assay. It is sometimes spoken of as the mint par.

**752.** The **rate of exchange** between countries and cities varies according to commercial and monetary conditions, but the intrinsic par of gold standard countries remains at a fixed ratio; *e. g.*, the United States gold dollar contains 23.22 troy grains and the English pound sterling 113.0016 troy grains or 4.8665 times the value of a dollar. **Customs duties** are computed on an intrinsic par basis. The premium or discount of exchange depends primarily on the balance of trade, but is also affected by the ownership in one country of the investment securities of another, etc. Exchange in the debtor country or city will be at a premium and creditor country or city at a discount; consequently, the buying or selling of exchange is determined by cost of exporting or importing gold bullion, which is about $\frac{1}{2}\%$ to $\frac{5}{8}\%$ between the United States and Great Britain, and a correspondingly lower rate between cities of the same country.

**753. Commercial quotations** of foreign exchange are given by means of equivalents, without reference to the par value.

**754. Sterling exchange** is quoted by giving the value of £1 in dollars and cents; as, "Sterling exchange, 4.86½ @ 4.88" means \$4.86½ = £1 for 60-day bills and \$4.88 = £1 for sight bills.

**755. Exchange on France, Belgium or Switzerland** is quoted by giving the value of \$1 in francs and centimes; as, "Paris exchange, 5.15 @ 5.13⅛" means \$1 = 5.15 francs for 60-day bills and \$1 = 5.13⅛ francs for sight bills.

**756. Exchange on Holland** is quoted by giving the value of 1 guilder in cents.

**757. Exchange on Germany** is quoted by giving the value of 4 marks in cents; as, 94⅜ @ 94⅝ means 94⅜ cents = 4 marks for 60-day bills and 94⅝ cents = 4 marks for sight bills.

**758.** It is provided by an Act of Congress that the values of the standard coins of the nations of the world shall be estimated at stated intervals by the Director of the Mint, and be proclaimed by the Secretary of the Treasury. The following table shows the values of the foreign moneys of account as published April 1, 1914.

## Table of Foreign Coins, April 1, 1914

### VALUES OF FOREIGN COINS

| COUNTRY | Standard | Monetary Unit | Value in terms of U. S. gold dollar | COINS |
|---|---|---|---|---|
| Argentine Republic | Gold | Peso | $0.9647 | Gold: argentine ($4.824) and ½ argentine. Silver: peso and divisions. |
| Austria-Hungary | Gold | Crown | .203 | Gold: 10 and 20 crowns. Silver: 1 and 5 crowns. |
| Belgium | Gold | Franc | .193 | Gold: 10 and 20 francs. Silver: 5 francs. |
| Bolivia | Silver | Boliviano | .389 | Silver: boliviano and divisions. |
| Brazil | Gold | Milreis | .546 | Gold: 5, 10 and 20 milreis. Silver: ½, 1 and 2 milreis. |
| Canada | Gold | Dollar | 1.000 | |
| Central Amer. States— | | | | |
| Costa Rica | Gold | Colon | .465 | |
| British Honduras | Gold | Dollar | 1.000 | |
| Guatemala, Honduras, Nicaragua, Salvador | Silver | Peso | .422 | Silver: peso and divisions |
| Chile | Gold | Peso | .365 | Gold: escudo ($1.825), doubloon ($3.650), and condor ($7.300). Silver: peso and divisions. |
| China | Silver | Tael‖ / Dollar¶ | | |
| Colombia | Gold | Dollar | 1.000 | Gold: condor ($9.647) and double-condor. Silver: peso. |
| Denmark | Gold | Crown | .268 | Gold: 10 and 20 crowns. |
| Ecuador | Gold | Sucre | .497 | Gold: 10 sucres ($4.8665). Silver: sucre and divisions. |
| Egypt | Gold | Pound, 100 piasters | 4.943 | Gold: pound (100 piasters), 5, 10, 20 and 50 piasters. Silver: 1, 2, 5, 10 and 20 piasters. |
| Finland | Gold | Mark | .193 | Gold: 20 marks ($3.859), 10 marks ($1.93). |
| France | Gold | Franc | .193 | Gold: 5, 10, 20, 50 and 100 francs. Silver: 5 francs. |
| German Empire | Gold | Mark | .238 | Gold: 5, 10 and 20 marks. |
| Great Britain | Gold | Pound sterling | 4.866½ | Gold: sovereign (pound sterling) and ½ sovereign. |
| Greece | Gold | Drachma | .193 | Gold: 5, 10, 20, 50 and 100 drachmas. Silver: 5 drachmas. |
| Haiti | Gold | Gourde | .965 | Gold: 1, 2, 5 and 10 gourdes. Silver: gourde and divisions. |
| India (British) | Gold | Rupee* | .3244½ | Gold: sovereign (pound sterling). Silver: rupee and divisions. |
| Italy | Gold | Lira | .193 | Gold: 5, 10, 20, 50 and 100 lire. Silver: 5 lire. |
| Japan | Gold | Yen | .498 | Gold: 5, 10 and 20 yen. Silver: 10, 20 and 50 yen. |
| Liberia | Gold | Dollar | 1.000 | |
| Mexico | Gold | Peso† | .498 | Gold: 5 and 10 pesos. Silver: dollar‡ or (peso) and divisions. |
| Netherlands | Gold | Florin | .402 | Gold: 10 florins. Silver: 2½, 1 florin and divisions. |
| Newfoundland | Gold | Dollar | 1.014 | Gold: 2 dollars ($2.028). |
| Norway | Gold | Crown | .268 | Gold: 10 and 20 crowns. |
| Panama | Gold | Balboa | 1.000 | Gold: 1, 2½, 5, 10 and 20 balboas. Silver: peso and divisions. |
| Persia | Silver | Kran | .1704 | Gold: ½, 1 and 2 tomans ($3.409). Silver: ¼, ½, 1, 2 and 5 krans. |
| Peru | Gold | Libra | 4.866½ | Gold: ½ and 1 libra. Silver: sol and divisions. |
| Philippine Islands | Gold | Peso | .500 | Silver peso: 10, 20 and 50 centavos. |
| Portugal | Gold | Milreis | 1.080 | Gold: 1, 2, 5 and 10 milreis. |
| Russia | Gold | Ruble | .515 | Gold: 5, 7½, 10 and 15 rubles. Silver: 5, 10, 15, 20, 25, 50 and 100 copeks. |
| Spain | Gold | Peseta | .193 | Gold: 25 pesetas. Silver: 5 pesetas. |
| Straits Settlements | Gold | Dollar sterling§ | 1.00 / 4.866½ | Gold: sovereign (pound sterling). Silver: dollar and divisions. |
| Sweden | Gold | Crown | .268 | Gold: 10 and 20 crowns. |
| Switzerland | Gold | Franc | .193 | Gold: 5, 10, 20, 50 and 100 francs. Silver: 5 francs. |
| Turkey | Gold | Piaster | .044 | Gold: 25, 50, 100, 250 and 500 piasters. |
| Uruguay | Gold | Peso | 1.034 | Gold: peso. Silver: peso and divisions. |
| Venezuela | Gold | Bolivar | .193 | Gold: 5, 10, 20, 50 and 100 bolivars. Silver: 5 bolivars. |

NOTE.—The coins of silver-standard countries are valued by their pure silver contents, at the average market price of silver for the three months preceding.

* The sovereign is the standard coin of India, but the rupee ($0.3244½) is the current coin, valued at 15 to the sovereign.

† Seventy-five centigrams fine gold. ‡ Value in Mexico, $0.498.

§ The current coin of the Straits Settlements is the silver dollar issued on Government account and has been given a tentative value of $0.567758½.

‖ The value of the tael varies in the different provinces. The values are as follows: Amoy $.692, Canton .690, Chefoo .681, Chin Kiang .676, Fuchau .640, Haikwan (customs) .704, Hankow .647, Kiaochow .670, Nankin .685, Nischwang .649, Ningpo .665, Peking .674, Shanghai .632, Swatow .639, Takau .648, Tientsin .670.

¶ The dollar has the following values in the respective provinces: Hongkong $.423, British .423, Mexican .427.

The rates in the foregoing table are used at the Custom House in estimating the value of foreign merchandise on invoices made out in foreign currencies.

**759. To find the cost or value of a foreign bill.**

EXAMPLE.—Find the value of a draft for £120 15s. at 4.867.

$4.867
$120\frac{3}{4}$
584 040
3 650¼
$587.690 Result, $587.69

**760.** RULE.—*Multiply face of bill by unit quotation of exchange.*

### WRITTEN PROBLEMS

**761.** Find the value in United States money of

1. £375 at 4.88
2. 2340 francs at 5.15
3. 978 guilders at 41½
4. 1200 marks at 96½
5. £562 8s. 6d. at 4.86½
6. £782 15s. at 4.87¼
7. £524 7s. 4d. at 4.86¾
8. 7643.12 francs at 5.13½
9. £1142 18s. 3d. at 4.88½
10. 3789 guilders at 41⅞

**762. To find the face of a foreign bill.**

EXAMPLE.—Find the face of a draft on London costing $587.69, exchange 4.867.

£120 7499+
4.867 ) 587.690 0000
486 7
100 99
97 34
3 6500
3 4069
24310
19468
48420
43803
46170

£.7499
20
14.9980

Result, £120 15s.

NOTE.—The exact result is not obtained, owing to the ignoring of the small fraction in finding cost.

**763.** RULE.—*Divide value of bill by unit quotation of exchange.*

### WRITTEN EXERCISE

**764.** Find the face of a bill that costs

1. $738.72 at 4.89 (London)
2. $975.18 at 5.14½ (Paris)
3. $840.75 at 41⅜ (Amsterdam)
4. $1238.29 at 94⅞ (Berlin)
5. $3472.30 at 5.17 (Antwerp)
6. $2575.84 at 4.85⅞ (London)
7. $395.97 at 4.88⅜ (London)
8. $1892.33 at 95⅛ (Hamburg)
9. $5698.77 at 5.16⅛ (Geneva)
10. $7535.44 at 96⅜ (Bremen)

### WRITTEN PROBLEMS

**765.** *1.* Find the value of a draft on Vera Cruz for 3426 dollars at .662.

*2.* Find the cost of a draft on Shanghai for 6847 taels at .901.

*3.* What will a draft on St. Petersburg for 13625 rubles cost at .488?

*4.* Find the value of a draft on Yokohama for 24677 yen at .658.

*5.* What is the cost of a draft on Havana for 36355 pesos at .926?

*6.* A Liverpool merchant buys 40000 bushels of American wheat at 69 cents a bushel; how much does it cost him to remit by draft in settlement, exchange 4.88½?

*7.* If U. S. 4s are quoted in New York at 112, what is the equivalent London quotation, exchange 4.88?

NOTE.—American securities are quoted in London on a fixed basis of $5 = £1.

*8.* What is the equivalent New York quotation of stock quoted in London at 87⅞, exchange 4.88½?

*9.* How much must a London merchant pay for a draft on Bombay for 30000 rupees, exchange at 1s. 3⅛d. (for 1 rupee)?

*10.* A cargo of wine invoiced at 23642.80 milreis in Lisbon is worth more than a cargo of coffee invoiced at 25000 milreis in Rio Janeiro. How much more in U. S. money?

*11.* A manufacturer in England drew a sight draft on an importer in New York for £978 15s. 9d. What will it cost the importer to pay the draft at $4.865?

*12.* A grain firm in Philadelphia drew a bill of exchange on a Paris firm for $32750. If exchange in Paris on Philadelphia is quoted at 5.175, how many francs are required to pay the draft?

*13.* A Baltimore merchant drew a 60-day draft on a debtor in France for 18700 francs. What should a Baltimore banker pay for the draft, allowing interest at 4%, if demand exchange is 5.18¾?

*14.* I purchased a draft for 18675 francs, and another for £35 8s. 10d., the former at 5.17½, and the latter at $4.845. What must be the face of a 60-day note, dated and discounted to-day, that will yield the needed money?

*15.* During a trip through Europe, A spent 3 days in Naples, 25 lire per day, 4 days in Rome, 30 lire per day, 5 days in Berlin, 26 marks per day, 2 weeks in Paris, 20 francs per day, 12 days in London, 28 shillings per day. What were his total expenses in these cities (United States money)?

# TAXES

**766.** A **tax** is a sum of money assessed on the person or property of a citizen by the government to defray public expenses; as, a city, borough or town tax, county tax, state tax, etc.

**767.** The **public revenues** of the various state and local governments in the United States are raised by *direct* taxation upon the property, and in some of the states upon the polls and personal incomes.

**768.** The **public revenues** of the **National government** are raised by *indirect* taxation in the form of duties on imported goods and the internal revenue taxes on liquors and tobacco. Recently a *direct* tax has been levied on incomes above a certain amount.

**769.** A **capitation** or **poll tax** is a specified sum levied on the person of every adult male citizen.

**770. Property taxes** are divided into two classes—(a) taxes on *real property* or *real estate*, i. e., lands, houses, etc.; and (b) taxes on *personal property* or *personal estate*, i. e., horses, cattle, vehicles, furniture, money, stocks, bonds, mortgages, merchandise, etc.

**771. To find amount of tax.**

EXAMPLE.—What is the amount of tax on a property assessed at **$2250**, rate **13** mills?

$$2250 \times .013 = \$29.250$$

**772.** RULE.—*Multiply valuation by rate expressed decimally.*

**773.** **TAX DUPLICATE**

| | Valuation | County Tax 15 mills | RoadTax 8 mills | School Tax 10 mills | Poor Tax 2 mills | Total |
|---|---|---|---|---|---|---|
| *1.* | $5000 | | | | | |
| *2.* | $3000 | | | | | |
| *3.* | $2500 | | | | | |
| *4.* | $12500 | | | | | |
| *5.* | $8000 | | | | | |
| *6.* | $6500 | | | | | |
| *7.* | $800 | | | | | |
| *8.* | $7250 | | | | | |
| *9.* | $4300 | | | | | |
| *10.* | $1350 | | | | | |
| *11.* | $950 | | | | | |
| *12.* | $4000 | | | | | |
| *13.* | $2000 | | | | | |
| *14.* | $1700 | | | | | |
| *15.* | $3900 | | | | | |

**774. To find [redacted] tax required to raise given amount of tax.**

EXAMPLE.—Required tax rate on assessed valuation of $1700000 to raise a school tax of $18000, 10% of levy being uncollectable and cost of collection being 2%.

| | | |
|---|---|---|
| $1.00 of tax levied | .882 )18000.0000(20408.163 | total amount |
| .10 of tax uncollectable | 1764 | to be levied |
| .90 of tax collectable | 3600 | |
| .018 amount paid collector on each $1 of tax levied | 3528 | |
| | 7200 | |
| .882 net amount received on each $1 of tax levied | 7056 | |
| | 1440 | |
| | 882 | |
| 1700000 ) 20408.163 ( .012 | 5580 | |
| 17000 00 | 5292 | |
| 3408 163 | 2880 | |
| 3400 000 | 2646 | Result, 12 mills |

NOTE.—The result is nearly 12 mills, which would be the rate to levy to cover the amount needed.

**775.** RULE.—*Divide amount necessary to levy by assessed valuation.*

### WRITTEN PROBLEMS

**776.** *1.* A city whose assessed valuation is $2500000 must raise $30000 for city purposes; $12500 for highways; $20000 for schools; $5000 for the support of the poor: these sums include collectors' fees. What rate must be levied for each purpose?

*2.* What is the tax on $14500 of real estate and $23275 of personal property, at $2.07 per $100, less a discount of 2% for prompt payment?

*3.* If A's tax is $541.64, at the rate of $19.75 on $1000, what is the assessed valuation of his property?

*4.* What is the rate of taxation per $100, if property assessed at $3850 pays a tax of $73.42?

*5.* The assessed valuation of all the taxable property in a certain town is $1987690. The number of polls is 418, at 50 cents each. The estimated expenses of the town are $33210. What rate of taxation will raise this amount (assuming that all the taxes can be collected)?

*6.* What must be the rate of taxation to yield $5289.70, after paying collector's commission of 2%, if the assessed value of the property is $321500?

NOTE.—The tax collector receives his commission upon the gross amount of tax collected.

*7.* Mr. Brown was assessed as follows: Real estate, $30000; personal property, $3500; money at interest, $25000; income from occupation, $3000; and two gold watches. He obtains an abatement of one-third on real estate, one-fourth on personal property, $3000 on money at interest, two-fifths for occupation, and one gold watch. The tax rate was 2¼ mills, and $1.50 for each watch. What was Brown's tax after the abatement, and how much was it lessened?

# DUTIES

**777.** **Duties** or **customs** are taxes levied by the Government upon imported goods for revenue for the support of the general government and for the protection of home industries.

**778.** Duties are of two classes—*ad valorem* duties and *specific* duties.

**779.** An **ad valorem duty** is a tax assessed at a certain per cent. on the cost of the goods in the country from which they are imported.

NOTE.—*Ad valorem* duties are computed on the *invoice cost* of goods "packed and ready for shipment," exclusive of subsequent expenses, such as freight, insurance, etc. In custom house calculations, duties are not reckoned on fractions of a dollar; a fraction of a dollar if less than one-half is rejected, if over one-half is counted as another dollar.

**780.** A **specific duty** is a tax assessed at a certain sum per pound, ton, gallon, foot, yard, or other weight or measure, without regard to value.

**781.** **Tare** is an allowance made, in estimating specific duties, by way of deduction from the gross weight of goods on account of the weight of the box, cask, etc., in which they are contained.

**782.** **Leakage** is an allowance made for waste of liquids in barrels or casks.

**783.** **Breakage** is an allowance for waste of liquids in bottles.

**784.** **To find ad valorem duty.**

EXAMPLE.—What is the duty on goods invoiced at £72 12s. at 30% ad valorem?

$4.8665 × 72.6 = 291990 + 3503880 = $353.3079 value of invoice

$353 × .30 = $105.90 duty

NOTE.—Take duty on dollars only, dropping cents if less than 50, adding $1 if 50 cents or more.

**785.** RULE.—*Find value of goods in United States money at government value and multiply by rate of duty.*

NOTE.—Specific duty is simply a tax on quantity and is found accordingly. Ad valorem duty is always expressed at a certain rate per cent.

### WRITTEN PROBLEMS

**786.** *1.* What is the duty on an invoice amounting to 12673.10 francs at 40% ad valorem?

*2.* Find the duty at 7 cents per pound on an invoice of goods weighing 32478 lbs., tare 2%.

*3.* What is the duty on an importation of goods invoiced at 12643 marks at 40%?

*4.* Find the duty on goods invoiced at 7846 francs at 60%.

*5.* At 50%, what is the duty on an invoice amounting to 6955 lire?

*6.* Find the duty on goods invoiced at 23632 florins at 25%.

*7.* Find the duty on an invoice of £593 17s. 10d. at 45%.

*8.* At 35 cents per gallon, what is the duty on 100 cases of olive oil, each case containing 2 dozen quarts?

*9.* Find the duty on 40 blocks of marble, each $1\frac{1}{2} \times 2 \times 4$ ft. at 50 cents per cubic foot.

*10.* At 10%, what is the duty on an invoice of 12684 lbs. of leather, imported from Liverpool at 1s. 2½d. per pound?

*11.* At 35%, what is the duty on an invoice of chinaware from Paris, valued at 14274 francs?

*12.* Find the duty on 300 dozen penknives, valued at 3s. per dozen, at 25 cents per dozen and 25% ad valorem; and 400 dozen penknives, valued at 4s. 6d. per dozen, at 40 cents per dozen and 25% ad valorem.

*13.* At 30 cents per square yard, what is the duty on 5694 yards of cloth 27 in. wide?

*14.* A merchant received an invoice of silk from Paris dutiable at 60% on 1760 francs; and another invoice of German toys, the total of which was 13785 marks, duty 55%. What was the total duty? What will it cost to pay the former invoice at 5.16¾ and the latter at 94¾?

*15.* Imported woolen clothing from England, invoiced at £728 13s. 9d., and weighing 1915 lbs.; duty 44 cents per lb. and 60% ad valorem. What was the entire cost of the clothing if the draft was purchased at $4.8585?

*16.* Imported decorated china valued at 1295.38 marks; commission charges (not dutiable) 5%; other charges were 250 marks. If the ad valorem duty was 55%, find the entire cost of the invoice, exchange being 95¾.

*17.* An importer received 3265 yds. Brussels carpet (27 in.) valued at 3 marks per yd.; charges, 485 marks; duty 44 cents per sq. yd., and 35% ad valorem. If exchange is bought at 95¼, at what price per yard must the carpet be sold to net a profit of 27½ per cent.?

*18.* Make the extensions and calculate the duty on the following invoice:

| | | £ | s. | d. |
|---|---|---|---|---|
| 876½ yds. Broadcloth | @ 15s. 8d. | | | |
| 278¾ " | 18s. 6d. | | | |
| Discount 2% | | | | |
| Boxes and packing | | | 17 | 4 |
| Cartage | | | 6 | |
| Duty, 50% | | | | |

# RATIO AND PROPORTION

## RATIO

**787. Ratio** is a measure of relation between quantities of the same kind. There are two kinds of ratio—*arithmetical* and *geometrical.*

**788. Arithmetical ratio** expresses the difference between two quantities.

**789. Geometrical ratio** is the division of one term by another. The usual way of expressing a geometrical ratio is by placing two points, one above the other, between the quantities compared. Thus 2 : 3 signifies the ratio of 2 to 3, and is read 2 *is to* 3. The quantities compared are called the *terms* of the ratio. The first is called the *antecedent*; the second, the *consequent.*

**790.** The **value of a ratio** is found by dividing the antecedent by the consequent. Since the antecedent of the ratio is always the dividend and the consequent the divisor, the ratio may be written in the form of a fraction, the antecedent being the numerator and the consequent the denominator. It follows that the terms of a ratio may be multiplied or divided in the same manner as the terms of a fraction, without changing the value of the ratio.

**791.** A **simple ratio** consists of one antecedent and one consequent; as, 10 : 12.

**792. To find the value of a simple ratio.**

EXAMPLE.—What is the ratio of $1000 to $300?

$$\$1000 : \$300 = \frac{1000}{300} = \frac{10}{3}, \text{ or } 3\tfrac{1}{3}$$

**793.**—RULE.—*Divide the antecedent by the consequent.*

### ORAL EXERCISE

**794.** *1.* Find the ratio of 9 : 6

$9 : 6 = \frac{9}{6}$, or $1\frac{1}{2}$

*2.* 5 : 8

*3.* 7 : 6

*4.* 12 : 26

*5.* 15 : 5

*6.* $3\frac{1}{2} : 2\frac{1}{4}$

*7.* 17 : 4

*8.* 25 : 6

*9.* $15\frac{1}{2} : 4\frac{1}{2}$

*10.* $6\frac{1}{4} : 9\frac{1}{2}$

*11.* $4\frac{3}{4} : 2\frac{7}{8}$

*12.* $8\frac{1}{3} : 17\frac{1}{2}$

*13.* $5\frac{3}{8} : 14\frac{3}{8}$

*14.* $8 : $10 (men : men)

*15.* 27 : 9 (yd. : yd.)

*16.* $16\frac{2}{3} : 33\frac{1}{3}$ (horses : horses)

*17.* 54 : 18

*18.* $3\frac{1}{3}$ bu. : 20 bu.

*19.* 4 mo. 6 da. : 8 mo. 12 da.

*20.* 25 mi. : 75 mi.

**795.** A **compound ratio** consists of two or more simple ratios; as, $\left.\begin{matrix} 9 : 12 \\ 8 : 14 \end{matrix}\right\}$

**796.** The **value of a compound ratio** is found by dividing the product of the antecedents by the product of the consequents; thus, in the above illustration,

$$\frac{\overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}} \times \underset{7}{\cancel{14}}} = \frac{3}{7}$$

## PROPORTION

**797.** A **proportion** is the expression of equality between two ratios. Proportion is indicated by placing four points between the ratios—thus, 6 : 12 : : 24 : 48

**798.** The first and last terms of a proportion are called the **extremes**; the second and third, the **means.**

**799.** In any proportion, *the product of the extremes equals the product of the means.* It follows from this that either *extreme* is found by multiplying the means and dividing this product by the given extreme; or, either *mean* is found by multiplying the extremes and dividing by the given mean.

**800.** A **simple proportion** expresses the equality of two simple ratios.

**801. To find the fourth term of a proportion.**

EXAMPLE 1.—25 : 42 : : 70 : ? 2.—$\frac{7}{8} : 1\frac{1}{2} : : 2\frac{1}{4} : ?$

(1) $\frac{\overset{14}{\cancel{42}}\times\overset{}{70}}{\underset{5}{\cancel{25}}} = \frac{588}{5} = 117\frac{3}{5}$

(2) $\frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{7}}}\times\frac{11}{\underset{3}{\cancel{12}}}\times\frac{\overset{3}{\cancel{21}}}{\underset{2}{\cancel{44}}} = \frac{1}{2}$

NOTE.—Since in Example 2 the first term is the divisor, we simply invert the fraction and multiply.

The operation may frequently be abridged by cancelation.

**802.** RULE.—*Divide the product of the means by the given extreme.*

**803.** **WRITTEN EXERCISE**

1. 17 : 32 : : 51 : ?
2. $\frac{1}{2} : \frac{3}{4} : : \frac{5}{8} : ?$
3. 9 : 24 : : 7 : ?
4. 33 : 46 : : 3 : ?
5. 5 : 7 : : 9 : ?
6. $7\frac{1}{2} : 13\frac{1}{4} : : 3\frac{1}{3} : ?$
7. $7 : 4\frac{1}{2} : : 9\frac{1}{4} : ?$
8. 15 : 27 : : 8 : ?
9. $14\frac{1}{4} : 5 : : 9\frac{1}{2} : ?$
10. $3 : 7\frac{1}{2} : : 21 : ?$
11. 4 : 26 : : 9 : ?
12. $7\frac{3}{4} : 15 : : 7\frac{3}{8} : ?$

**804. To determine the statement of a simple proportion.**

EXAMPLE 1.—If 15 men can do a piece of work in 22 days, in how many days can 24 men do the same work?

EXPLANATION.—If 15 men can do the work in 22 days, 24 men can do the work in less time; in this case the smaller term of the complete ratio is placed as the second term. The proportion is 24 : 15 : : 22 : ?

$$\frac{\overset{5}{\cancel{15}}\times\overset{11}{\cancel{22}}}{\underset{4}{\cancel{24}}} = \frac{55}{4} \text{ or } 13\tfrac{3}{4} \text{ days}$$

EXAMPLE 2.—If 19 men can earn $410 in a given time, how much could 28 men earn in the same time?

EXPLANATION.—If 19 men can earn $410, 28 men can earn more; in this case the larger term of the complete ratio is placed as the second term. The proportion is 19 : 28 : : 410 : ?

$$\frac{28\times410}{19}=\frac{11480}{19} \text{ or } \$604.21$$

**805.** RULE.—*Place for the third term the term which is of the same kind as the required result. Then from the conditions of the problem, determine whether the fourth term, or result, should be more or less than the given third term. If the fourth term should be less than the third, place the smaller of the remaining terms second; if more, place the larger of the remaining terms second.*

## WRITTEN PROBLEMS

**806.** *1.* If 29 tons of coal cost $130.50, what would 35 tons cost at the same rate?

*2.* If 32 horses can do a given amount of work in 46 days, in what time could 54 horses do the same amount of work?

*3.* If $326 produce $1980 interest in a given time, how much interest would $1920 produce in the same time at the same rate?

*4.* If $15\frac{3}{4}$ pounds of butter cost $5.04, what will $23\frac{7}{8}$ pounds cost at the same rate?

*5.* If $44\frac{5}{8}$ yards of cloth cost $145.03, what should $37\frac{5}{8}$ yards cost?

*6.* If the freight on a car load of coal from one point to another is $8.75 and it is carried $36\frac{1}{2}$ miles on one company's road and $46\frac{7}{8}$ miles on another, what share of the freight should each receive?

*7.* If the liabilities of a bankrupt amount to $11726 and the assets to $9427, how much should A receive to whom he owes $1247?

NOTE.—The entire liability is to the liability of each creditor as the entire assets are to that creditor's share.

*8.* The collectable accounts of an insolvent merchant are: notes, $4675; personal accounts, $11625; he has $2362 worth of stock on hand. His liabilities are: notes, $13654; and other debts amounting to $15065. How much can he pay A, to whom he owes $1290; B, to whom he owes $2265; and C, to whom he owes $4475?

*9.* The total assessment of a borough is $3457500; the tax to be raised for 1915 is $14750. How much of this should a property holder pay whose property is assessed at $18500?

NOTE.—The total assessment is to each individual's assessment as the tax to be raised is to the amount to be paid by that individual.

*10.* If $\frac{7}{8}$ of a yard of silk cost $1.85, what should $1\frac{3}{4}$ yards cost at the same rate?

11. A and B form a partnership. A invests $12500 and B invests $16750. They gain during the year $7250. Find each partner's share of the gain.

Note. The total investment is to each partner's investment as the whole gain is to each partner's gain.

12. The total tax to be raised by a certain town for the year 1915 is $17650. The assessed valuation of the town is $4724000. What tax should A pay, whose property is assessed at $7500, B, whose property is assessed at $12000, and C, whose property is assessed to the amount of $17500?

13. Three boys buy a bicycle, each paying as follows: the first $40, the second $45, the third $40. How long should each use it, every day of ten hours?

14. A works 15 hours at a certain job, B 22 hours, C 28 hours. If the amount received for the work is $62.50, what should each receive?

15. A traveler purchased a railroad ticket for $24. To get to his destination he travels over 125 miles of one company's road, 216 miles of another and 475 miles of a third. Find the sum due each road.

16. If 42 men or 56 boys can do a piece of work in 24 days, how long would it take 18 men and 20 boys to do the same work?

Solution. Since 42 men do as much as 56 boys, find how many boys are equal to 18 men. Then proposition can be stated in terms of boys only.

17. If 25 men can do a piece of work in 36 days, in what time could 42 boys do the same work, provided 7 boys can do as much as 5 men?

18. If a grocer's passing weight is $\frac{1}{2}$ of an ounce light, what would his sales amount to in a week, if during this time he has defrauded his customers of $5?

19. If I have coal for $4.25 a long ton and wish to sell by the short ton so as to gain 25%, what must I ask per ton?

20. If $49.75 is the interest of a given principal at 5%, what is the interest on the same principal for the same time at 7½%?

21. If $64.2 produce $2.675 interest in a given time, what principal, for the same time and rate will produce $92.96 interest?

22. If a merchant gains 16⅔%, by selling goods for $14, what would be his gain per cent by selling them for $17?

Solution. The selling per cent is 116⅔, the ratio between the selling prices equals the ratio between the selling per cents.

23. If by selling goods for $87 there is a loss of 1%, what would be the per cent of loss or gain by selling them for $102?

24. If by selling goods for $\frac{3}{4}$ of the marked price I gain $\frac{1}{5}$ of the cost, what part of the cost would I gain by selling them for $\frac{2}{3}$ of the marked price?

## COMPOUND PROPORTION

**807.** A **compound proportion** is the expression of equality between ratios, one of which is compound. Thus:

$$\left.\begin{matrix}4:5\\8:12\\35:15\end{matrix}\right\} ::25:?$$

**808. To find the fourth term in a compound proportion.**

EXAMPLE.—

$$\left.\begin{matrix}3\frac{3}{4}:4\frac{1}{2}\\\frac{7}{8}:\frac{5}{9}\\13\frac{1}{2}:13\frac{3}{5}\end{matrix}\right\} ::3\tfrac{3}{8}:?$$

$$\frac{4\times8\times\ 2\times9\times5\times68\times29}{15\times7\times27\times2\times9\times\ 5\times\ 9}=2\tfrac{13}{25}\tfrac{9}{35}\tfrac{7}{18}\tfrac{4}{5}$$

NOTE.—The fourth term in a compound proportion is found by multiplying the means, and dividing this product by the product of the extremes.

**809.** RULE.—*Divide the product of the means by the product of the given extremes.*

### WRITTEN EXERCISE

**810.** Find the fourth term in

*1.* $\left.\begin{matrix}4:9\\7:6\\8:5\\20:21\end{matrix}\right\} ::9:?$

*4.* $\left.\begin{matrix}7\frac{1}{2}:35\\3\frac{1}{2}:12\\5\frac{4}{5}:22\end{matrix}\right\} ::6:?$

*6.* $\left.\begin{matrix}14:26\\25:65\\33:72\\25:40\\125:140\end{matrix}\right\} ::16:?$

*2.* $\left.\begin{matrix}9:14\\7:6\\15:36\end{matrix}\right\} ::24:?$

*5.* $\left.\begin{matrix}14:26\\8:28\\7\frac{1}{2}:45\\3\frac{3}{4}:15\end{matrix}\right\} ::7\tfrac{1}{2}:?$

*7.* $\left.\begin{matrix}4\frac{1}{2}:18\\5\frac{1}{4}:16\\6\frac{2}{3}:42\end{matrix}\right\} ::15:?$

*3.* $\left.\begin{matrix}40:65\\75:20\\32:8\\9:27\\14:7\end{matrix}\right\} ::300:?$

*8.* $\left.\begin{matrix}60.5\text{ ft.}:120\text{ ft.}\\18\text{ in.}:12.75\text{ in.}\\25.25\text{ ft.}:27.5\text{ ft.}\end{matrix}\right\} ::15:?$

**811. To determine the statement of a compound proportion.**

EXAMPLE. If 16 men, working 9 hours a day, can build a wall 27 feet long, 18 inches thick and 28 feet high in 18 days, in how many days of 10 hours each could 22 men build a wall 38 feet long, 2 feet thick and 12 feet high?

| Statement | men | hrs | long | thick | high | in |
|---|---|---|---|---|---|---|
| | 16 | 9 | 27 ft | 18 in | 28 ft | 18 |
| | 22 | 10 | 38 ft | 2 ft | 12 ft | ? |

22 : 16
10 : 9
27 : 38 : : 18 : ?
18 : 24
28 : 12

$$\frac{2}{11} \cdot \frac{19}{5} \cdot \frac{6}{9} \cdot \frac{14}{27} \cdot \frac{48}{38} \cdot \frac{24}{2} \cdot \frac{12}{12} \cdot \frac{14}{?} = 8\frac{1}{11} \text{ days}$$

**812. RULE.** *Write as the third term the quantity which is of the same kind as the result. Then, as in simple proportion, determine whether the answer depending upon each ratio should be more or less than the third term; if more, place the larger term of ratio as second the third term; if less, place the smaller term of the ratio as second the third term. Proceed in this way, reasoning with each ratio entirely independently of the others, until you have disposed of them and the unknown term.*

## WRITTEN PROBLEMS

**813.** 1. If the freight on 32 barrels of sugar for a distance of 54 miles is $4.25, what should be the freight on 64 barrels for 25 miles?

2. If a 5-pound loaf of bread cost 10 cents when flour is worth $4.75 a barrel, what should be the price of a 6-pound loaf when flour is worth $5.50 a barrel?

3. If 15 apples can be bought for 25 cents when apples are worth $4.25 a barrel, what should be the cost of 45 apples when a barrel costs $6?

4. If the tax on a property assessed at $5,000 is $125.25 at a tax rate of 6¼, what should be the tax on a property assessed at $4,000 if the rate is 2½?

5. If I pay $2.25 a yard for cloth ⅞ of a yard wide, what should I pay for 32 yards of a similar quality ⅝ of a yard wide?

6. If I pay $42.25 for a pile of wood 16 feet long, 4 feet wide and 7 feet high, how much should a pile be worth that is 14 feet in length and 8 feet high?

7. If a pile of wood 18 feet long, 4 feet wide and 6 feet high cost $32, what should be paid for a pile 21 feet long, 4 feet wide and 11 feet high?

8. If 42 men can earn $1,500 in 18 days by working 11 hours a day, how much should 30 men earn in 20 days of 9 hours each?

9. If the cost of pasturing 75 head of cattle for 11 weeks is $72, what should be the cost of pasturing 42 head of cattle for 4 weeks?

10. If it costs $19.20 to carry 7,642 pounds 120 miles, what should be the cost of carrying 12,841 pounds 60 miles at the same rate?

*11.* If the interest of \$367 at 5% for 4 years, 7 months, 18 days, is \$85.02, what is the interest of \$19.72 at 4½% for 2 years, 5 months and 18 days?

SUGGESTION.—The time in an interest question must be considered as a unit; as, 4 yrs. 7 mos. 18 da. = 1768 days; 2 yrs. 5 mos. 18 da. = 888 days.

*12.* If \$765 produce \$144.59 interest in 2 years, 8 months and 12 days at 7%, what principal would produce \$146.75 in 3 years, 2 months and 18 days at 5%?

*13.* If \$1365, in 3 years, 9 months and 14 days, at 6%, produce \$310.31 interest, at what rate would \$1065 produce \$78.41 in 1 year, 7 months and 19 days?

*14.* If 52 men can earn \$1850 in 19 days by working 10 hours a day, how much should 29 men earn in 23 days of 8 hours each?

*15.* If the cost of pasturing 85 head of cattle for 13 weeks is \$75, what should be the cost of pasturing 99 head of cattle for 4 weeks?

*16.* If \$1426, at 5%, will produce \$163.99 interest in 2 years, 3 months and 18 days, in what time will \$1820 produce \$726.80 interest at 4½%?

*17.* If there are 20 perches of masonry in a wall 33 feet long, 5 feet high and 3 feet thick, how many perches are there in a wall 32 feet long, 14 feet high and 2 feet thick?

*18.* If a wall 122 feet long, 12 feet high, 2½ feet thick contains 148 perches of masonry, how high should a wall 136 feet long and 3 feet thick be to contain 136 perches?

*19.* If 45 tons of hay will be sufficient for 85 horses 96 days, how many tons must be added to be sufficient for 125 horses 112 days?

*20.* If 3 men or 5 boys can bind 5 acres of oats in 2½ days, how many acres of the same average yield could 5 men and 7 boys bind in 3¾ days?

*21.* How many Belgian blocks 3½ inches wide, 7 inches long, will be required to cover a space 36 feet wide, 88 feet long, if 5120 blocks 9 inches long, 4½ inches wide, are required to cover a space 32 feet wide and 45 feet long?

*22.* How many hours a day must 64 men work to dig a canal 2 miles long, 24 feet wide and 7 feet deep in 528 days, if 42 men, working 10 hours a day, can dig a canal 1½ miles long, 26 feet wide and 6 feet deep in 610 days?

*23.* If 26 men, working 10 hours a day, can dig a ditch 142 feet long, 4 feet wide and 12 feet deep in 96 days, in how many days of 8 hours each could 92 men dig a ditch 108 feet long, 3 feet wide and 16 feet deep, if the digging in the second case is estimated to be 20% harder?

*24.* If 4 horses, working 9 hours a day, can plow a field 372 feet long, 348 feet wide, in 2½ days, the walking gait of the horses being 4 miles an hour, in how many days, working 10 hours a day, could 6 horses, whose walking gait is 3 miles an hour, plow a field containing 14 acres, 142 perches? (A perch = 1 sq. rod)

# ALLIGATION

**814. Alligation,** or **medial proportion,** is the process of combining two or more quantities of different values so as to make a combination of a given mean value.

**815. To find the average value when the values of the several ingredients are given.**

EXAMPLE.—A grocer mixes 45 pounds of tea worth 50 cents a pound, 85 pounds worth 60 cents, 125 pounds worth 75 cents, and 90 pounds worth $1. Find the average price.

| | | |
|---|---|---|
| 45 lbs. @ 50c. | = | $22.50 |
| 85 lbs. @ 60c. | = | 51.00 |
| 125 lbs. @ 75c. | = | 93.75 |
| 90 lbs. @ $1 | = | 90.00 |
| 345 lbs. | = | $257.25 |

$257.25 ÷ 345 = 74$\frac{5}{69}$ cents, value of 1 lb.

**816. To find the amount of the several ingredients taken when their respective values and the mean or average value are given.**

EXAMPLE.—It is required to know how many pounds of coffee at the following prices, 25 cents, 32 cents, 38 cents, 40 cents and 45 cents, may be taken to form a mixture worth 35 cents a pound.

| | | | | | | Totals | Proof |
|---|---|---|---|---|---|---|---|
| 35 | 25 | 10c. gain | 1 lb. | | 1 lb. | 2 lbs. | @ 25c = 50 cents |
| | 32 | 3c. gain | | 1 lb. | | 1 lb. | @ 32c. = 32 " |
| | 38 | 3c. loss | | 1 lb. | | 1 lb. | @ 38c. = 38 " |
| | 40 | 5c. loss | | | 2 lbs. | 2 lbs. | @ 40c. = 80 " |
| | 45 | 10c. loss | 1 lb. | | | 1 lb. | @ 45c. = 45 " |
| | | | | | | 7 lbs. | ) 245 cents |
| | | | | | | | Average, 35 cents |

EXPLANATION.—Arrange the prices in a vertical column with the desired price before it. If one pound worth 25c. is mixed and is then worth 35c., there is a gain of 10 cents; on a pound worth 32c., there is a gain of 3 cents; on the pound worth 38c., a loss of 3c.; on the pound worth 40c., a loss of 5c.; and on the pound worth 45 cents, a loss of 10 cents. In order to have gain and loss equal, 1 lb. each of the 25c. and 45c. kind must be taken; also 1 lb. each of the 32c. and 38c. kind; then 2 pounds of the 40c. kind will cause a loss of 10 cents, which may be made up by 1 lb. at 25c., on which there is a gain of 10 cents. Carrying out the results obtained we have the answers as above.

**817. To find the number of each ingredient when one or more of the ingredients are limited.**

EXAMPLE.—It is required to know how many gallons of alcohol 65% proof and how many gallons of water must be mixed with 90 gallons of alcohol 80% proof to produce alcohol 60% proof.

| | | | | |
|---|---|---|---|---|
| 60% | 65% | 5% reduced | 8 gals. = 40% reduced | |
| | 80% | 20% reduced | 7 gals. = 140% reduced | 180 reduced |
| | 00% | 60% increased | 3 gals. = 180% increased | 180 increased |

EXPLANATION.—Arrange as in Art. 816, and compare each with the desired per cent. of proof. Since 7 gallons of the 80% grade must be used, that will reduce the test 140%; the nearest multiple of 60% increase is 180, to produce which takes 8 gals. at 65%.

**818. To find the number of pounds of each ingredient when the total amount is limited.**

EXAMPLE.—A grocer wishes to fill an order for 680 pounds of tea which shall cost him 55 cents a pound. To produce this grade of tea, he blends the following grades: 40 cent, 50 cent, 75 cent, 90 cent and $1.00. How many pounds of each does he take?

| | | g. and l. on 1 lb. of 1 and 5 | g. and l. on 1 lb. of 2 and 4 | g. and l. on 1 lb. of 2 and 3 | Amt. of 1 and 5 g. and l. equal | Amt. of 2 and 4 g. and l. equal | Amt. of 2 and 3 g. and l equal | Relative Amt. of each | | Average of lbs. | Proof |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 55 | 40 | 15c. | | | 3 lbs. | | | 3 | 40 × = | 120 | @ 40c. = 4800 |
| | 50 | | 5 | 5 | | 7 lbs. | 4 lbs. | 11 | | 440 | @ 50c. = 22000 |
| | 75 | | | 20 | | | 1 lb. | 1 | | 40 | @ 75c. = 3000 |
| | 90 | | 35 | | | 1 lb. | | 1 | | 40 | @ 90c. = 3600 |
| | 1.00 | 45c. | | | 1 lb. | | | 1 | | 40 | @ 1.00 = 4000 |
| | | | | | | | | 17 × 40 | = | | 680 ) 37400 |
| | | | | | | | | | | | 55 |

EXPLANATION.—Proceed as in Art. 816. Divide the number of pounds required by the total number obtained by comparison, and multiply each quantity by this number.

## WRITTEN PROBLEMS

**819.** *1.* What is the average price of the following mixture of tea: 95 pounds at 38 cents, 150 pounds at 45 cents, 68 pounds at 95 cents and 75 pounds at $1.00?

*2.* In a certain school there are 150 children 6 year of age; 105 children 7 years of age; 96 children 9 years of age; 168 children 11 years of age, and 214 children 12 years of age. Find the average age of the children of the school.

9 The average price of a certain blend of tea, consisting of teas worth 50, 60, 65, 75, 80, 85, 90, 95 cents and $1, is 80 cents. Find the number of pounds of each kind required to produce this grade of tea.

10 A broker bought 221 shares of stock par $50 at an average premium of 10%. He bought some at 6% discount, some at 2% discount, some at a premium of 10%, and some at a premium of 20%. How many shares of each kind did he buy?

11 A merchant mixed 95 pounds of coffee worth 35 cents with coffees worth 40, 42 and 48 cents a pound. How many pounds of each did he take, if the average price was 45 cents a pound?

12 What relative quantities of alcohol 62%, 68%, 80%, 92%, 96%, and 98% proof are required to form alcohol 85% proof?

13 A grocer has an order for 800 pounds of tea, the price to be 80 cents a pound. To fill the order he mixes the following kinds: 100 pounds at 42 cents, some at 50 cents, some at 60 cents and some at $1. How many pounds of each kind does he take?

14 I want to fill an order for 1000 pounds of coffee which I have agreed to sell at 60 cents a pound, a gain of 10%. I use 200 pounds at 42 cents, some at 55 cents, some at 70 cents and some at 80 cents. Find the number of pounds of each kind.

15 I want to mix wine costing $2.50 a gallon with 60 gallons of wine costing $3.00 a gallon and with water, so that I may be able to sell the mixture for $2.40 a gallon and gain 20%. How many gallons of water and how many gallons of the wine at $2.50 must I take?

16 A feed merchant mixes bran worth 85 cents a hundred, middlings worth $1.05 a hundred, cracked corn worth 92 cents a hundred, with oats worth 62 cents a bushel of 32 pounds. How many pounds of each of these ingredients shall he take to fill an order for 3 tons, 2000 pounds each, at an average price of $30 a ton?

17 I want to mix coffees worth, respectively, 28, 32, 35, 38 and 40 cents a pound so as to be able to sell for 40 cents a pound and gain 10%. How many pounds of each kind must I take?

18 I have on hand teas worth 65, 70, 75, 80 cents and $1 a pound. I have an order for 2000 pounds at 72 cents a pound. I use 200 pounds of the $1 tea. How many pounds of the remaining kinds shall I take?

19 A man has $1800 in 10-cent pieces which he wishes to exchange for 5-cent, 3-cent, 2-cent and 1-cent pieces. How many of each kind will he take?

20 I want to mix alcohol 90% proof, 75% proof and 98% proof with water so as to get alcohol 70% proof. Find the number of gallons of each kind I must take, if in all I want 1000 gallons.

21 I have teas worth 48, 54, 72, 80 and 85 cents a pound. I want to mix 1200 pounds of tea which I can sell at 80 cents a pound and gain 20%. By taking 240 pounds of the 85-cent tea, how many pounds of the remaining kinds must I take?

## GENERAL AVERAGE

**820. Average** is of two kinds—*general average* and *particular average*.

**821. General average** arises when sacrifices have been voluntarily made, or expenditures incurred, for the preservation of a ship, cargo and freight, from some peril of the sea or from its effects. It implies a subsequent contribution from all the parties concerned, ratable to the values of their respective interests, to make good the loss thus occasioned.

**822. Particular average** signifies the damage or partial loss happening to the ship, goods or freight, by some fortuitous or unavoidable accident. It is shared by the persons whose property is destroyed or by their insurers.

**823.** The fundamental **principle** of general average is that a loss incurred *for the advantage of all the coadventurers* should be made good by all in equitable proportion to their stakes in the venture.

**824.** All general average losses may be divided into two principal classes—(1) sacrifices of part of the cargo and freight (as when part of a cargo is thrown overboard to save the ship from foundering in a storm), or of part of the ship (masts, etc.) for the general safety; (2) extraordinary expenditures incurred with the same object (as when a ship is obliged to put into a port of refuge in consequence of damage received in the course of the voyage).

**825. Jettison** is the throwing overboard of goods or cargo, in stress of weather or to prevent foundering.

**826. Salvage** is the compensation allowed to persons by whose voluntary exertions a vessel, her cargo, or the lives of those belonging to her are saved from danger or loss in case of wreck, capture, or other marine misadventure.

**827.** A **general average loss** may include the following items:

(*a*) Jettison: damage to cargo by water getting down the hatches during jettison; damage by breaking or chafing after jettison; freight on cargo jettisoned.

(*b*) Sacrifices of ship's materials, by the cutting away of masts, spars, rigging, etc. One-third of the cost of repairs to a vessel is a special charge on the ship, the new work being considered better than the old; this leaves the remaining two-thirds only of such cost to be included in the general average.

(*c*) Expense of floating a stranded ship; salvage in general.

(*d*) Expense of entering a port of refuge, whether disability were caused by accident or sacrifice.

(*e*) Expense of discharging cargo to make repairs, reloading, etc.

(*f*) Wages and provisions of crew from time vessel deviates from its course until it resumes its voyage.

**828. The contributory interests** are as follows:

a. The *ship* contributes on what was its full value before the loss.

b. The *cargo* contributes on its net market value at port of destination, including value of part jettisoned or damaged.

c. The *freight* contributes on its full amount, less ½ for the wages, etc., of the crew. (In New York, and some other States, less ⅓.)

**829. Insurers** indemnify the owners of contributory interests for such proportion of the contribution of each as the amount insured on such contributory interest bears to the full value of said contributory interest.

**830. An adjuster** is a person who apportions the losses and expenses of a general average.

**831. To apportion a loss by general average.**

EXAMPLE. The schooner *Swallow* from Savannah for Philadelphia, with a cargo of cotton valued at $37500 and a deck load of pine lumber valued at $18760, encountered heavy gales off Cape Hatteras which made it necessary to throw overboard the deck load and cut away masts and rigging, after which the vessel was rescued by a steamer and towed into Wilmington, N. C. The salvage amounted to $1500; repairs to the vessel, $2871, wages and provisions of steamer from date of the disaster, $874.60, the other expenses, $1486.38. The value of the vessel was $12000, the total freight, $6439, of which $2132 was the freight on the deck load. Apportion loss and show settlement.

| GENERAL AVERAGE LOSS | | | CONTRIBUTORY INTEREST | |
|---|---|---|---|---|
| Jettison | 18760 | | Vessel | 12000 |
| Freight on jettison | 2132 | | Cargo of cotton | 37500 |
| Repairs ½ of $2871 | 1916 | | Cargo of lumber | 18760 |
| Salvage | 1500 | | Freight ($6439 less ½) | 4727 |
| Wages, etc., of steamer | 874 | 60 | | |
| Other expenses | 1486 | 38 | | |
| Total | 66469 | 98 | Total | 73987 |

$73987 : $66469.98 :: $12000 : Vessel's contribution = $10786.31
$73987 : $66469.98 :: $37500 : Cotton's contribution = 33683.05
$73987 : $66469.98 :: $18760 : Lumber's contribution = 16841.51
$73987 : $66469.98 :: $4727 : Freight's contribution = 4248.11
$66469.98

| RECEIVED | | | OWNER'S SETTLEMENT | | |
|---|---|---|---|---|---|
| Owners of cotton | 33683 | 05 | Owners of lumber | 1918 | 49 |
| Freight contributory | 214 | 60 | Repairs | 2871 | |
| Owners of vessel | 7571 | 31 | Salvage | 1500 | |
| | | | Wages, etc., of steamer | 874 | 60 |
| | | | Other expenses | 1486 | 38 |
| | 39469 | 96 | | 39469 | 96 |

EXPLANATION OF ADJUSTER'S SETTLEMENT.—The adjuster settles with the owners of the cotton by receiving from them their contribution to the general average, $20841.05. He settles with the freight company (the charterers of the vessel) by receiving from them the difference between their contribution and the freight on jettison, $2366.40 − $2132 = $234.40. He settles with the owners of the vessel by receiving from them their contribution plus their ⅓ of the repairs, $6616.20 + $957 = $7573.20. He settles with the owners of the lumber by paying them the value of the lumber jettisoned, less their contribution, $18760 − $10343.33 = $8416.67. He pays the full amount for repairs, $2871; for salvage, $15000; and other expenses, $1486.38. He pays the general average portion of seamen's wages, etc., the charterers paying their regular wages. Of course, the charterers receive their freight charges from the owners of the cotton, and they pay the owners of the vessel for the charter of it, entirely aside from the general average settlement.

**832.** RULE.—*Find the total of the contributory interests and the total loss subject to general average.*

*Calculate the proportionate contribution of each contributory interest by the following formula:*

*Total interest : Total loss : : Each owner's interest : Each owner's contribution.*

## WRITTEN PROBLEMS

**833.** *1.* A steamer having on board $47923 worth of goods shipped by A, $25437 worth shipped by B, and $11926 worth shipped by C, experienced such rough weather that $20624 worth of goods was jettisoned for the safety of the ship. Of the goods jettisoned, $8370 worth was from goods owned by A, and the remainder from B's shipment. The vessel having been damaged, the repairs amounted to $2145.38, and the maintenance of the crew during delay was $537.80. The loss of freight on cargo jettisoned amounted to $2085.50; the freight paid on the remainder was $6249.80. The steamer, before starting on the voyage, was valued at $57500. Apportion the loss among the different contributory interests, and state how settlement should be made.

*2.* The bark *Bonnie Jean* started from Glasgow to New York with an assorted cargo. During the voyage, she encountered a severe storm, which drove her out of her course and caused considerable damage. For the safety of the ship her masts and rigging had to be cast away; also a portion of her cargo, valued at $2375. By rigging up jury masts, she was enabled to reach the port of St. John, N. B., for repairs. Replacing the masts and rigging cost $5438; repairing other damage, $675. The freight on cargo jettisoned amounted to $435.25. Port charges for entering, with expense of discharging cargo, etc., amounted to $835.15; wages and provisions for crew, $715; adjuster's fee, $175. The value of the vessel on arriving at New York was $28000; value of cargo delivered, $50860. The total freight (including freight on cargo jettisoned) was $17150. The consignors of the cargo were: James Thompson, $9500; William Irvine, $21275; Alexander McPherson, $7325; Joseph Montgomery, $15135. The jettisoned goods were part of Irvine's consignment. How ought the settlement to be made?

NOTE.—The value of the vessel on arriving at New York being $28000, to find the value before it was damaged (which will be the value upon which its contribution is computed) deduct one-third of the charges for repairs, this being reckoned as the superior value of the new material. That is to say, the vessel, after being repaired, is worth so much more than [illegible] was sustained.

3. James Thompson's consignment was insured for $8000, and Joseph Montgomery's consignment for $12000. How much did each receive from the underwriters?

4. During a storm off the coast of Maine, the schooner *Sarah Jane*, valued at $25000, having a cargo consigned to Frazer & Co., Baltimore, had to jettison $5000 worth of the cargo for the general safety. The cargo amounted to $50500, part of which was consigned by Jones & Brown and invoiced at $15750, of which $3000 was jettisoned. The remainder of the jettison, $2000, was taken from goods consigned by Bradley & Co. By the shifting of the cargo, and the entering of water through the hatchway while throwing the goods overboard, the remainder of Jones & Brown's consignment was damaged to the extent of $1\frac{1}{4}\%$. Freight expected on cargo was $12500, including that on jettison, $315. The adjuster's fee was $175. How much does Jones & Brown and Bradley & Co. each contribute in the general average?

5. If Bradley & Co. carried $30000 insurance on their consignment, how much do they receive from the insurance company?

6. The brig *Arcotta*—owned by Jacob Orr, Wm. Coates and Henry Fox, general traders, whose respective interests in the vessel were: Orr $\frac{1}{2}$, Coates $\frac{3}{8}$, and Fox $\frac{1}{8}$—delivered cargoes at several of the principal seaports of the Mediterranean, shipped a cargo of oranges and lemons at Messina for the return voyage, each of the owners putting in his own venture. The value of the cargo was $43575, which was as follows: Jacob Orr $14525, William Coates $7262.50, Henry Fox $9078.13, and a consignment by Wessenberg & Co. to Penn Fruit Co., Philadelphia, $12709.37. On the homeward voyage the brig was struck by a violent storm, during which the vessel was damaged to a considerable extent. Some of the cargo had to be thrown overboard, and having sprung a leak, the vessel had to put into Bordeaux for repairs. Expenses for repairing loss and damages, port charges for entering, wages and provisions for crew while detained, unloading, reloading, etc., amounted to 7532.75 francs, which was paid by the owners, Wessenberg's share to be collected from the Penn Fruit Co. Of the total expenses, 1236.20 francs was the amount paid for repairing damage to vessel caused by the storm.* The goods jettisoned were valued at $8500—$4500 being from J. Orr's consignment and $4000 from Wessenberg's consignment. Freight calculated at $8400, including $345 freight on jettison. Value of brig in Philadelphia, $25000. Adjuster's fee, $225. How was settlement made?

7. The cargo of the brig *Arcotta* was insured for $38000. How much did the insurance company pay, and how apportioned?

* This item is, of course, a *particular average* loss to be shared by the owners.

# EQUATION OF ACCOUNTS

**834. Equating an account** is finding a date on which settlement of two or more debts, due at different dates, can be made without loss to the debtor or creditor.

**835.** The date thus found is called the **average date** or **equated time.**

**836.** It is necessary to assume some common date of comparison. This date is called the **focal date.**

REMARK.—Any date conceivable may be taken as a focal date, and interest may be computed at any rate per cent. without varying the result, provided only that the dates of all items be compared with such focal date, and uniformity in rate and manner of computing interest be observed throughout.

In practise it is well to observe a simple method, by assuming the latest date in the account as a focal date, computing all interest at 6% by the short method on a 360-day basis.

**837.** The **term of credit** is the time allowed for the payment of a debt after the date of contracting it; if given in days, it is counted on from the date of purchase or sale, the *exact* number of days of the term; if given in months, it is counted on from date of purchase or sale the number of months regardless of the number of days thus included.

**838.** The **average term of credit** is the average time allowed for the payment, in one sum, of the total of two or more debts, due at different dates.

REMARK.—Book accounts bear legal interest after they become due, and notes, even if not containing an *interest clause*, bear interest after maturity.

**839.** The importance of a thorough knowledge of both the **theory** and **practise** of Equation of Accounts, on the part of bookkeepers and accountants, can hardly be overrated, as much of this class of work is to be found in every wholesale and commission business.

**840.** The **equity** of the **settlement** of an account by equation rests in the fact that, by a review of such account, one of the parties owes the other a balance to which certain interest should be added or from which certain interest (discount) should be subtracted.

**841.** Accounts having entries on but one side, either debit or credit, are appropriately called simple accounts; accounts having both debit and credit items may appropriately be called compound accounts.

**842.** It is the custom in many lines of business to charge **interest** on items that have matured. Brokers also charge customers interest for stock carried or allow it on deposits made. Private and state banks frequently allow interest on daily balances on deposit. These various business customs lead to interest calculations of special kinds, with which the business man should be familiar. There are several methods in use.

**843. To find the average date of sales.**

EXAMPLE.—In the following account find the average date of sales: on June 8, 1907, John Bowen sold to William Lorigan mdse. to the amount of $960; on June 16, to the amount of $1240; and on June 30, to the amount of $497.

| 1907 | | | Items | Days | Products |
|---|---|---|---|---|---|
| June | 8 | Mdse. | 960 00 | 22 | 21120 |
| June | 16 | " | 1240 00 | 14 | 17360 |
| June | 30 | | 497 00 | 0 | 00000 |
| | | | 2697 00 | | )38480(14 days |
| | | | | | 2697 |
| | | | | | 11510 |
| | | | | | 10788 |
| | | | | | 722 |

June 30
14
16 Average date of sales, and in this case average due date of total indebtedness.

EXPLANATION.—We assume the latest date of sale, June 30, as the time of settlement. If the first item is paid on that date, it bears interest for a period of 22 days, or $1 has an interest period of 21120 days. The second item, purchased June 16, bears interest for a period of 14 days, or $1 has an interest period of 17360 days. If all items are paid June 30, $1 has an interest period of 38480 days, or $2697 for 14 days. Since Mr. Lorigan, by paying June 30, owes the interest on $2697 for 14 days, it is evident that the day on which he owes no interest must be 14 days earlier than June 30, or June 16.

**844.** RULE.—*Assume as a focal date the latest date of sale in the account. Multiply each item in the account by the number of days between the date of sale and the focal date. The sum of these products divided by the sum of the items will give the average credit before the focal date.*

NOTE.—A fraction of a day, if less than one-half, is discarded; if one-half or more, it is called a day. If an item contains cents, they may be discarded if less than one-half dollar. One-half dollar or more may be called one dollar.

**845.** Find the average date of sale of

| 1914 | | 1914 | | 1914 | |
|---|---|---|---|---|---|
| 1. May 26 | Mdse. $250 | 2. Jan. 21 | Mdse. $450 | 3. Sept. 9 | Mdse. $1125 |
| June 18 | Mdse. 300 | Mar. 26 | Mdse. 500 | Oct. 29 | Mdse. 970 |
| Aug. 6 | Mdse. 750 | Apr. 15 | Mdse. 625 | Dec. 14 | Mdse. 1350 |

**846. To find the equated date or average due date.**

EXAMPLE.—In the following statement find the equated date (1) by the product method; (2) by the interest method.

| 1914 | | | | |
|---|---|---|---|---|
| Mar. | 7 | Mdse. 30 da. | 200 | 40 |
| May | 9 | " 2 mos. | 375 | 00 |
| Aug. | 17 | " 60 da. | 400 | 60 |

PRODUCT METHOD

| 1914 | | | DUE | ITEMS | | DS. | PRODUCTS |
|---|---|---|---|---|---|---|---|
| Mar. | 7 | Mdse. 30 da. | April 6 | 200 | 40 | 193 | 38600 |
| May | 9 | " 2 mos. | July 9 | 375 | 00 | 99 | 37125 |
| Aug. | 17 | " 60 da. | Oct. 16 | 400 | 60 | 0 | 00000 |
| | | | | 976 | | | 976 ) 75725 ( 77.58 = 78 da. |

Oct. 16—78 days
Oct. 16—30 days = Sept. 16
Sept. 16—30 days = Aug. 17
Aug. 17—18 days = July 30 equated date
78 days

Or,

78
16 Oct. 32
62 31 Aug.
30 Sept. 1 31 July
32 1
30 "

EXPLANATION.—First find the due date of each item. Assume October 16, the latest due date, as the date of settlement for all the items. If the first item is paid on that date, it bears interest for a period of 193 days; or $1 has an interest period of 38600 days. The second item, due July 9, has an interest period of 99 days; or $1 has an interest period of 37125 days. If all items are paid October 16, $1 has an interest period of 75725 days, or $976 for 78 days. Since the purchaser owes the interest on $976 for 78 days by paying October 16, not to owe this interest he must pay 78 days before October 16, or July 30.

**847.** RULE.—*Assume the latest due date of the account as a focal date. Multiply each item in the account by the number of days between the due date of the item and this focal date. The sum of the products divided by the sum of the items gives the average credit (in days) before the focal date.*

INTEREST METHOD

| 1914 | | | DUE | DS. | ITEMS | | INTEREST | |
|---|---|---|---|---|---|---|---|---|
| Mar. | 7 | Mdse. 30 da. | April 6 | 193 | 200 | 40 | 6 | 4462 |
| May | 9 | " 2 mos. | July 9 | 99 | 375 | 00 | 6 | 1875 |
| Aug. | 17 | " 60 da. | Oct. 16 | 0 | 400 | 60 | 0 | 0000 |
| | | | | | 976 | 00 | )12 | 6337 ( 77.58 = 78 da. |

.16266 ) 12.6337 ( 77.58

$.000\frac{1}{6}$
.16266

EXPLANATION.—Assume as a date of settlement the latest due date, October 16. If he did not pay $200.40 (due April 6) until October 16, he owes the interest on that amount for 193 days, or $6.4462. In like manner $375 (due July 9) was paid October 16, or 99 days later; therefore, the interest due is $6.1875. All the interest (due October 16) amounts to $12.6337. Not to owe this interest October 16, he should have paid *as many days earlier* as the interest on the total for one day ($0.16266) is contained times into $12.6337, or 78. 78 days earlier than October 16, 1914, equals July 30.

**848.** RULE.—*Find interest for regular time on each item and divide the total interest by interest on total amount for one day.*

NOTE.—To verify, use the earliest due date, and explain as discounts.

## WRITTEN PROBLEMS

**849.** When are the following accounts due by equation?

*1.* JOHN HARDEGG

*To* WILLIAM PRICE, *Dr.*

| 1915 | | | | |
|---|---|---|---|---|
| March | 2 | To Mdse. | 375 | 00 |
| " | 18 | " " | 1565 | 00 |
| " | 31 | " " | 742 | 00 |

*2.* JAMES MARTIN

*To* PHILIP SHELLENING, *Dr.*

| 1915 | | | | |
|---|---|---|---|---|
| Feb. | 6 | To Mdse. | 496 | 72 |
| " | 23 | " " | 962 | 85 |
| Mar. | 10 | " " | 1742 | 10 |

*3.* HENRY HALL

*To* JAMES R. MICKLEY, *Dr.*

| 1915 | | | | |
|---|---|---|---|---|
| June | 8 | Mdse., 60 da. | 492 | 67 |
| " | 16 | " 10 " | 1275 | 43 |
| " | 30 | " 15 " | 674 | 92 |

*4.* PACKER H. GOODHALL

*To* MCFETRIDGE & HALL, *Dr.*

| 1915 | | | | |
|---|---|---|---|---|
| July | 16 | Mdse., 2 mo. | 1129 | 63 |
| " | 28 | " 1 " | 976 | 42 |
| Aug. | 4 | " 3 " | 497 | 12 |
| " | 18 | " 60 da. | 794 | 13 |

*5.* First find equated date, then find amount due July 6, 1916.

GOODYEAR & CO.

*To* WILLIAM H. HEANEY, *Dr.*

| 1916 | | | | |
|---|---|---|---|---|
| April | 8 | Mdse., 120 da. | 796 | 45 |
| " | 13 | " 3 mo. | 1726 | 50 |
| " | 28 | " 60 da. | 475 | 22 |
| May | 12 | " 30 " | 762 | 95 |
| June | 6 | " 1 mo. | 742 | 94 |

**850. To find the equated date for the payment of an account.**

EXAMPLE 1.—Find the time when the balance is due, or "equate" the account by the product method.

DR. JOHN K. YARDHAM CR.

| 1915 | | | | 1915 | | | |
|---|---|---|---|---|---|---|---|
| Feb. | 15 | To Mdse. | 600 | Mar. | 10 | By Cash | 350 |
| May | 25 | " " | 400 | June | 12 | " " | 300 |

DR. PRODUCT METHOD CR.

| 1915 | | DS. | ITEMS | PRO. | 1915 | | DS. | ITEMS | PRO. |
|---|---|---|---|---|---|---|---|---|---|
| Feb. 15 | To Mdse. | 117 | 600 00 | 70200 | Mar. 10 | By Cash | 94 | 350 00 | 32900 |
| May 25 | " " | 18 | 400 00 | 7200 | June 12 | " " | 0 | 300 00 | 00000 |
| | | | 1000 | 77400 | | | | 650 | 32900 |
| | | | 650 | 32900 | | | | | |
| | | | 350) | 44500(127 | | | | | |

127 days before June 12 or Feb. 5 = equated date

June 12—30 days = May 13
May 13—30 days = Apr. 13
Apr. 13—30 days = Mar. 14
Mar. 14—30 days = Feb. 12
Feb. 12— 7 days = Feb. 5 equated date

EXPLANATION.—In the above illustration we take the latest date of the account, June 12, as the assumed date of settlement.

Suppose the first item ($600) to have been paid on this assumed date, it would have been paid 117 days after it was due, and Yardham would owe the interest on $600 for 117 days, which is equal to an interest period on $1 for 70200 days. Had the second item ($400) been paid on this assumed date, he would owe interest on $400 for 18 days, which is equal to an interest period on $1 for 7200 days. Yardham, therefore, on June 12, owes $1000 and the interest on $1 for a period of 77400 days.

On the credit side of the account, by using the same date of comparison, since he paid $350 on March 10, or 94 days before the date of settlement, he is entitled to a discount on $350 for 94 days, or on $1 for a period of 32900 days. The second item was paid June 12, and is not entitled to a discount.

Subtracting the number of days for which he is to be allowed the discount on $1 from the number of days for which he owes the interest on $1 (viz., 77400 − 32900 = 44500, the number of days for which, on June 12), he owes the interest on $1. If he owes the interest on $1 for 44500 days, he owes the interest on $350, the balance of the account, for as many days as $350 is contained times in 44500, or 127. If, on June 12, 1915, he owes the interest on the balance for 127 days, this balance was due 127 days before June 12, or February 5.

**851.** RULE.—*Assume the latest due date in the account as a focal date. Multiply each item on both sides of the account by the number of days between the due date of the item and this focal date. Divide the difference between the sums of the products by the difference between the sums of money. If these differences are on the same side of the account, count the number of days obtained by the division backward from the focal date; if on opposite sides, count forward from the focal date.*

NOTE.—To verify the above result, select a different date as a focal date.

EXAMPLE 2.—Find the equated date by the interest method.

DR. J. H. BUTLER CR.

| 1915 | | | | | 1915 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| May | 15 | To Mdse., 10 da. | 285 | 00 | June | 12 | By Cash | 200 | 00 |
| June | 18 | " " 30 " | 340 | 00 | July | 1 | Note, 60 da. | 300 | 00 |

DR. INTEREST METHOD CR.

| 1915 | | DUE | D. | INT. | ITEMS | 1915 | | DUE | D. | INT. | ITEMS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| May 15 | Mdse., 10 da. | May 25 | 97 | 4 6075 | 285 00 | June 12 | Cash | June 12 | 79 | 2 6333 | 200 00 |
| June 18 | " 30 " | July 18 | 43 | 2 4366 | 340 00 | July 1 | Note, 60 da. | Aug. 30 | 0 | 0 0000 | 300 00 |
| | | | | 7.0441 | 625.00 | | | | | 2.6333 | 500.00 |

$7.0441 Dr. int. $625
2.6333 Cr. " 500
$4.4108 ½) 125
.0208 int. for 1 da.

4.4108 ÷ .0208 = 212 or 212 days

212 days before Aug. 30 = Jan. 30

| | | |
|---|---|---|
| Aug. | 30—30 days | = July 31 |
| July | 31—30 days | = July 1 |
| July | 1—30 days | = June 1 |
| June | 1—30 days | = May 2 |
| May | 2—30 days | = Apr. 2 |
| Apr. | 2—30 days | = Mar. 3 |
| Mar. | 3—30 days | = Feb. 1 |
| Feb. | 1— 2 days | = Jan. 30 |
| | 212 days | |

**852.** RULE.—*Assume the latest due date in the account as a focal date. Find the interest (to four places) on each item of the debit side, supposing each to be paid on this assumed date. Find the interest on each item of the credit side (to four places), supposing each to be due on this assumed date. Find the difference between the sum of the debit interests and the sum of the credit interests. Divide this difference by the interest on the balance of the account for 1 day. The quotient expresses the number of days between the assumed date and the date on which the balance of the account is due. If the difference between the sums of money and the difference between the interests are on the same side, count backward; if on opposite sides, count forward from the focal date.*

REMARK.—It is more practical to use the date of the latest transaction than to assume a date. The number of days is less and there is one less item to calculate. A great advantage in using the interest method is that it gives excellent drill in calculating interest; and by the use of the 60-day method it is as quickly worked as by the product method. It has the advantage also of being more easily explained and more readily understood. It is also noted that many business houses use the interest method exclusively, in rendering accounts current, especially in foreign trade. The form of account current used shows the items of days and interest in detail, having separate columns for each.

## WRITTEN PROBLEMS

**853.** *1.* Find when the balance of this account is due by equation.

| *Dr.* | | | GEORGE SCARLET | | | | | *Cr.* |
|---|---|---|---|---|---|---|---|---|
| 1914 | | | | | 1914 | | | |
| Sept. | 12 | Mdse. | 450 00 | | Oct. | 1 | Cash | 450 00 |
| Nov. | 10 | " | 525 00 | | Dec. | 1 | " | 500 00 |

*2.* When is the balance of this account due by equation?

| *Dr.* | | | I. MERCHANT | | | | | *Cr.* |
|---|---|---|---|---|---|---|---|---|
| 1915 | | | | | 1915 | | | |
| Oct. | 1 | Cash | 450 00 | | Sept. | 13 | Mdse. | 450 00 |
| Dec. | 1 | " | 500 00 | | Nov. | 10 | " | 525 00 |

*3.* Find the equable date of settlement of the balance of this account.

| *Dr.* | | | DAVID DEAL & BRO. | | | | | *Cr.* |
|---|---|---|---|---|---|---|---|---|
| 1916 | | | | | 1916 | | | |
| Apr. | 1 | Cash | 1400 00 | | Jan. | 3 | Mdse. | 726 00 |
| June | 1 | " | 1200 00 | | Feb. | 2 | " | 574 00 |
| | | | | | Mar. | 4 | " | 267 00 |
| | | | | | Apr. | 5 | " | 472 00 |

*4.* Find when due by equation.

| *Dr.* | | | | BEYER BROTHERS | | | | | *Cr.* |
|---|---|---|---|---|---|---|---|---|---|
| 1916 | | | | | 1916 | | | |
| Jan. | 3 | Mdse., | 30 da. | 679 25 | Feb. | 3 | Cash | 600 00 |
| Feb. | 18 | " | 30 da. | 794 20 | Mar. | 18 | " | 750 00 |

*5.* When is the balance of this account due by equation?

| *Dr.* | | | | FRANK S. MISLEY | | | | | *Cr.* |
|---|---|---|---|---|---|---|---|---|---|
| 1915 | | | | | 1915 | | | |
| Mar. | 3 | Mdse., | 30 da. | 796 14 | Apr. | 1 | Note, 60 da. | 750 00 |
| " | 17 | " | 2 mo. | 1722 11 | " | 28 | " 30 " | 850 00 |
| " | 31 | " | 60 da. | 1946 75 | May | 1 | Acceptance, 2 mo. | 1 |
| May | 19 | " | 30 da. | 472 11 | June | 26 | Cash | |

date when the balance is due by equation.

FRANK B. HUGHES Cr.

| | | 1915 | | | |
|---|---|---|---|---|---|
| Balance | 911 72 | Mar. | 27 | Cash | 900 00 |
| Mdse., 60 days | 375 95 | Apr. | 22 | Note, 10 days | 370 00 |
| " 2 mos. | 702 10 | May | 10 | Draft, 30 days | 600 00 |
| " 1 mo. | 387 98 | | | | |

find the equated date for payment of the balance of this account.

HAROLD S. WARNER Cr.

| | | 1913 | | | |
|---|---|---|---|---|---|
| Mdse., 60 days | 2314 75 | June | 9 | Cash | 2000 00 |
| " 30 days | 6218 90 | July | 15 | " | 5000 00 |
| " net | 572 20 | Aug. | 27 | Note, 30 days | 2000 00 |
| " 30 days | 376 19 | Nov. | 12 | Cash | 250 00 |
| " net | 1042 18 | | | | |

When should the balance of this account in equity be paid?

FRANCIS E. HOTT Cr.

| | | 1916 | | | |
|---|---|---|---|---|---|
| Mdse., 30 days | 800 00 | May | 20 | Cash | 150 00 |
| " 60 days | 463 29 | July | 1 | Note, 1 mo. | 412 50 |
| " net | 377 89 | Sept. | 2 | Cash | 200 00 |

## CASH BALANCE

**The Cash Balance** of an account current is the amount due at a specified

**An Account Current** is a written statement of the gross debits and credits from business transactions, within a certain period, between two *houses*. be rendered monthly, quarterly, semi-annually or annually. Interest is though not invariably, charged; that being regulated by a previous under- or by the custom of each particular business. Since the interest that is so earns interest, the oftener an account is adjusted and interest added, the he balance or amount due. Some merchants, however, who render accounts ftener than once a year, do not carry the interest balance to the main column ccount until the end of the year.

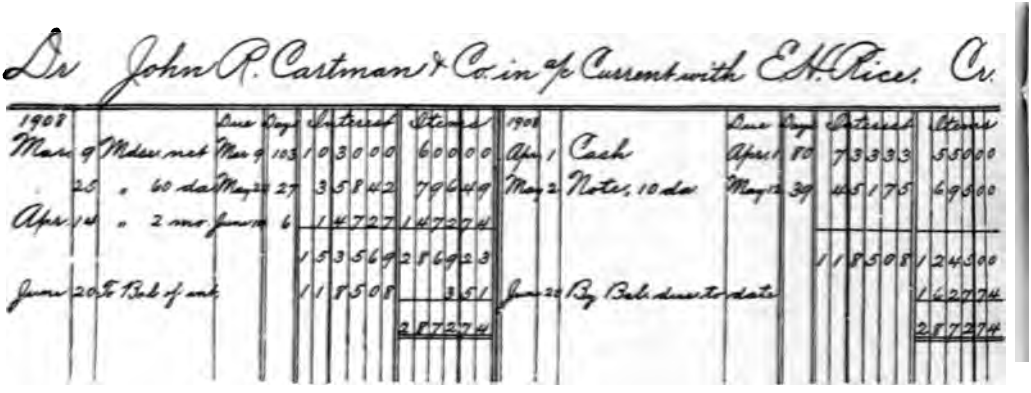
**A Cash Balance** may be ascertained in either of two ways: (1) by the **method; (2) by the** equation method.

**857. To find the cash balance of an account.**

EXAMPLE 1.—Find cash balance June 20, 1908.

*Dr.* JOHN R. CARTMAN & CO., IN % CURRENT WITH E. H. RICE *Cr.*

| 1908 | | | | 1908 | | | |
|---|---|---|---|---|---|---|---|
| Mar. 9 | Mdse., net | 600 00 | Apr. 1 | Cash | 550 00 |
| " 25 | " 60 da. | 796 49 | May 2 | Note, 10 da. | 695 00 |
| Apr. 14 | " 2 mo. | 1472 74 | | | |



EXPLANATION.—In this operation, after maturing the debit and credit items, we first find the number of days that each debit and each credit item has been due prior to June 20, 1908, and compute the interest on the same for this time at 6%. We then take the difference between the interests of the debit and the credit items; and, as the excess of balance is against Cartman & Co., we charge it to them. To find the cash balance, take the difference between the debit and the credit amount of the account. In business practise it is the custom of bookkeepers to place the interest of items which fall due after the date of taking the cash balance in the opposite interest column; such entries being known as *contra* entries.

**858.** RULE.—(*1*) *Find the interest on each item on the debit side from its due date to the date of settlement; then find the interest on each item on the credit side from its date to the date of settlement. Add the sums of these interest items to their respective sides, and the balance will be the amount due on the date of settlement.*

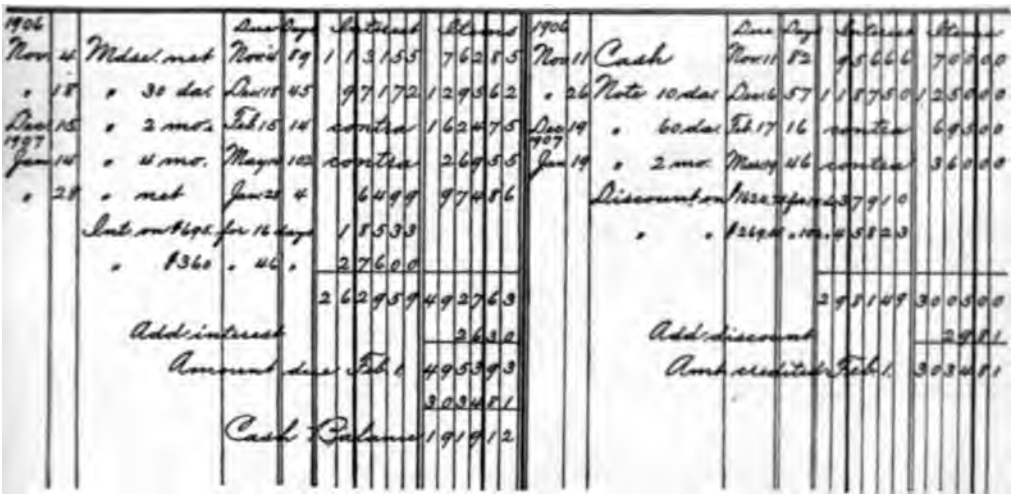
(*2*) *Find the interest on the balance of the account from the equated date to the date of settlement; if the date of settlement is later than the equated date, add this interest to the balance; if before, subtract it.*

NOTE.—The first method is exact; the second, sufficiently accurate for many practical purposes.

EXAMPLE 2.—Find cash balance February 1, 1907.

| *Dr.* | | | | BERNHARD F. LEWIS | | | | | *Cr.* |
|---|---|---|---|---|---|---|---|---|---|
| 1906 | | | | | 1906 | | | | |
| Nov. | 4 | Mdse., net | | 762 85 | Nov. | 11 | Cash | | 700 00 |
| " | 18 | " | 30 da. | 1295 62 | " | 26 | Note, 10 da. | | 1250 00 |
| Dec. | 15 | " | 2 mo. | 1624 75 | Dec. | 19 | " | 60 da. | 695 00 |
| 1907 | | | | | 1907 | | | | |
| Jan. | 14 | " | 4 mo. | 269 55 | Jan. | 19 | " | 2 mo. | 360 00 |
| " | 28 | " | net | 974 86 | | | | | |

*Dr. Bernard F. Lewis in ℔ Current with Field & Co. Cr.*



**859. To find cash balance for account current.**

EXAMPLE 1.—Vim Brothers *in account current with* Enterprise Mill Co., July 1, 1921.

| *Dr.* | | | | *Cr.* | | | | |
|---|---|---|---|---|---|---|---|---|
| 1921 | | | | | 1921 | | | |
| Jan. | 5 | Mdse. 30 days (as per bill rendered) | 470 00 | | Mar. | 3 | Cash | 600 |
| Mar. | 1 | " " " " | 650 00 | | May | 7 | Note 2 months | 1500 |
| May | 2 | " " " " | 580 00 | | | | | |
| June | 25 | " " " " | 750 00 | | | | | |

PRODUCT METHOD

Vim Brothers *in account current with* Enterprise Mill Co., July 1, 1921.

| *Dr.* | | | | | | | | *Cr.* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Date | | | Due | Amt. | | Days | Prod. | Date | | | Due | Amt. | Days | Prod. |
| 1921 | | | | | | | | 1921 | | | | | | |
| Jan. | 5 | Mdse. 30 da. (as per bill rendered) | Feb. | 4 | 470 | 147 | 69090 | Mar. | 3 | Cash | Mar. | 3 600 | 120 | 72000 |
| Mar. | 1 | " 30 da. " " | Mar. | 31 | 650 | 92 | 59800 | May | 7 | Note 2 mo. | July | 7 1500 | (Contra) | |
| May | 2 | " 30 da. " " | June | 1 | 580 | 30 | 17400 | | | | | | | |
| June | 25 | " 30 da. " " | July | 25 | 750 | (Contra) | | | | | | | | |
| | | Contra | July | 7 | | | 9000 | | | Contra | July | 25 | | 18000 |
| | | | | | 2450 | 00 | 155290 | | | | | 2100 | 00 | 90000 |
| | | Balance of interest | | | 10 | 88 | | | | *Balance* | | *360* | *88* | |
| | | | | | 2460 | 88 | | | | | | 2460 | 88 | |
| | 1 | To balance | | | 360 | 88 | | | | | | | | |

$\frac{65290}{60}$ = $10.88 bal. of int.

155290 − 90000 = 65290 balance of products

## WRITTEN PROBLEMS

**860.** *1.* When is the following account due by equation?

*Dr.* J. L. LIPPINCOTT *Cr.*

| 1916 | | | | 1916 | | | |
|---|---|---|---|---|---|---|---|
| Jan. 7 | Mdse., net | | 826 54 | Jan. 15 | Cash | | 675 00 |
| " 15 | " | 30 da. | 1692 74 | " 28 | Note, 3 mo. | | 1000 00 |
| Feb. 12 | " | 2 mo. | 1749 89 | Mar. 18 | Draft, 30 da. | | 1200 00 |
| " 25 | " | 90 da. | 1845 29 | Apr. 22 | Note, 4 mo. | | 2000 00 |
| Mar. 8 | " | 1 mo. | 749 86 | | | | |

*2.* In the following, find the equated date, prove it, and then find the cash balance Sept. 15, 1915.

*Dr.* JOHN N. CHAPMAN *Cr.*

| 1915 | | | 1915 | | | |
|---|---|---|---|---|---|---|
| May 6 | Cash | 1000 00 | May 6 | Mdse., net | | 1620 70 |
| " 20 | Note, 2 mo. | 1620 00 | June 12 | " | 60 da. | 922 40 |
| June 15 | Acceptance, 60 da. | 2240 00 | Aug. 19 | " | 2 mo. | 1452 49 |
| July 22 | Cash | 1490 00 | " 27 | " | 4 mo. | 2975 14 |

*3.* Find cash balance June 1, 1916, find also the equated date.

*Dr.* JOHN R. CARTMAN & Co. *Cr.*

| 1916 | | | 1916 | | | |
|---|---|---|---|---|---|---|
| Mar. 9 | Mdse., net | 600 00 | Apr. 1 | Cash | | 550 00 |
| " 25 | " | 60 da. 796 49 | May 2 | Note, 10 da. | | 695 00 |
| Apr. 14 | " | 2 mo. 1472 74 | | | | |

*4.* Find cash balance April 15, 1916.

*Dr.* JAMES J. KING & Co. *Cr.*

| 1915 | | | 1915 | | | |
|---|---|---|---|---|---|---|
| Aug. 12 | Mdse., 30 da. | 796 41 | Sept. 23 | Cash | | 1000 00 |
| Sept. 8 | " | 1 mo. 949 56 | Nov. 17 | Note, 2 mo. | | 1200 00 |
| Nov. 17 | " | 60 da. 1264 75 | Dec. 22 | Acceptance, 90 da. | | 750 00 |
| Dec. 15 | " | 2 mo. 694 86 | 1916 | | | |
| 1916 | | | Mar. 6 | Note, 30 da. | | 1420 00 |
| Feb. 10 | " | 90 da. 1543 75 | | | | |

*5.* When is the following account due by equation, and what amount is due June 1, 1916?

*Dr.* L. T. SPENCER *Cr.*

| 1915 | | | 1915 | | | |
|---|---|---|---|---|---|---|
| Jan. 1 | Balance | 694 87 | Feb. 20 | Cash | | 550 00 |
| " 28 | Mdse., 30 days | 1400 00 | " 27 | Draft, 20 days | | 700 00 |
| Mar. 20 | " | net 510 40 | Mar. 31 | Cash | | 845 50 |
| Apr. 12 | " | 10 days 383 83 | | | | |

*6.* What is the cash balance due Jan. 27, 1915; when due by equation?

*Dr.* JOHN MILLS & Co. *Cr.*

| 1914 | | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|---|
| Sept. | 5 | To Mdse., 60 da. | 758 90 | | Oct. | 5 | By Cash | 1000 00 |
| " | 18 | " " 30 " | 387 40 | | Nov. | 7 | " " | 520 00 |
| Oct. | 7 | " " 4 mo. | 500 00 | | " | 30 | " Draft, 30 da. | 100 00 |
| " | 20 | " " 10 da. | 400 50 | | Dec. | 18 | Cash | 650 75 |
| Nov. | 6 | " " 3 mo. | 515 70 | | 1915 | | | |
| " | 27 | " " net | 400 00 | | Jan. | 15 | By acceptance, 60 da. | 584 00 |
| Dec. | 8 | " " 90 da. | 274 60 | | Feb. | 9 | " Cash | 478 90 |
| " | 25 | " " 2 mo. | 309 75 | | Mar. | 10 | " " | 350 40 |
| 1915 | | | | | | | | |
| Jan. | 14 | " " 1 mo. | 357 80 | | May | 3 | " " | 150 00 |
| Feb. | 5 | " " 30 da. | 200 00 | | | | | |

*7.* When are the proceeds of the following account sales due by equation?

Philadelphia, Pa., Apr. 7, 1915.

Account sales of flour sold for account of J. S. HARLINGTON & Co.

| 1915 | | | | |
|---|---|---|---|---|
| Feb. | 11 | 420 bbls. | at $4.10 | |
| " | 25 | 245 " | " 4.62 | |
| Mar. | 18 | 296 " | " 4.48 | |
| " | 31 | 369 " | " 4.32 | |
| Apr. | 7 | 162 " | " 4.25 | |
| | | *Charges* | | |
| Feb. | 13 | Freight | | 89 45 |
| " | 18 | Cartage | | 16 75 |
| Mar. | 20 | Remittance | | 2000 00 |
| Apr. | 7 | Storage | | 100 00 |
| " | 7 | Cooperage | | 12 00 |
| | | Commission 4% | | |

NOTE.—Date the commission on the last date of sales, and equate the account in the ordinary way, taking the charges as debits and the sales as credits.

8. When are the proceeds of the following account sales due by equation?

1. Account sales of shoes

Sold for account of Wm. B. Munich & Co.

| | | | | |
|---|---|---|---|---|
| 7 | 325 pairs | at $2.10, cash | | |
| 22 | 456 " | " 2.55, 3 mo. | | |
| 19 | 267 " | " 1.75, 60 da. | | |
| 31 | 412 " | " 2.18, cash | | |
| 22 | 564 " | " 2.95, 90 da. | | |
| | *Charges* | | | |
| 7 | Freight | | 102 | 00 |
| 10 | Drayage | | 5 | 60 |
| 26 | Remittance | | 1000 | 00 |
| 24 | Storage | | 110 | 00 |
| | Commission 5% | | | |

2. Account sales of broadcloth

Sold for account of John Detts' Sons

| | | | | |
|---|---|---|---|---|
| 5 | 375 yards | at $3.25, cash | | |
| 20 | 642 " | " 3.92, 60 da. | | |
| 17 | 895 " | " 4.15, 3 mo. | | |
| 30 | 268 " | " 3.68, net | | |
| 15 | 495 " | " 4.10, 30 da. | | |
| 14 | 722 " | " 4.05, 60 da. | | |
| | *Charges* | | | |
| 5 | Inspection | | 9 | 00 |
| 19 | Freight | | 59 | 20 |
| 2 | Cartage | | 10 | 50 |
| 10 | Remittance | | 2500 | 00 |
| 14 | Storage | | 90 | 25 |
| | Commission 3% Guaranty 2% | | | |
| | Net proceeds due | | | |

3. Account sales of apples

Sold for account of Joshua Haley

| | | | | |
|---|---|---|---|---|
| 8 | 228 bbls. | at $2.25, cash | | |
| 22 | 245 " | " 2.30, 10 da. | | |
| 2 | 280 " | " 2.40, 30 da. | | |
| 18 | 175 " | " 2.28, 10 da. | | |
| | *Charges* | | | |
| 5 | Freight and drayage | | 92 | 50 |
| 18 | Storage | | 18 | 56 |
| 5 | Advertising and Insurance | | 25 | 00 |
| | Commission 2½% Guaranty 2% | | | |
| | Net proceeds due | | | |

# ACCOUNTS BEARING INTEREST

### Daily Balances

**861.** Some banks allow interest on daily balances of their depositors on amounts exceeding a certain figure which has been agreed upon by the depositor and the bank. Small balances subject to check usually draw no interest.

**862.** It is to be noticed that the interest rates differ according to the customs of various lines of business. Trust companies commonly pay 2% on daily balances on active accounts; on those that require notice to withdraw money, it is customary to allow 3% on daily balances.

**863. To find interest on daily balances.**

THE FRUGALITY SAVINGS COMPANY

*In account with* JOHN WISE

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| July | 1 | To Balance | 155 00 | July | 1 | By Check | 33 33 |
| " | 9 | " Cash | 50 00 | " | 15 | " " | 15 00 |
| " | 23 | " " | 50 00 | Aug. | 1 | " " | 33 33 |
| Aug. | 6 | " " | 50 00 | " | 15 | " " | 12 00 |
| " | 20 | " " | 50 00 | Sept. | 1 | " " | 33 33 |
| Sept. | 10 | " " | 50 00 | " | 15 | " " | 10 00 |
| " | 24 | " " | 50 00 | *Oct.* | *1* | " *Balance* | *319 70* |
| Oct. | 1 | " Int. | 1 69 | | | | |
| | | | 456 69 | | | | 456 69 |
| 1914 | | | | | | | |
| Oct. | 1 | To Balance | 319 70 | | | | |

**Operation**

| Balances | | | | | | Days | |
|---|---|---|---|---|---|---|---|
| $121.67 | from | July | 1 | to July | 9 | 8 | $ 968 |
| 171.67 | " | " | 9 | " " | 15 | 6 | 1026 |
| 156.67 | " | " | 15 | " " | 23 | 8 | 1248 |
| 206.67 | " | " | 23 | " Aug. | 1 | 9 | 1854 |
| 173.34 | " | Aug. | 1 | " " | 6 | 5 | 865 |
| 223.34 | " | " | 6 | " " | 15 | 9 | 2007 |
| 211.34 | " | " | 15 | " " | 20 | 5 | 1055 |
| 261.34 | " | " | 20 | " Sept. | 1 | 12 | 3132 |
| 228.01 | " | Sept. | 1 | " " | 10 | 9 | 2052 |
| 278.01 | " | " | 10 | " " | 15 | 5 | 1390 |
| 268.01 | " | " | 15 | " " | 24 | 9 | 2412 |
| 318.01 | " | " | 24 | " Oct. | 1 | 7 | 2226 |
| | | | | | | | $20235 |

Total daily balance, $20235 for 1 day
Interest at 3% $1.69

**Form with Balance Columns**

PENN TRUST COMPANY

*In account with* SMITH & JONES

| Date | | Dr. | | Cr. | | Daily Balance | | Days | Total Balance | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1916 | | | | | | | | | | |
| Jan. | 1 | 8000 | 00 | | | 8000 | 00 | 14 | 112000 | 00 |
| " | 15 | 2000 | 00 | | | 10000 | 00 | 16 | 160000 | 00 |
| " | 31 | | | 5000 | 00 | 5000 | 00 | 15 | 75000 | 00 |
| Feb. | 15 | 6000 | 00 | | | 11000 | 00 | 13 | 143000 | 00 |
| " | 28 | | | 4500 | 00 | 6500 | 00 | 15 | 97500 | 00 |
| Mar. | 15 | 4000 | 00 | | | 11000 | 00 | 16 | 176000 | 00 |
| " | 31 | | | 5000 | 00 | 6000 | 00 | 1 | 6000 | 00 |
| | | 20500 | 00 | 14500 | 00 | | | | 769500 | 00 |
| Apr. | 1 | 42 | 75 | | | | | | | |
| | *Bal.* | | | *6042* | *75* | | | | | |
| | | 20542 | 75 | 20542 | 75 | | | | | |
| Apr. | 1 | 6042 | 75 | | | | | | | |

Interest at 2% on $769500 for 1 day = $42.75

## WRITTEN PROBLEMS

**864.** *1.* What is the net amount due on the following account July 1, 1917?

*Dr.* I. M. PORTER *in account current with* VICI BROTHERS *Cr.*

| 1916 | | | | 1916 | | | |
|---|---|---|---|---|---|---|---|
| July | 1 | Balance | 1200 00 | Nov. | 1 | Mdse., 3 mo. | 600 00 |
| Sept. | 12 | Draft | 900 00 | 1917 | | | |
| 1917 | | | | Mar. | 14 | " 60 da. | 600 00 |
| Jan. | 3 | " | 900 00 | Apr. | 25 | " 30 da. | 850 00 |
| May | 16 | " | 400 00 | June | 4 | " net | 500 00 |

*2.* What is the balance of the following account May 1, 1915, interest at 6%?

*Dr.* HILL & SON *in account with* GROVE LAND COMPANY *Cr.*

| 1915 | | | | 1915 | | | |
|---|---|---|---|---|---|---|---|
| Jan. | 15 | Draft | 950 00 | Jan. | 1 | Balance | 3500 00 |
| Feb. | 1 | " | 860 00 | " | 22 | Sales at 3 mo. | 1200 00 |
| Mar. | 1 | " | 1800 00 | Feb. | 15 | " 3 " | 700 00 |
| " | 24 | " | 960 00 | Mar. | 2 | " 3 " | 2400 00 |

*3.* The City Trust Company allows 2% interest on daily balances of $100 or more. What is the balance to the credit of Charles Careful on December 31, 1915, after crediting him with the interest due?

Deposits: March 5, 1915, $50; April 1, 1915, $40; April 15, 1915, $60; April 30, 1915, $50; May 14, 1915, $50; June 1, 1915, $35; June 15, 1915, $50; June 25, 1915, $40; June 26, 1915, $35; July 16, 1915, $52.92; July 9, 1915, $62.50; July 30, 1915, $50, August 3, 1915, $38; August 16, 1915, $50; August 31, 1915, $50; September 15, 1915, $40; September 30, 1915, $40; October 15, 1915, $50; October 30, 1915, $50; November 25, 1915, $50.

Checks: March 19, 1915, $12; April 9, 1915, $10; April 15, 1915, $25; April 21, 1915, $20; April 23, 1915, $12; May 3, 1915, $10; May 7, 1915, $11; May 14, 1915, $25; May 20, 1915, $10; May 25, 1915, $10; June 10, 1915, $10; June 15, 1915, $25; June 21, 1915, $10; June 24, 1915, $20; July 1, 1915, $100; July 5, 1915, $33.33; July 20, 1915, $25; July 27, 1915, $7; August 2, 1915, $33.33; August 5, 1915, $20; August 10, 1915, $11.50; August 12, 1915, $10; August 13, 1915, $10; August 17, 1915, $15; August 24, 1915, $15; September 1, 1915, $33.33; September 1, 1915, $10; September 1, 1915, $19.55; September 15, 1915, $10; September 21, 1915, $15; September 30, 1915, $3.25; October 4, 1915, $33.33; October 7, 1915, $16.80; October 8, 1915, $10; October 14, 1915, $10; October 21, 1915, $12; October 26, 1915, $20; October 28, 1915, $8; November 2, 1915, $5; November 3, 1915, $33.33; November 6, 1915, $10; November 9, 1915, $4.70; November 12, 1915, $6.

## SAVINGS-BANK ACCOUNTS

**865. Savings-banks** are institutions of a **mutual** and **semi-public** nature for receiving and investing savings. They have no other capital than the money deposits and are conducted solely for the benefit of the depositors. They allow usually 3% to 4% interest, compounded, if not withdrawn, at regular fixed periods, annually, semi-annually, quarterly or monthly.

**866.** The **deposits** in savings-banks are not subject to check, but may be withdrawn on presentation of bank book and proper notice.

**867.** The **interest** term is the interval between interest periods.

**868. To calculate the interest on a savings-fund account.**

In the Western Saving Fund Society of Philadelphia interest is allowed at the rate of 3.65 per cent. per annum on accounts of five dollars and over. No interest is allowed on fractional parts of a dollar. Interest is calculated by calendar months, and is allowed on deposits made on or before the fifth of the month; in the case of withdrawals, interest is allowed for one-half month on money withdrawn on or after the sixteenth of the month. At the end of the calendar year the interest is reckoned, and is added to the principal.

NOTE.—Practises vary somewhat in different institutions.

## WRITTEN PROBLEMS

**869.** *1.* William Trask deposited in the Western Saving Fund Society of Philadelphia, as follows: January 2, 1914, $20; February 16, 1914, $24; March 14, 1914, $18; April 27, 1914, $60; May 29, 1914, $12; June 3, 1914, $15; July 20, 1914, $36; August 24, 1914, $72; November 14, 1914, $24; December 7, 1914, $50. On March 18, 1914, he withdrew $16; April 10, 1914, $15; September 9, 1914, $25; October 3, 1914, $15. What was the balance of his account January 1, 1915?

*2.* Mary Jones deposits in the Western Saving Fund Society as follows: February 3, 1915, $20; May 6, 1915, $32; July 7, 1915, $18; October 22, 1915, $22; December 15, 1915, $25; February 10, 1916, $15; April 20, 1916, $17; June 30, 1916, $24; August 17, 1916, $12; September 26, 1916, $20; November 3, 1916, $15; December 20, 1916, $21; February 9, 1917, $16; April 14, 1917, $25; June 12, 1917, $18; August 22, 1917, $20; October 11, 1917, $16; December 5, 1917, $25; January 26, 1918, $12; March 12, 1918, $26; May 18, 1918, $17; July 16, 1918, $30; September 18, 1918, $12; November 7, 1918, $21; December 20, 1918, $14; January 24, 1919, $10; March 28, 1919, $20. What is the balance due her on July 1, 1919?

*3.* On January 1, 1915, William Smith's balance in a savings bank, in which interest at 3% per annum is allowed on the smallest monthly balance and compounded quarterly, was $180. During the year he made the following deposits: January 18, $24; February 6, $16; March 4, $28; March 6, $14; March 30, $15; April 16, $13; May 1, $60; May 8, $24; May 17, $25; June 3, $12; June 19, $18; July 10, $18; August 2, $14; August 20, $13; September 3, $16; October 25, $28; November 5, $22; December 3, $30. He withdrew January 21, $25; February 26, $30; April 20, $60; August 20, $25; September 16, $12; October 30, $15; December 24, $12. What was the balance of the above account January 1, 1916?

*4.* Mr. W. T. Zaner made the following deposits and withdrawals at a savings bank, the by-laws of which provide for an allowance of interest at three per cent. per annum on the smallest balance for interest terms of three months ending, respectively, January 1, April 1, July 1, and October 1, interest to be compounded semi-annually. (That is, a deposit begins todraw interest from the first day of each quarter, and only the balance left the entire quarter draws interest in the event of withdrawals.) He deposited January 1, 1915, $600; February 2, 1915, $145; February 15, 1915, $180; March 16, 1915, $160; March 26, 1915, $240; April 20, 1915, $160; May 1, 1915, $120; June 21, 1915, $180; July 16, 1915, $125; August 26, 1915, $280; September 2, 1915, $460; October 4, 1915, $240; October 18, 1915, $160; November 12, 1915, $180; He withdrew January 22, 1915, $100; February 26, 1915, $200; March 25, 1915, $200; May 20, 1915, $150; July 1, 1915, $50; October 23, 1915, $20; December 24, 1915, $200. What was due Mr. Zaner January 1, 1916?

# BANKRUPTCY

**870.** A **bankrupt** or **insolvent** is a debtor whose property is taken to be divided among his creditors by a court under the operation of an insolvent law.

**871.** An **insolvent law** provides that when a person has been judicially ascertained to be insolvent, and has surrendered his property for distribution among his creditors under a decree of court, he is to be discharged from his indebtedness.

**872.** An **assignee in bankruptcy,** or an **assignee in insolvency,** is a person to whom the property of a bankrupt or insolvent is transferred in order that he may dispose of the same for the benefit of the creditors.

**873.** A **dividend** is a sum arising out of the bankrupt's available assets, which the assignee divides among the creditors *pro rata.*

**874. To find each creditor's share.**

EXAMPLE.—The statement of a bankrupt's affairs was as follows:

*Assets:* Cash, $2870.13; Bills Receivable, $3750; Merchandise, $8439.70; Real Estate, $4200; Stocks and Bonds, $3430.75; Personal Accounts, $7833.48.

*Liabilities:* Due A, $9238.17; due B, $12375; due C, $20453.38; due D, $17216.85; due E, $4114.29. The expenses of the assignment were $917.34. What was the rate of dividend, and how much should each creditor receive?

$2870.13
3750.
8439.70
4200.
3430.75
7833.48
$30524.06 Total assets
917.34
$29606.72 Net assets

$9238.17
12375.
20453.38
17216.85
4114.29
$63397.69 Total liabilities

$29606.72 ÷ $63397.69 = .467, or 46⅞%, rate of dividend.

$9238.17 X .467 = $4314.23 A's dividend
12375. X .467 = 5779.12 B's "
20453.38 X .467 = 9551.73 C's "
17216.85 X .467 = 8040.27 D's "
4114.29 X .467 = 1921.37 E's "
$29606.72 Total dividend

*Or,*

$63397.69 : $9238.17 :: $29606.72 : A's div. = $4314.23
63397.69 : 12375.00 :: 29606.72 : B's " = 5779.12
63397.69 : 20453.38 :: 29606.72 : C's " = 9551.73
63397.69 : 17216.85 :: 29606.72 : D's " = 8040.27
63397.69 : 4114.29 :: 29606.72 : E's " = 1921.37

NOTE.—Usually it is necessary to carry the decimal to eight places.

**878. Rule.** — *Deduct expenses of the assignment from the total assets, and divide by the total liabilities. The quotient will be the rate of dividend. Then multiply the amount of each creditor's claim by the rate of dividend.* Or by proportion.

*Total liabilities : Each liability :: Net assets : Each creditor's share of assets.*

## WRITTEN PROBLEMS

**879.** 1. If a firm fails with liabilities amounting to $24000 and assets $4300, how much should each creditor receive on the dollar, expenses of assignment being $412? What sum should a creditor receive whose claim amounted to $6480.75?

2. A bankrupt's assets are as follows: Cash, $1263.18; Merchandise, $6492.72; Bills Receivable, $1850; Personal Accounts, $2658.47. His liabilities are: Due to Jones, $3316.24; Brown, $11624.25; Smith, $7422.96; Clark, $1127.90; Expenses of settlement, $412.70. How much does each creditor receive?

3. After converting all the available assets into cash and paying all expenses, an assignee has in his hands $73462.17 to distribute among creditors whose claims amount to $104392.64. How much does A receive, whose claim is $21622.75?

4. An insolvent merchant owes A, $3122.75; B, $2560.38; C, $9421.67; D, $248.90; E, $647.38; F, $427.32. His assignee has realized $8962.60 from the assets, and has $264.77 expenses to pay. How much will each of the creditors receive?

5. Reed & Co. fail in business with liabilities amounting to $65850. The assignee sells the real estate for $19500 and the stock of goods for $27000, collected debts owing them amounting to $7230, and expended $1250 in settling up the business. What will William Harper receive if his claim is $12720?

6. Henkel & Co. made an assignment for the benefit of their creditors with the following resources and liabilities. *Resources*: Cash, $1263.17; Mdse., $17324.90; Real Estate, $12000; Bills Receivable, $2340; Accounts Receivable, $12725.62. *Liabilities*: Bills Payable, $2287.50; Jones & Hoyt, $2578.32; Clark & Co., $1872.90; Hansen & Adams, $1487.18; William F. Davis, $6284.36; Henry Thompson, $1937.62; Lavelle & Wilson Co., $2716.40; Morton & Haskell, $7750.92. The expenses of the assignment were $1214.85. Show how much each creditor receives.

7. A merchant failed in business and rendered the following schedule. *Assets*: Cash, $1560; Real Estate, $2750; Personal Accounts, $1125; Merchandise, $1125. *Liabilities*: Notes, $6000; due John Boyd, $1225; R. S. Mann, $1130; J. A. Lee, $5675. The expenses of assignment were $1375. How much should each creditor receive?

## PARTNERSHIP SETTLEMENTS

**877.** A **partnership** is an association of individuals formed by an **agreement** made between them to combine their money, labor and skill in some business enterprise, the profits or losses of which are divided in certain proportions. This association is generally called a *firm* or *house*.

**878.** The **capital** of a partnership consists of the money, real estate, or any other property invested.

**879.** The **resources** of a firm consist of the money and other property it owns, and the debts due the firm. In theory, the firm is distinct from the individuals composing it; hence, a partner may owe the firm, and his obligation to the firm is a resource of the firm, as in the case of withdrawals.

**880.** The **liabilities** of a firm are the debts it owes. For accounting purposes the firm may be considered as owing the individuals composing the firm for their investment; hence, the capital of the business is a *quasi* liability of the firm.

**881.** The **net capital** of a firm, or **present worth,** is the excess of the resources over the liabilities. While the excess of resources over liabilities is called either net capital or present worth, still, strictly speaking, the former is the appropriate term to use when opening books and the latter when closing.

**882.** When a firm's resources exceed its liabilities, it is said to be *solvent.* When the liabilities exceed the resources, the firm is said to be *insolvent.*

**883.** The **net insolvency** of a firm is the excess of liabilities over resources.

**884.** The **net gain** is the excess of gain over loss for a given period; or the excess of present worth over the invested capital.

**885.** The **net loss** is the excess of loss over gain for a given period; or the excess of invested capital over present worth.

**886.** An **adventure, enterprise,** or **venture,** is usually a co-operation for a single business arrangement for a limited time or for separated business arrangements, and they are not partnerships in the strict use of that term. They are settled in accordance with the terms of the agreement or in accordance with the custom in such cases.

**887.** Partnerships are settled in accordance with the terms of the agreement; this should preferably be written, but may be oral. The acts of the partners may modify, and, in fact, entirely abrogate the terms of the agreement. In general, where there is no distinct agreement as to the sharing of gains or losses, partners share equally, no matter what the respective amounts of investments may be. Differences in investment are usually adjusted through interest allowances. Where one or some of the members of a firm are active in its business, devoting their entire time and energies to its interests, while others are passive, these differences are frequently adjusted by allowing the former salaries.

**888.** When two classes of accounts are furnished, one showing resources and liabilities, and the other gains and losses, and the results obtained do not agree, the result of the class showing resources and liabilities is to be taken, as the accounts of this class can be verified by taking inventories, sending out statements, etc.

**889. To find the net gain or net loss of a business when the investment is given, and the present worth.**

EXAMPLE 1.—What is the net gain of a business whose net capital at starting was $4000 and whose present worth is $4500?

| | |
|---|---|
| Present worth | $4500 |
| Net capital | 4000 |
| Net gain | $500 |

EXAMPLE 2.—What is the loss of a business whose net capital was $15000, now showing a present worth of $13500?

| | |
|---|---|
| Net capital | $15000 |
| Present worth | 13500 |
| Net loss | $1500 |

**890.** RULE.—*Find the difference between the net capital and thepresent worth If the present worth is the greater, the difference shows a gain; if the capital is the greater, the result shows a loss.*

### WRITTEN PROBLEMS

**891.** Find the net gain or net loss in the following:

1. Net capital $2500, present worth $3750.
2. Insolvency $1100, present insolvency $1500.
3. Net capital $2500, insolvency $1500.
4. Insolvency $500, present worth $1200.

**892. To find the net gain or net loss of a business when the items of gain and loss are given.**

EXAMPLE.—The total gains of a business are $5400 and the total losses are $1700; what is the net gain?

| | |
|---|---|
| Total gains | $5400 |
| Total losses | 1700 |
| Net gain | $3700 |

**893.** RULE.—*Find the difference between the total gains and total losses; the excess of gain shows the net gain and the excess of loss the net loss.*

### WRITTEN PROBLEMS

**894.** 1. What is the net gain or loss, if the total gain is $3500 and the total loss $1800?

2. Total gain, $7200; total loss, $3750?
3. Total gain, $825; total loss, $653?
4. Total gain, $12500; total loss, $14200?
5. Total gain, $1700; total loss, $1900?
6. Items of gain: merchandise, $500; discount and interest, $80—Item of loss: expense, $420?
7. Items of gain: merchandise, $1250; rent, $400; discount and interest, $120—Items of loss: expense, $650; furniture and fixtures, $35?

**895. To find the present worth of a business when the net capital is given, and gain or loss.**

EXAMPLE.—A merchant invested $12000 in business. Upon closing, his Loss and Gain account shows the following items: Gains, Merchandise, $3500; Discount and Interest, $375. Losses, Expense, $1500. What is the present worth of the business?

Loss and Gain

| | | | |
|---|---|---|---|
| Expense | $1500.00 | Merchandise | $3500.00 |
| Net gain | 2375.00 | Discount & Interest | 375.00 |
| | $3875.00 | | $3875.00 |
| | | Net capital | $12000 |
| | | Net gain | 2375 |
| | | Present worth | $14375 |

REMARK.—The principles of accounting should be carefully applied in working the problems that follow, and the statement should be presented in such form that the results may be readily seen, and the process by which they are attained readily followed. Students frequently fail to get accurate results because of lack of system and method in arranging the work.

**896.** RULE.—*Add the net gain to the net capital, or deduct the net loss from it.*

### WRITTEN PROBLEMS

**897.** *1.* What is the present worth, if net capital was $3500, gain $550?

*2.* Net capital $7500, loss $700? Net capital $6500, gain $800?

*3.* Net capital $1700, gain $3500? Insolvency $300, gain $600?

*4.* Insolvency $700, loss $300? Insolvency $1200, gain $900?

*5.* A business was started with a capital of $17500; the totals of the Loss and Gain account show debits $5625, credits $6000. What is the present worth?

*6.* The net capital of a business was $8000. The Loss and Gain account shows debits: Expense, $325; Discount and Interest, $78; Real Estate, $200—and the following credits: Merchandise, $2250; Stocks, $250. What is the present worth of the business?

*7.* A invested $6000 and B $3000 in a partnership. At the end of a year the items of expense were: General Expense, $1200; Discount and Interest, $300; Furniture and Fixtures, $150. The items of gain were: Merchandise, $750; Rent, $300. What is the present worth of the business?

*8.* A, B and C form an equal partnership. During the year A invests an additional $2000; B withdraws $1500; their purchases of merchandise were $12500; their sales, $14600; goods on hand at closing, $3160; expenses have been $2590; advertising has cost $675; insurance, $460; inventory of insurance at closing, $210; notes and acceptances on hand, $8950; notes outstanding, $175; accounts payable, $1416; real estate is Dr. $5410, which is also inventory value; accounts receivable, $10106, of which $460 is considered worthless. Make a statement of losses and gains and determine the present worth of each partner.

**888. To find the present worth of a business when the resources and liabilities are given.**

EXAMPLE. The resources of a business are Cash, $1500; Merchandise, $2420; Bills Receivable, $310; Accounts Receivable, $850; and the liabilities are Bills Payable, $1500; Accounts Payable, $240. What is the present worth?

| Resources | | Liabilities | |
|---|---|---|---|
| Cash | $1500 | Bills Payable | $1500 |
| Merchandise | 2420 | Accounts Payable | 240 |
| Bills Receivable | 310 | *Present Worth* | [illegible] |
| Accounts Receivable | 850 | | |
| | $7870 | | $7870 |

889. RULE. *Find the difference between the resources and the liabilities. The excess of resources over liabilities shows the present worth, the excess of liabilities over resources shows the insolvency.*

**890. To distribute gain or loss when periods of investment are equal.**

EXAMPLE. A, B and C form a partnership, agreeing to share the gains or losses in proportion to their investments. A invests $6500, B $12500, C, $7500. They gain $2500. Find each partner's share.

**Operation by Fractional Method**

A $6500
B 12500
C 7500
$26500

$\frac{6500}{26500}$ of $2500 = $613.71 A's gain
$\frac{12500}{26500}$ of $2500 = $1179.25 B's gain
$\frac{7500}{26500}$ of $2500 = $1564.29 C's gain

**Operation by Proportion**

The ratio of the total investment to each man's investment equals the ratio of the total gain to each man's gain.

$26500 : $6500 :: $2500 : A's gain = $613.71
$26500 : $12500 :: $2500 : B's gain = $1179.25
$26500 : $7500 :: $2500 : C's gain = $1564.29

**Operation by Percentage Method**

$2500 ÷ $26500 = 21 4/53%

$6500 × 21 4/53% = $613.71
$12500 × 21 4/53% = $1179.25
$7500 × 21 4/53% = $1564.29

### WRITTEN PROBLEMS

**901.** *1.* The resources of a business are: Merchandise, $4340; Cash, $2810; Bills Receivable, $1300; Accounts Receivable, $1500; and the liabilities are: Bills Payable, $8420; Accounts Payable, $3200. What is the condition of the business? If the partners invested in the ratio 2 to 3, find each partner's share.

*2.* A and B form a partnership, agreeing to divide the gains or losses in proportion to investments. A invests $7500 and B invests $9600. Divide a gain of $3750.

*3.* Three men rent a pasture for three months for $96. One puts into it 14 head of cattle, the second 22 head, the third 19 head. What should each be required to pay?

*4.* A, B and C form a partnership, agreeing to share gains or losses in proportion to investments. A invests $4600. They gain $3800. B receives as his share of the gain $1500 and C receives $\frac{1}{8}$ of the gain. Find B's and C's investments.

*5.* Two men, M and N, hired a team to go a distance of 10 miles and return; the charge for the team was $8. On arriving at their destination M proposed to return by another conveyance, to which N agreed. They also agreed that M should only be required to pay in proportion to the number of miles he had ridden. Find what each must pay.

**902. To distribute gain or loss when periods of investment are unequal.**

EXAMPLE.—A and B form a partnership, agreeing to divide the losses or gains in proportion to investments.

On January 1, A invests $5200 and B invests $4800. A, on May 1, invests $2200 additional, and on July 1, $3000. He withdraws on August 1, $2800 and on November 1, $3300. B invests on March 1, $2600, and on June 1, he withdraws $3000. They gain $3860. Find each partner's share.

A

| *Dr.* | | | | *Cr.* | | | |
|---|---|---|---|---|---|---|---|
| Aug. 1 | Cash | 2800 | | Jan. 1 | Cash | | 5200 |
| Nov. 1 | Cash | 3300 | | May 1 | Cash | | 2200 |
| | | | | July 1 | Cash | | 3000 |
| | $2800\times5=14000$ | | | | | $5200\times12=62400$ | |
| | $3300\times2=\ 6600$ | | | | | $2200\times\ 8=17600$ | |
| | 20600 | | | | | $3000\times\ 6=18000$ | |
| | | | | | | 98000 | |
| | | | | | | 20600 | |
| | | | | A's net investment for 1 mo. = | | 77400 | |

| *Dr.* | B | | | | *Cr.* |
|---|---|---|---|---|---|
| June 1 Cash | 3000 | Jan. 1 Cash | | | 4800 |
| | | Mar. 1 Cash | | | 2600 |

$3000 \times 7 = 21000$

$4800 \times 12 = 57600$
$2600 \times 10 = 26000$
$83600$
$21000$
B's net investment for 1 mo. $= 62600$

$77400 A's net investment for 1 mo.
62600 B's " " " 1 "
$140000 total investment for 1 mo.
$140000 : $77400 : : $3860 : A's gain = $2134.03
$140000 : $62600 : : $3860 : B's gain = $1725.97

EXPLANATION.—A's investment on January 1, $5200, was invested for 12 months. An investment of $5200 for 12 months is equal to an investment of $62400 for 1 month. The $2200 was invested for 8 months, which is equal to an investment of $17600 for 1 month. The $3000 was invested for 6 months, which is equal to an investment of $18000 for 1 month. A's total investment is equal to an investment of $98000 for 1 month. Considering his withdrawals, we find the $2800 to have been out of the business for 5 months. A's withdrawal of $2800 for 5 months is equal to a withdrawal of $14000 for 1 month; and the $3300 for 2 months is equal to $6600 for 1 month. His total withdrawals are equal to a withdrawal of $20600 for 1 month. Subtracting the total withdrawal for 1 month from the total investment for 1 month, he has a net investment for 1 month of $77400.

We proceed in the same way with B's account, and find that he has a net investment for 1 month of $62600. Now these net investments for one month will form a true basis for the division of the gain. Taking the sum of these investments as the first term of a proportion, A's investment for the second term, the total gain for the third, we find the fourth term, or A's gain, to be $2134.03. In the same manner we find B's gain to be $1725.97.

**903.** RULE.—*Multiply each investment by the time it was invested, and each withdrawal by the time it was withdrawn. The difference between the sums of these products is the investment for one month, which forms a basis for the division of gain or loss. Then by proportion find gain or loss of each partner.*

**Formula** for determining each partner's share of loss or gain

*Whole investment : Each partner's investment : : Whole gain : Each partner's share of gain.*

## WRITTEN PROBLEMS

**904.** *1.* A, B and C enter into partnership. A puts in $1000 for 8 months; B, $1200 for 10 months; C, $800 for 12 months. They gain $1500. What was the share of each?

*2.* Armor and Baker engaged in a partnership for four years; Armor put in $6000 and Baker $8000. At the close of the second year Armor took out $2000, and Baker put in $2000; at the close of the fourth year they divided $8890 as net gain. What was the share of each?

**905. To find each partner's capital when interest is considered.**

EXAMPLE.—A and B form a partnership, agreeing to share equally the losses or gains. It is also agreed that each partner is to receive interest on his investment at 8% and pay interest on his withdrawals at the same rate. The following is a statement of their investments and withdrawals. They gain $4250. Close their accounts.

*Dr.* A *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| Apr. | 9 | Cash | 1800 00 | Jan. | 1 | Cash | 2500 00 |
| Nov. | 6 | Cash | 600 00 | Mar. | 16 | Cash | 3600 00 |
| *Dec.* | *31* | *Balance* | *8738 19* | Sept. | 9 | Cash | 2900 00 |
| | | | | Dec. | 31 | Interest | 391 09 |
| | | | | " | 31 | Gain | 1747 10 |
| | | | 11138 19 | | | | 11138 19 |
| | | | | 1915 | | | |
| | | | | Jan. | 1 | Balance | 8738 19 |

Int. on $1800 for 266 da. = $106.40
" " 600 " 55 " = 7.33
$113.73

Int. on $2500 for 1 yr. = $200.00
" " 3600 " 290 da. = 232.00
" " 2900 " 113 " = 72.82
$504.82
113.73
A's net credit interest $391.09

*Dr.* B *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| Mar. | 6 | Cash | 900 00 | Jan. | 1 | Cash | 3200 00 |
| Sept. | 19 | Cash | 1200 00 | Apr. | 9 | Cash | 1600 00 |
| *Dec.* | *31* | *Balance* | *8411 81* | Aug. | 26 | Cash | 3600 00 |
| | | | | Dec. | 31 | Interest | 364 71 |
| | | | | " | 31 | Gain | 1747 10 |
| | | | 10511 81 | | | | 10511 81 |
| | | | | 1915. | | | |
| | | | | Jan. | 1 | Balance | 8411 81 |

Debit interest $87.47
NOTE.—Method as in A's account

Credit interest $452.18
87.47
B's net credit interest $364.71

$391.09 + $364.71 = $755.80; $4250.00 − $755.80 = $3494.20
$3494.20 ÷ 2 = $1747.10, each partner's share of the net gain

**906.** RULE.—*Find the interest on the investments for the time for which they were made. Find also the interest on the withdrawals for the time they were withdrawn. The difference between the sums of these interest items is the interest with which the account should be either credited or debited. Then divide the remainder of the gain or loss. Enter this on the proper side of the account and find the balance.*

## WRITTEN PROBLEMS

**267.** 1. A set of books shows a net gain of $[illegible]. A's investment, $2500.00; B's investment, $[illegible]; A withdrew $3016.28; B withdrew $[illegible]. Interest is to be allowed on the investment of each partner, but is not to be charged on withdrawals, and the balance of net gain is to be equally divided. What is the balance of each partner's account?

2. A set of books shows the following results. Loss and Gain, Dr., $7395.60; Cr., $9673.21; A's Capital, Cr., $5175.29, withdrawals, $946.71; B's Capital, Cr., 4295.35, withdrawals, $125.15. B is to have a salary of $2000; an interest account to be kept to adjust the differences in capital. After crediting salary and one year's interest at 6% on investments, the balance is shared by A and B equally. What are the proper balances for A and B to commence the new period?

3. A set of books show: Loss and Gain, Dr., $[illegible], Cr., $[illegible]; A's account, Dr., $4567.28, Cr., $24758.62; B's account, Dr., $3867.19, Cr., $[illegible]. Before settling, credit A's salary, $2500; B's salary, $4500; also credit A and B interest 6 per cent. on the original investment; then close Loss and Gain, giving each partner half of net gain. State each partner's balance at closing.

4. Close books containing the following: Gain, $8937.65. Partners' capital—Cr., $9654.62; B—Cr., $6978.78; C—Cr., $11964.65. Partners were debited as follows: A—Dr., $2245.67; B—Dr., $1463.79; C—Dr., $1296.55. The articles of agreement provide that the partners shall receive salary for their services: A, $2000; B, $1500; and C, $3000. They shall have interest at 6% on their capital; the net balance, gain or loss, shall then be equally divided.

5. Close a set of books which shows the following results, and state each partner's balance. Net credit to Loss and Gain account before salary and interest are credited, $25000. White, Dr., $1800, Cr., $12000. Redd, Dr., $2500, Cr., $10000; Redd is to be allowed $1000 for services, and each partner is to be allowed interest at 5% on his capital, and the net gain to be divided equally.

6. The Loss and Gain account in a set of books is credited $31084.80. Abel's account (senior partner), debited $2937.63, credited $60758.93. Bearer's account (junior partner), debited $925.76, credited $17896.49. Determine and credit the interest, ignoring interest on withdrawals. Credit each partner $3000 for services. Divide the net profits equally and bring down the new balances to the credit of partners.

7. A and B formed a partnership, agreeing to share losses and gains in proportion to average net investments. On January 1, A invested $3200, and B on the same date invested $2650. A on March 1 invested $1600 and on September 1, $2000, and withdrew on May 1, $1600 and on October 1, $550. B invested on April 1, $1800, and on August 1, $1950, and withdrew on June 1, $1000 and on December 1, $900. The present worth at the close of the year is $12500. How much was due each partner?

## REVIEW PROBLEMS

**908.** *1.* A, B and C traded in company; A put in \$1 as often as B put in \$3, and B put in \$2 as often as C put in \$5. B's money was in twice as long as C's and A's twice as long as B's; they gained \$5250; how much was each man's share of the gain if divided in proportion to average net investment?

*2.* A and B purchase a house on which expenses for a year are \$105, and which rents for \$420 a year. B pays \$2200 of the purchase money, and A receives \$295 as his share of the rent. Find the amount of the purchase money B pays. If rent and expenses are shared in proportion to investment, how much of the expense should each pay? How much of the rent should each receive?

*3.* Three boys buy a bicycle, A paying \$32, B \$48 and C \$20. They agree that in every week of six days (10 hours each) each shall have the use of it for a time proportionate to his payments. If A begins to use it at six o'clock Monday morning, when should B receive it? When should C receive it?

*4.* Three men take an interest in a coal mine. B invests his capital for four months, and claims one-tenth of the profits; C's capital is in eight months; D invests \$6000 for six months and claims two-fifths of the profits; how much did B and C put in? Losses and gains shall be divided in proportion to average net investments.

*5.* X, Y and Z entered into business as partners, each putting in \$5000 as capital. At the end of two years X took out \$1000, Y \$2000, and Z \$3000. At the end of the fourth year they closed the business with a loss of \$3600. What was the loss of each if losses and gains were divided in proportion to average net investments?

*6.* Loss and gain, credit in excess of debit, \$9346.21; Ames's debits, \$1300.00, credits, \$15265.75; Bell's debits, \$900.00, credits, \$6875.94; Carr's credits, \$10000.00. The agreement allows Bell \$1000 for services, and each partner is to be paid with interest before the profits are divided. Bell is to pay 12 per cent. interest to Carr for the use of his money; Carr is not a partner, but loans money to the firm of Ames & Bell, at the expense of Bell alone; what will be the balance of Ames's and Bell's accounts after settlement? 18390.55 9697.34

*7.* A certain ledger, after all the work was posted and the accounts closed, showed the following results: Loss and Gain, debits, \$2829.46, credits, \$18765.40; A. Bard, debits, \$2526.89, credits, \$26345.48; C. Dash, debits, \$3687.95, credits, \$19875.69. The articles of copartnership require an interest account kept with the partners, crediting each with interest on capital invested, but not charging interest on withdrawals; C. Dash, outside partner, is to be credited \$1000 a year personal expenses. Complete the closing of the ledger and determine the balance to the credit of each partner, if losses or gains are shared equally? 3148.66 24461.61

*8.* A and B form a partnership. At the close of the year A's account is overdrawn \$2900, and B's \$2300. How may they settle if no other resources?

*9.* A and B form a partnership, agreeing to share gains and losses in proportion
› average net investments. A on January 1 invests $4800, and B on the same date
vests $5300. On May 1, A invests $2200, and on August 1 withdraws $1200. B on
pril 1 invests $4300, and on September 1 he withdraws $1900. They gain $6200.
hey take in C as a partner on July 1, and he invests a sufficient amount to entitle
m to ⅖ of the gain. Find A's and B's gain and C's investment.

*10.* A and B form a partnership on January 1, 1914, agreeing to share the gains
ıd losses in proportion to their average net investments. A on January 1 invests
600, and B $4800. A invests on July 6, $2800, and withdraws on September 1,
200. B, on September 1, withdraws such an amount as will make his average
vestment sufficient to entitle him to ⅜ of the gain, and C on June 1 invests a sufficient
nount for the rest of the year to entitle him to ⅜ of the profits. They gain $5800.
nd B's withdrawal, A's gain, and C's investment.

*11.* A and B form a partnership, agreeing to share losses and gains in propor-
›n to average net investments. A on January 1 invests $5500, and B on the same
ıte invests $4200. A withdraws on June 1, $1200, and he invests on August 1, $1400.
withdraws on September 1, $1500, and invests on November 1, $2500. C is
lmitted to the partnership and invests $3800 for a time sufficient to entitle him to
of the gain. Find the date on which C made his investment.

*12.* Three men, A, B and C rent a pasture for 4 months, for which they agree
pay $250. A puts in 25 cattle for 2 months, and 32 additional for the remaining
months. B puts in 18 cattle for 1 month, and for the remaining three months has
26. C puts in 28 for 3 months, and for the remaining month has in 34. Find
e amount each should pay.

*13.* William Reid and John Day agree to form an equal partnership. They
ıy a planing mill, the price of which is $15000, $7000 of which Reid pays and the
ılance is paid by Day. They buy rough lumber to the amount of $3900, half of
hich is paid by each. Reid pays $3950 during the year for wages and other expenses,
ıd Day pays $2620. Their sales of finished product during the year amount to
7320, and they make an additional purchase of rough lumber to the amount of $1850.
heir taxes and insurance during the year amount to $310. At the time of settle-
ıent, at the close of the year, they have on hand rough lumber to the amount of $920
ıd finished product to the amount of $1750. Day is to receive $1200 for conducting
ıe business, and Reid $1200 for managing the mill. It is required to know, first,
ıeir gain or loss; second, in case Reid wishes to sell out, how much Day should pay
m for his interest.

*14.* A, B and C form a partnership, agreeing to share losses and gains equally.
t the close of the year it is found that A has to his credit $9200, that B has over-
awn his account $1500, and that C has overdrawn his account $2150. There being
› other resources or liabilities, determine an equitable dissolution.

*15.* A and B are doctors, and agree to combine their interests and share equally in the net amount made during a year. A owns the office which they use, and values the yearly rental at $750. B supplies the horses and carriages necessary to the business and pays $1000 for their care and keep; their value is $900. He charges 15% of this value for their use. A collects during the year $4225, and B $5006. A's incidental expenses connected with the business are $112.50, and B's are $212.75. Determine each one's share of the net earnings and how they shall settle.

*16.* A company engaged an agent to do business for them for four months at a salary of $175 a month. They shipped him merchandise amounting to $3920.50, and sent him also $920 cash. During the four months the agent purchased goods amounting to $2140.50. At the end of the time the agent's sales amounted to $3950.82, and the goods on hand amounted to $1005.26. Find gain or loss. Find also the amount due the company at the end of that time.

*17.* The firm of Platt & Son find upon closing their books a net loss of $8700. A statement of resources and liabilities shows Cash, $1800; Merchandise, $11000; Bills Receivable, $500; Accounts Receivable, $1500; Bills Payable, $4000; Accounts Payable, $7500. What was the net capital of Platt & Son at the opening of the season?

*18.* Henry Jones has his business incorporated. His assets are Cash, $9000; Real Estate, $15000; Mdse., $10000; Bills Receivable, $560; Accounts Receivable, $8950. He owes Bills Payable, $1200; and Accounts Payable, $3500. The business is incorporated under the title of the "Linen Manufacturing Company," with a capital of $200000, divided into 2000 shares at $100 each. Henry Jones takes 500 shares for his share; George Long takes 500 shares; William Day takes 500 shares; William Smith takes 400 shares; Leonard Hay receives 100 shares for his services as attorney. William Day buys his shares at par and pays one-half cash and note for balance. George Long buys his at 95 and pays cash for them. William Smith buys at 105, paying one-half cash and turns over real estate for balance. What does each pay for his interest in the corporation? What is the real value of each one's interest?

*19.* The firm of Sharp & Dull have just closed their books, which show a gain of $550. A statement of resources and liabilities shows the following: Cash, $8000; Merchandise, $13000; Real Estate, $5000; Bills Receivable, $3500; Accounts Receivable, $6750; Bills Payable, $8000; Accounts Payable, $15200. What was Sharp's investment, Dull having invested $2500?

*20.* A merchant is doing business and keeps his books by single entry. He began with Cash $10000. He bought Mdse. from A, $7000; B, $3000; C, $4000; D, $1000. Paid A $4000 and note $3000. Sold E $5000 and paid B in full. Sold F $7500, gave C on account $3000 and paid D in full. Sold G $5300. Received from E $4000. Sold H $4200. Received from F cash $5000 and note for $2500, and from G cash $3000 and note $1300. Received from H cash $1000 and note $1200. Wishing to change his books to double entry, he took an account of stock and made the change. Inventory, Mdse., $2500. What is the present worth of the business?

*21.* A and B are partners. A having $\frac{3}{4}$ interest and B $\frac{1}{4}$ interest in gains and sses. They have disposed of everything belonging to the business and have on nd $5000 cash. They wish to settle with each other. A's account shows a net edit of $6000, while B's account shows a net debit of $3500. What is the final id equitable settlement between them?

*22.* A Philadelphia firm opened a branch store in Bethlehem, Pa., and put J. M. ayth in charge. He is to receive 5% commission on all sales and incomes of the siness. He employs an assistant at a salary of $1500, which is paid by himself. he firm supplied the branch as follows: Merchandise, $15000; Cash, $5000. During e year Fayth bought for the branch $175325 and the sales were $222000. He ceived a consignment of goods invoiced at $1200, from Wilmington, Del., upon hich he paid freight $100 and storage $75. The consignment was disposed of at i advance of 10% above invoice value. A commission of 3% was received and proeds remitted. The running expenses of the business, rent, etc., outside of assistant's lary, were $2300; these come out of the business and have been paid. The Merchanse inventory is $4500, and there is $6500 cash on hand. What is the result of the siness? What has been Fayth's income from the business?

*23.* A and B form a copartnership, agreeing to share gains or losses in proportion average net investment. On January 1 each invests $6000. On February 1 A insts $2000 additional, and B withdraws $500. On March 1 A withdraws $750. On pril 1 C is admitted to the firm, and invests $10000. On May 1 B invests $1000 ldditional. On June 1 A withdraws $500. On July 1 D is admitted to the firm, and ivests $7500. On August 1 C withdraws $2500. On September 1 B withdraws $400. n October 1 D withdraws $500. On November 1 A withdraws $750, B withdraws 100, C withdraws $1500, and D withdraws $1000, leaving their investments equal r the remainder of the year.

On December 31 their books show the following resources and liabilities: Cash, 10000; Mdse., $19000; Bills Rec., $5000; Accts. Rec., $7650; Bills Pay., $5000; Rent ue by us, $1800; Accts. Pay., $4000. Allow interest at 6% on all investments id charge interest on all withdrawals. Find net gain of each, and find present orth of each partner.

# BUILDING AND LOAN ASSOCIATIONS

**909. Building and Loan Associations** are co-operative organizations which receive small deposits from members at stated periods and loan the capital to its members or to others on good security, principally mortgages. The primary object is (1) to encourage thrift and the purchase of homes, and (2) to afford a safe and profitable investment for small amounts of money.

NOTE.—The statutes of some states restrict loans to members.

**910.** The **Association** is composed of and controlled by the shareholders, who are made up of borrowers and non-borrowers, or investors.

**911.** The **authorized capital** is set forth in the charter, and the value of each share is usually $200.

**912.** The **shares** are usually issued in series at regular times fixed by the by-laws and terminated when a determined fixed value (usually $200) has been reached. Some associations have but one series.

**913.** The **dues** are regular instalments, usually $1 a month per share.

**914.** The **profits** accrue from interest, premiums, fines, etc.

**915.** The **periodical profits** of the Association are apportioned annually, semi-annually, quarterly, monthly, etc.

**916.** The **withdrawal value** of partially paid shares is the amount paid in and a portion of the accrued profits.

**917. Interest** and **Premiums** are charged by several plans, known as **gross, net** and **instalment.** (1) By the gross plan, the borrower receives the par value less premium, but pays interest on the par value; (2) by the net plan, the par value less premium, and pays interest on net amount; (3) by the instalment plan, par value of share and premium is paid in monthly instalments along with interest and dues.

ILLUSTRATION.—By plan 1, a buyer of one share of $200 at 15% premium receives the net sum of $185, and pays interest in advance on the gross sum of $200; by plan 2, a buyer pays interest on the net sum, but in advance as in plan 1; by plan 3, a buyer receives $200 and pays premium in monthly instalments when dues and interest are paid.

**918.** The Profits are **apportioned** among the series by a variety of methods, but the ones explained generally obtain.

**919. To apportion profits.**

EXAMPLE.—An association has issued six series of shares at intervals of 6 months. The number of shares in the series are respectively 53, 48, 110, 235, 191 and 125. The dues in each series are $1 per month per share. At the end of the third year from the date of the first series the entire profits are $1183.34. What is the value of a share in each series?

| Series No. | Total Payments | Number of Shares | Average Time Invested | $1 Invested for Equivalent of | Total Gain for Series | Gain Per Share | Present Value Per Share |
|---|---|---|---|---|---|---|---|
| 1 | $36 | 53 | 18.5 months | 35298 months | $281.61 | $5.31 | $41.31 |
| 2 | 30 | 48 | 15.5 months | 22320 months | 178.07 | 3.71 | 33.71 |
| 3 | 24 | 110 | 12.5 months | 33000 months | 263.27 | 2.39 | 26.39 |
| 4 | 18 | 235 | 9.5 months | 40185 months | 320.59 | 1.36 | 19.36 |
| 5 | 12 | 191 | 6.5 months | 14898 months | 118.86 | .62 | 12.62 |
| 6 | 6 | 125 | 3.5 months | 2625 months | 20.94 | .16 | 6.16 |
| | | | Total | 148326 months for investment of $1 | | | |

$\frac{35298}{148326}$ of $1183.34 = $281.61 or profits apportioned to Series No. 1

$\frac{22320}{148326}$ of $1183.34 = $178.07 or " " " " 2

Or, by proportion

$X$ : $1183.34 : : 33000 : 148326 = $263.27 Series 3
$X$ : $1183.34 : : 40185 : 148326 = $320.59 " 4
$X$ : $1183.34 : : 14898 : 148326 = $118.86 " 5
$X$ : $1183.34 : : 2625 : 148326 = $20.94 " 6

NOTE.—The average time of one share in any series is ½ of the sum of the time of the first and that of the last payment.

**920.** RULE.—*Multiply the total payment in a series by the number of shares in the series, and that result by the average time the money was invested; then apportion the gain. Divide the gain for each series by the number of shares in the series, and the gain per share is obtained.*

**921.** By the plan commonly called the **"power plan"** the fractional parts of a month are omitted, and the pro rata gain is determined by casting out the common factors of the whole months of each series, squaring the remaining factors and multiplying by the number of shares. The gain, divided by the total thus obtained, will give the common multiplier. This gives a business, though not exact, result.

EXAMPLE.—An association has issued three series of shares, 30, 40 and 50 respectively, at intervals of one year. The gain for the third year is $120. Apportion it among the different series.

EXPLANATION.—An investment of $36 for 18 mos. is equivalent to $1 for 648 mos.
" " 24 " 12 " " 1 " 288 "
" " 12 " 6 " " 1 " 72 "

Therefore, the first series should receive proportionately 9 times and the second series 4 times as much as the last series.

| Payments | Powers | | Shares | | Total | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $36 | 9 | × | 30 | = | $270 | × | .25 | = | $67.50 |
| 24 | 4 | × | 40 | = | 160 | × | .25 | = | $40.00 |
| 12 | 1 | × | 50 | = | 50 | × | .25 | = | $12.50 |
| | | | | | 480 | | | | |

Total Gain $120 ÷ $480 = $ .25

$67.50 ÷ 30 shares = $2.25 gain per share on 1st series
40.00 ÷ 40 " = 1.00 " " " 2d "
12.50 ÷ 50 " = .25 " " " 3d "

### WRITTEN PROBLEMS

**922.** *1.* A building association has issued a new series of shares March 1st of each year for 4 years. In the first series there are 281 shares; in the second, 404 shares; in the third, 449 shares; and in the fourth, 612 shares. Dues in all series are \$1 per month per share. At the end of the fourth year the total profits are \$5680.98. What is the value of a share in each series at that time?

*2.* I have 10 shares in 4th series of a building and loan association which has been running 3 years, series being issued April and October of each year. Dues are \$1 per month per share. My 10 shares are valued at \$191.34; what is the value of a share in each of the other series at the same time?

NOTE.—Apportion profits by both methods.

**923. To find the rate of interest received on an investment.**

EXAMPLE.—I have paid dues on Building & Loan shares for $4\frac{1}{2}$ years; the shares are then worth \$64.10 each. What interest am I receiving on my money?

\$64.10 present value
\$54.00 dues paid in
\$10.10 earnings per share
$\$54 \times .01 \times 27\frac{1}{2}$ months $= \$1.23\frac{3}{4}$ int. at 1%
\$1.2375)\$10.1000
.0816, or 8.16%

EXPLANATION.—Dues paid, \$54 per share. Present value, \$64.10; interest, or earnings, on my money equals \$10.10. The average time is $27\frac{1}{2}$ months; the payments were \$54 per share. At 1 per cent. the earnings on \$1 for $27\frac{1}{2}$ months equals $2\frac{7}{4}$ cents, and on \$54 the earnings would be 54 times $2\frac{7}{4}$ cents or $\$1.23\frac{3}{4}$, and to produce \$10.10 the rate is as much as $\$1.23\frac{3}{4}$ is contained into \$10.10, or 8.16 per cent.

**924.** RULE.—*Divide the earnings per share by the earnings for the average time at 1 per cent. on the paid-in value.*

### WRITTEN PROBLEMS

**925.** *1.* A shareholder has paid dues of \$1 per month per share for 9 years, when his shares are worth \$151.65 each. What has been his total earnings for the 9 years and what rate per annum is he getting on his investment?

*2.* After paying dues 5 years and 8 months at \$1 per month, my value per share is \$73.47; my sister has paid dues for the same time and rate and her value per share is \$78.55. How much more per cent. does her investment pay per annum than mine?

**926. To find the withdrawal value.**

EXAMPLE.—Baker owns 10 shares in a Building & Loan Association, and has paid dues of $1 per month per share for 4 years and 8 months, when he wishes to withdraw. If he is allowed profits at 4% per annum, to what amount is he entitled?

4 years 8 months are 56 months
$10 per month for 56 months are $560 paid-in dues

$$\frac{\$560 \times .04 \times 28\frac{1}{2} \text{ months}}{12} = \$53.20 \text{ profits}$$

$560 + $53.20 = $613.20 withdrawal value

EXPLANATION.—The first dues $10 have been earning interest for 56 months at 4 per cent.; the second payment, for 55 months; the third, for 54 months; the last, for 1 month; this would be equal to each payment of $10 earning for an average $\frac{56-1}{2}$ or $28\frac{1}{2}$ months or $2\frac{1}{4}$ years. There were 56 payments, or a total of $560 earning at the rate of 4 per cent. for $2\frac{1}{4}$ years, or earning $53.20. This earning plus the $560 paid in makes the withdrawal value $613.20, or $61.32 per share.

927. RULE.—*Find the interest on the total paid-in dues for the average number of years it is invested, and by the rate per cent. allowed; add this to the paid value to determine the withdrawal value.*

**WRITTEN PROBLEMS**

928. *1.* B desires to withdraw his 25 shares from his association after having regularly paid his dues of $1 per month per share for 10 years. If he is entitled to 6% interest, but is charged with $2.80 unpaid fines, what sum will he receive?

*2.* Jones, after 4 years, is unable to continue his payments of $1 per month per share on 35 shares of Building and Loan, and withdraws therefrom. He is allowed interest at $3\frac{1}{2}$ per cent. per annum. If the association had been making 7 per cent. interest, how much does he lose by withdrawing?

*3.* An association having issued 3 series of shares at intervals of 3 years has 250 shares in 1st series; 325 in 2d series, and 290 in 3d series. When the 3d series is 3 years old the profits are $7865. A has 8 shares in the 1st series and is entitled to 80% of the profits upon withdrawing between the 8th and 10th years; B has 12 shares in the 2d series and is entitled to 70% of the profits upon withdrawal between the 5th and 8th years; C has 6 shares in the 3d series and is entitled to $\frac{2}{3}$ of the earnings by withdrawing between the 2d and 5th years; if each were to withdraw when C's shares have run 3 years, what is the withdrawal value of each man's shares?

**929. To find the cost of a loan.**

EXAMPLE.—A man having $500 wishes to buy a house costing $3500, and secures the balance of the money from a Building & Loan Association; if the series matures in 11 years, 6 months, what per cent. interest has he paid on his loan? Dues $1 per month per share, interest $1 per share per month.

EXPLANATION.—Total cost $276 × 15 = $4140

Loan = $3000

Interest = $1140

$3000 × 138 months = $414000, value received for 1 month

$138 × 15 × 69½ (average months) = $143865, value paid-in dues for 1 month

Net value received $270135 for 1 month

Interest at 1% = $2701.35 for 1 year or $225.1125 for 1 month

1140 ÷ 225.1125 = .0506 or 5.06+%

**930.** RULE.—*Divide the interest on the net value at 1% for 1 year into the total interest paid.*

### WRITTEN PROBLEMS

**931.** *1.* I secure the value of 35 shares from a Building and Loan Association, pay regular dues of $1 per month, and $1 per month interest; if the series matures in 142 months, what rate per annum has the loan cost me?

*2.* I rent a house at $30 a month and at the end of 10 years buy it for $4000; Gray buys a house for $4000 paying $400 cash and borrowing the remainder from building and loan which matures in 10 years. If his taxes are $45 per year, payable at the beginning of the year, who paid the more and how much in 10 years?

*3.* A borrower secured the par value of 20 shares of Building and Loan Association, paying regular dues and $1 per month interest per share. At the end of 130 months the series matured. What per cent. has the loan cost him?

*4.* Wishing to borrow $3000 on a second mortgage, I subscribe for Building and Loan shares, and must pay 20 cents per share premium, dues of $1 per month and interest at 6%. If the series matures in 130 months, what is the full cost of the loan to me?

## GENERAL INVESTMENTS

282. *1.* A $15000 property was purchased on the following terms: 15% cash, 55% on first mortgage at 5.3% interest, payable annually, the remainder on second mortgage at 6% interest; the second mortgage payable in 10 semi-annual payments. If taxes, insurance, etc., total $185 per year, what is the total outlay the first year? The third year?

*2.* After paying the second mortgage, in Problem 1, how long will it require to pay off the first mortgage, if $1300 per annum be used to pay the carrying cost of the property and mortgage?

*3.* Which would be the cheaper—to take 20 shares of Building and Loan stock which matures in 138 months, borrowing on these from the association, and purchasing a $4500 property; or to buy a $4500 property, paying $500 in cash and borrowing the balance on a 5% mortgage and creating a sinking fund out of your income each year, invested at 3% to pay off mortgage at the end of 11½ years?

*4.* January 16th, bought at 95 and interest five ($1000) New York State Railways 50-year First Consolidated Mortgage Gold 4½% Bonds Series "A," dated November 1, 1912, interest payable semiannually redeemable at option of corporation any interest date on 60 days' notice at 105 and interest, brokerage ⅛%. First explain the issue as to security and designation; second, find the cost of bonds and rate of interest on your investment; third, redemption price January 1, 1921.

*5.* A property valued at $175000 was insured in 4 companies as follows: Company A, for $20000 at ¾% premium; company B, for $35000 at 1¼% premium; company C, for $37425 at 1½% premium; company D, for $15875 at ⅞% premium; all policies contained the 80% co-insurance clause. The property was damaged to the amount of $40000. If money is considered worth 5%, what was each company's net loss, if they held the insurance 9 years before damage occurred?

*6.* A owns 50 shares of preferred stock and 35 shares of common stock in a traction company. A new company is formed in which the preferred stockholders are privileged to convert ⅘ of their preferred stock into bonds equal to the par value of the preferred stock thus converted upon (1) the payment of $24 per share for each share converted; and (2) upon exchange of remaining stock for new 5% preferred. The common stockholders can convert ⅘ of the common stock into bonds equal to par of common stock upon (1) payment of $50 per share; and (2) upon exchange of remaining stock for new common stock. The par value of preferred and common in each company $100, and the bonds $500 and $1000. In each case, A buys a fractional share of common and preferred stock for bond conversion.

Find the value of bonds he should receive; amount of preferred and common stock in the new company, and the amount of extra money he pays out for the conversion.

# INVOLUTION AND EVOLUTION

## INVOLUTION

**933. Involution** is the process of raising a number to any required power. Any power of a number is obtained by using the number as a factor as many times as are indicated by the exponent, which is a small figure written above and to the right of the number. As,

$3^1=3$, the first power of 3

$3^2=3\times3=9$, the second power, or square, of 3

$3^3=3\times3\times3=27$, the third power, or cube of 3

$3^4=3\times3\times3\times3=81$, the fourth power of 3

**934.** The process of raising numbers of two or more figures to a required power may be formulated into a rule by separating the number into its component parts and then indicating the multiplication instead of actually multiplying; thus,

$$\begin{array}{rl} & 37=30+7 \\ & 37=30+7 \\ \hline & (30\times7)+7^2 \\ 30^2+ & (30\times7) \\ \hline \text{The square} = & 30^2+2(30\times7)+7^2 \end{array}$$

Or,

$$\begin{array}{r} t+u \\ t+u \\ \hline (t\times u)+u^2 \\ t^2+(t\times u) \\ \hline t^2+2(t\times u)+u^2 \end{array}$$

**935.** From this we derive the following principles:

*1.* The square of a number of two figures (t+u) equals *tens*$^2$ + 2 times *tens* $\times$ *units* + *units*$^2$.

*2.* The square of a number of three figures (h+t+u) equals *hundreds*$^2$ + 2 times *hundreds* $\times$ *tens* + *tens*$^2$ + 2 times (*hundreds* + *tens*) $\times$ *units* + *units*$^2$.

NOTE 1.—Higher powers may be obtained by combining the lower; thus, $4^4=4^2\times4^2$; $4^5=4^3\times4^2$; $4^6=4^2\times4^2$, etc.

NOTE 2.—A fraction is raised to a required power by raising numerator and denominator separately; as $(\frac{2}{3})^2=\frac{4}{9}$, $(\frac{2}{3})^4=\frac{16}{81}$, etc.

### WRITTEN EXERCISE

**936.** Find the value of

| | | | |
|---|---|---|---|
| *1.* $14^2$ | *8.* $(\frac{3}{4})^4$ | *15.* $.09^2$ | *22.* $1.672^4$ |
| *2.* $22^3$ | *9.* $(5\frac{1}{2})^3$ | *16.* $.67^4$ | *23.* $(2.76\frac{1}{2})^3$ |
| *3.* $46^2$ | *10.* $(6\frac{3}{4})^4$ | *17.* $.003^3$ | *24.* $9.06^4$ |
| *4.* $29^3$ | *11.* $(3\frac{1}{2})^7$ | *18.* $(.17\frac{1}{4})^5$ | *25.* $91.67^3$ |
| *5.* $16^4$ | *12.* $(22\frac{1}{2})^5$ | *19.* $.27^4$ | *26.* $22.05^2$ |
| *6.* $17^5$ | *13.* $(19\frac{1}{3})^3$ | *20.* $.086^6$ | *27.* $(18\frac{3}{4})^5$ |
| *7.* $927^3$ | *14.* $(26\frac{3}{8})^2$ | *21.* $.0721^3$ | *28.* $(66\frac{2}{3})^3$ |

*29.* Cube the three highest digits and cube the sum of those powers.

*30.* Find the value of $(7^2+8^3+4^2+3^4+5^6)^3$.

# EVOLUTION

**937. Evolution** is the process of finding the root of a number.

**938.** A **root** of a number is one of its equal factors. The root is indicated by the use of the symbol $\sqrt{\ }$, called the *radical sign*.

NOTE.—The square root of 36 is 6 ($\sqrt{36}=6$), because $6\times6=36$. The cube root of 216 is 6 ($\sqrt[3]{216}=6$), because $6\times6\times6=216$.

**939.** When the radical sign is used alone, it indicates the *square* root. When any other root is required, the figure indicating the root is written in the angle of the radical sign. Thus, $\sqrt{\ }$ indicates square root, $\sqrt[3]{\ }$ indicates cube root, $\sqrt[4]{\ }$ indicates fourth root, etc.

**940.** A **perfect power** is a number of which the required root can be found exactly.

**941.** An **imperfect power** is a number whose root cannot be found exactly.

### Square Root

**942. Observe the following numbers and their squares.**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Numbers | 1, | 3, | 5, | 9, | 10, | 19, | 34, | 99, | 100, | 500 |
| Squares | 1, | 9, | 25, | 81, | 100, | 361, | 1156, | 9801, | 10000, | 250000 |

From this we learn

1. That the square of a number of one figure contains one or two figures.
2. That the square of a number of two figures contains three or four figures.
3. In general, the square of any number contains twice as many figures as the number, or twice as many less one.
4. That if a given number be separated into groups of two figures each, each group or partial group will be represented by a figure in the root.

**943. To extract the square root of a whole number.**

EXAMPLE.—Find the square root of 225.

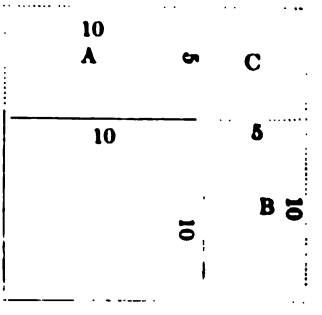
$$\begin{array}{r|l} 2'25 & 10 \\ 100 & 5 \\ \hline 125 & 15 \text{ Result} \\ \hline 125 & \end{array}$$

$10^2 = 100$

$10\times2=20$

$20)125$

$5)25$




EXPLANATION.—Separate the number into periods of two figures each. This determines that the root contains 2 figures. The first or left-hand period is 2, and the greatest square contained in it is 1 ten or 10 units. Place 10 in the root-place at the right, and subtract the square of 10 or 100 square units from 225, leaving 125 square units.

The largest part of these 125 square units is in rectangles A and B, each of which is ten units long, or together $2\times10$ or 20. This divided into 125 will give the width, 5 units.

The remaining square, C, is as large each way as the width of rectangles A and B, 5 units. Add this 5 units to 20 units, and we have the length of A, B, and C, 25 units. Multiply this length 25, by the width 5, and we have the area, 125 square units and no remainder.

The square root is $10+5$ or 15 units.

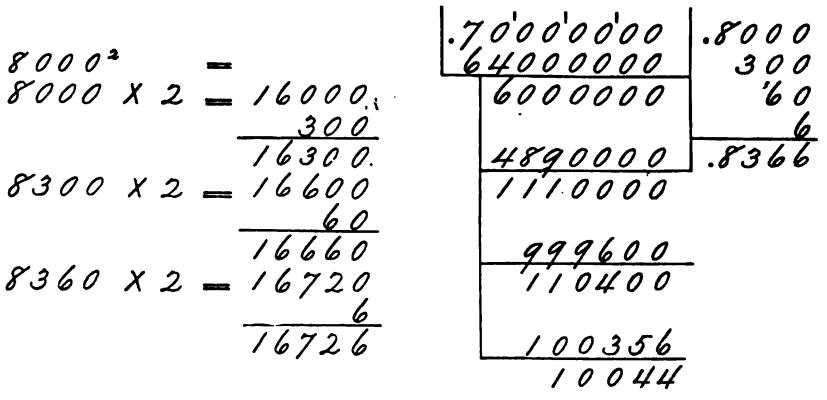
**944.** RULE.—*Beginning at units, separate the number into two-figure groups. Each period represents a figure in the square root.*

*Find the largest number whose square is contained in the first or left-hand period; write it in the first root-place, and put ciphers in the remaining root-places. Subtract the square of the entire root from the given number for a new dividend.*

*For a trial divisor, multiply the root by 2; divide the product into the dividend and write the quotient for the next figure of the root. Add the second part of the root to the trial divisor, and multiply the sum by the root just found. Subtract for the new dividend. Proceed in like manner until all the periods have been disposed of.*

**945. To extract the square root of a decimal.**

EXAMPLE.—Extract the square root of .7 to four decimal places.




**946.** RULE.—*Separate into two-figure periods, beginning at the decimal point, and supply ciphers to make the number of periods desired.*

**947. To extract the square root of a common fraction.**

EXAMPLE.—What is the square root of $1\frac{3}{4}\frac{1}{4}$?

$$\sqrt{1\tfrac{3}{4}\tfrac{1}{4}} = 1\tfrac{1}{2}$$

**948.** RULE.—*Take the square root of the numerator and of the denominator; or, if this cannot be done, reduce to a decimal and extract the root as above.*

### WRITTEN EXERCISE

**949.** Extract the square root of

| | | |
|---|---|---|
| *1.* 1369 | *7.* 74926 | *13.* .768 |
| *2.* 3136 | *8.* $\frac{1}{2}$ (to 3 places) | *14.* 43.1 |
| *3.* 277729 | *9.* $\frac{3}{4}$ (to 3 places) | *15.* 95.267 |
| *4.* 9339136 | *10.* 72645 | *16.* 43.14682 |
| *5.* $\frac{5}{8}$ (to 3 places) | *11.* 669847 | *17.* 9.00746 |
| *6.* 1429785 | *12.* 224.85 | *18.* 10.001 |

### WRITTEN PROBLEMS

**960.** *1* A square piece of ground contains 12 acres. What is the distance around it in yards?

*2* A carpenter has 9 boards, each 16 feet long and 18 inches wide. What is the largest square platform he can make of them, allowing nothing for waste?

*3* A rectangular field contains 9 acres. Its length is three times its width. Find the distance around it.

*4* A rectangular field is 10.7 rods long, 12.47 rods wide. What is the side of a square field that contains the same area?

*5* If it cost $120 to inclose a farm 104 rods long, 48 rods wide, how much less will it cost to inclose a square farm of the same area with the same kind of fence?

### Similar Figures

**961. Similar figures** are those which have the same form but which differ from each other in magnitude. Circles, squares, right angled triangles, etc., are similar figures.

**962.** RULE. *The areas of all similar surfaces are to each other as the squares of their like dimensions.*

*Like dimensions of similar surfaces are to each other as the square roots of their areas.*

### WRITTEN PROBLEMS

**963.** *1* The area of a circle, whose diameter is 5 rods, is 19.635 sq. rd. What is the diameter of a circle whose area is 40 sq. rd.?

*2* The area of a circle, whose diameter is 42 feet, is 1385.44 sq. ft. What is the area of a circle whose diameter is 14 feet?

*3* If a pipe 2 inches in diameter will empty a cistern in 9 hours, in what time will six pipes, each one-half inch in diameter, empty it?

*4* If a horse tied to a post by a rope 20 feet long can graze upon a certain area, how long should the rope be to allow him to graze upon an area four times as great?

*5* If a pipe 6 inches in diameter is 5 hours in running off a certain quantity of water, in what time will 3 pipes, each 3 inches in diameter, discharge the same quantity?

*6* How many pipes one-half inch in diameter are equal to one pipe 3 inches in diameter?

*7* If it cost $45 to paint a house 45 ft. long, what ought it cost to paint a similar house 25 ft. long?

*8* If $52 be paid to fence a field containing 12 acres, what should be paid for fencing a similar field with the same kind of fence, if its area is 30 acres?

## CUBE ROOT

**954.** The cube root of a number is one of its three equal factors. Thus, a cubic yard equals $3\times3\times3$, or 27 cubic feet; and since 3 is one of the three equal factors of 27, it is the cube root.

The cubes of 2, 4, 5, 7, 9, 10, 25, 3.5, 5.4
are 8, 64, 125, 343, 729, 1000, 15625, 42.875, 157.464

From this it is evident

1. That the cube of a number of a single figure will contain one, two or three figures.
2. That the cube of a number of two figures will contain four, five or six figures.
3. That the cube of any number will contain three times as many figures as the number itself, or three times as many less one or two.
4. That if a number be separated into three-figure groups, beginning at the right, or, in case of decimals, at the decimal point, the number of figures in the root will equal the number of periods or partial periods.

**955. To find the cube root of a number.**

EXAMPLE 1.—Extract the cube root of 1728.

EXPLANATION.—By separating the number into three-figure groups, we see that the cube root will contain two figures (tens and units). See figure on opposite page.

| | | | |
|---|---|---|---|
| | | 1 728 | 10 |
| $10^3$ | = | 1 000 | 2 |
| | | 728 | 12 cube root |
| $3\times(10)^2=300$ | | | |
| $3\times10\times2=60$ | | | |
| $2^2=4$ | 364 | 728 | |

EXAMPLE 2.—Extract the cube root of 78402.752.

EXPLANATION.—By commencing at the decimal point and separating the number into three-figure groups, we see that the cube root will contain three figures (tens, units and tenths).

| | | | |
|---|---|---|---|
| | | 78'402.752' | 40.0 |
| $40.0^3$ | = | 64 000.000 | 2.0 |
| $40.0^2 \times 3$ | = 4800.00 | 14 402.752 | .8 |
| $40.0\times3\times2.0$ | = 240.00 | | 42.8 cube root |
| $2.0^2$ | = 4.00 | | |
| | 5044.00 | 10 088.000 | |
| | | 4 314.752 | |
| $42.0^2 \times 3$ | = 5292.00 | | |
| $42.0\times3\times.8$ | = 100.80 | | |
| $.8^2$ | = .64 | | |
| | 5393.44 | 4 314.752 | |

The first period is 78 and the largest cube in 78 is 64, whose cube root is 4. This is represented by the cube in Figure 1. Write 4 in the tens place of the root and place ciphers in the remaining places, etc. See illustrated example, opposite page.

FIGURE 1—$t^3$

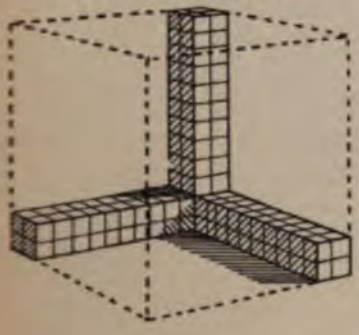
The first or left-hand period is 1, and the largest cube in 1 is 1. This is represented in Figure 1 by the cube. Write 1 in the tens place of the root and place a cipher in the remaining root-place.

This cube, having 10 units (1 ten) for each dimension, contains $(10)^3$ or 1000 cubic units ($t^3$). This subtracted from 1728 cubic units leaves 728 cubic units, the contents of Figure 2.

FIGURE 2—$3t^2 \times u$

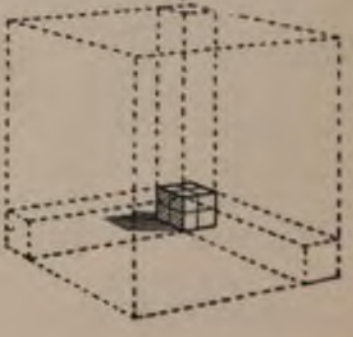
The largest part of Figure 2 is three square slabs, each 10 units square. The area of each is $10 \times 10$ ($t^2$), and of the three it is $3 \times (10 \times 10)$ or $300 = 3(t)^2$. Place this area to the left and use it as a trial divisor. Dividing 300 into 728, the approximate thickness of the slabs is obtained, which is 2 units. Place 2 under the former root in units place.

FIGURE 3—$3 (t \times u^2)$



After removing these three square slabs, there remain three rectangular slabs and a cube, Figure 3. Each slab is as long as the square slabs (10 units), and as wide and thick as the thickness of the square slabs, 2 units. The area of each rectangular slab is $10 \times 2$, and of the three it is $3 \times 10 \times 2$ or 60 ($3t \times u$). Place 60 under the trial divisor.

FIGURE 4—$u^3$



After removing the rectangular slabs, the small cube remains, Figure 4. Each dimension of this cube is the same as the thickness of the slabs, 2 units. The area of the cube is $2 \times 2$ or 4 ($u^2$). Place 4 under the former partial divisors.

The sum of these areas (364) is the true divisor. Multiplying this by the root-figure 2, gives 728. Since there is no remainder, our root is complete. Adding the partial roots we have 12, which is the cube root of 1728.

**956.** RULE.—*Separate the number into three-figure groups, counting left and right from units.*

*Determine the root of the largest cube in the left-hand period and write it for the first figure of the root. Place ciphers in the remaining root-places. Cube this entire root, and subtract from given number for a new dividend.*

*For a trial divisor, square the root and multiply by 3; determine how often it is contained in the new dividend, and place this number under the root already found.*

*To the trial divisor add 3 times the product of the first part of the root by the second part of the root, and to this result add the square of the second part of the root. Multiply this complete divisor by the second part of the root and subtract the product from the dividend.*

*For the next trial divisor, square the sum of the two partial roots and multiply by three; divide this product into the new dividend; the quotient is the next root figure.*

*To the trial divisor add three times the product of the third part of the root multiplied by the sum of the other two parts, and to this add the square of the third part.*

*Multiply this complete divisor by the third part of the root, and subtract for the new dividend.*

*If there are more root figures to be found, for a trial divisor square the sum of the partial roots and multiply by 3, and proceed as before.*

NOTE.—If there should be a remainder after all root places are filled, annex periods of ciphers, placing one cipher in the root for each period of 3 placed in the dividend.

The cube root of a fraction is the cube root of the numerator and of the denominator.

### WRITTEN EXERCISE

**957.** Find the cube root of

| | | | |
|---|---|---|---|
| *1.* 91125 | *5.* 10218313 | *9.* 633.839779 | *13.* $\frac{729}{8875}$ |
| *2.* 24389 | *6.* 131096512 | *10.* .997002999 | *14.* $\frac{1728}{21952}$ |
| *3.* 274.625 | *7.* 187149.248 | *11.* 118805247296 | *15.* $16\frac{656}{1331}$ |
| *4.* 103823 | *8.* 277167808 | *12.* 4.080659192 | *16.* $30\frac{1728}{21952}$ |

### WRITTEN PROBLEMS

**958.** *1.* A cubical box contains 64000 cubic feet. What are its dimensions?

*2.* A cellar 24 feet long and 12 feet wide was excavated to a depth of 6 feet. What would be the depth of a cellar of equal capacity if it were cubical?

*3.* A cubical tank holds 108.4156 barrels of water. What is the cost of lining the sides and bottom with tin costing 8 cents a square foot?

*4.* An open cubical bin holds 3810.24 bushels of grain. At $34 per M find cost of the 2-inch lumber required to make the bin.

*5.* What is the entire surface of a cube whose contents are 91125 cubic feet?

*6.* At 6 cents a square foot, what will it cost to plaster the sides and bottom of a cubical reservoir which contains 444.675 barrels of water?

*7.* What is the length of a cubical block containing 2 cubic feet, 1457 cubic inches?

*8.* The length of a square stick of timber is 32 times its width or thickness. If it contains $13\frac{1}{2}$ cubic feet, what are its dimensions?

*9.* A bin in a granary is 3 times as long as deep and $1\frac{1}{4}$ times as wide as deep. If it holds 270 bushels, find its dimensions.

*10.* A cubical pile of wood contains 500 cords. What decimal part of an acre does it cover?

### Similar Solids

**959. Similar solids** are those which have the same shape but which differ from each other in volume.

**960.** *The volumes of similar solids are to each other as the cubes of their like dimensions.*

**961.** *Like dimensions of similar solids are to each other as the cube roots of their volumes.*

### WRITTEN PROBLEMS

**962.** *1.* If a ball 3 inches in diameter weighs 12 pounds, how much will a ball 6 inches in diameter weigh?

*2.* If a haystack 15 feet in diameter contains 20 tons, how much hay should be in a similar stack 25 feet in diameter?

*3.* If a cubical bin 10 feet long contains 125 bushels, how much grain should a cubical bin 5 feet long hold?

*4.* The clay in a cubical bank 10 feet long contains 50 loads. How long should a cubical clay bank be to contain 120 loads?

*5.* The contents of a cubical box 5 feet long weigh 120 pounds, what should be the length of a similar cubical box to contain 700 pounds?

## ARITHMETICAL PROGRESSION

**963.** When any two consecutive numbers continually increases or decreases in a series by a common difference the numbers form an **arithmetical series.** If the numbers become greater from left to right, they form an **ascending** series; if they become less, a **descending** series.

**964.** The difference between any two consecutive numbers is the **common difference.**

**965.** The quantities considered are the **first term** (a), the **common difference** (d), the **number of terms** (n), the **last term** (l), and the **sum of the terms** (S).

**966. To find the last term of an arithmetical progression.**

EXAMPLE.—Given $a=10$, $d=5$, $n=20$, find l.

EXPLANATION.—First term $=10$; second term $=10+5$; third term $=10+5+5$; fourth term $=10+5+5+5$. Then the 20th term $=10+(19\times5)$ or 105, for in any term the common difference is used one time less than the number of that term. Substituting letters for terms used, the following formula is deduced:

FORMULA.—$l=a+(n-1)d$.

Substituting figures for the letters used, $l=10+(20-1)\ 5$, or $l=105$.

NOTE.—By the use of this formula, or the following ones, the first term or the common difference or the number of terms may be found by substitution.

$$a=l-(n-1)d$$

$$d=\frac{l-a}{n-1}$$

$$n=\frac{l-a}{d}+1$$

**967. To find the sum of an arithmetical series.**

EXAMPLE.—What is the sum of seven terms of the series 5, 8, 11, 14, etc.?

EXPLANATION.—Writing the series as follows:

Sum $=5+8+11+14+17+20+23$ : in reverse order
Sum $=23+20+17+14+11+8+5$

Adding $2S=28+28+28+28+28+28+28$; or, twice the sum $=28\times7$. Now 28 is the sum of first term (a) + last term (l), and 7 is the number of terms (n); hence the formula:

$$S=\frac{l+a}{2}\times n.$$

Substituting figures for the letters used, $S=\frac{23+5}{2}\times7=98$.

**968.** RULE.—*The last term of an arithmetical series equals the first term increased by the product of the common difference multiplied by one less than the number of terms.*

*The sum of an arithmetical series equals the sum of the first and last terms multiplied by one-half the number of terms.*

### WRITTEN PROBLEMS

**969.** *1.* Given a = 7, d = 4, and n = 10; find l and S.

*2.* Given n = 15, a = $2\frac{1}{2}$, and L = $44\frac{1}{2}$; find d.

*3.* Given l = $90\frac{3}{4}$, n = 22, d = $4\frac{1}{4}$; find a and S.

*4.* Given a = 10, l = 94, d = $3\frac{1}{2}$; find n.

*5.* Find the 8th term of the series $4\frac{1}{2}$, $6\frac{3}{4}$, $9\frac{1}{8}$, etc.

*6.* Find the sum of 12 terms of the series 2, 5, 8, 11, etc.

*7.* A boy worked 6 weeks, 6 days each, receiving 10 cents the first day, and an increase of 5 cents each day. What were his wages the last day? His total wages?

*8.* A man paid his debt in 15 years by paying $30 the first year, $38 the second year, $46 the third year and so on; what was the debt?

*9.* A merchant added $250 annually to his capital for 20 years, and then had $6000. What was his original capital?

*10.* Fourteen properties were valued each $225 higher than the preceding one. The eighth property is valued at $4575. What is the total valuation?

*11.* A man travels 15 miles the first day and increased the distance 10 miles each succeeding day. The last day he traveled 85 miles. How many days did he travel? How far?

*12.* A clerk receives a salary of $15 the first month, $25 the second month, $35 the third month and so on for two years. What was his salary the last month, and the average monthly salary?

*13.* A man owing $3240 paid it in a year by paying $50 the first month, and increasing each monthly payment by a specified sum. What was the sum?

*14.* A boy, who earns 15 cents the first day, has expenses averaging 95 cents daily; if his wages increase 10 cents daily, what will be his total net earnings in 40 days? On which day will he be most in debt?

*15.* Fifteen horses were purchased whose values are in arithmetical progression. The fifth horse cost $149, the twelfth cost $191. What was the total cost of the horses purchased?

*16.* Find the sum of 20 consecutive odd numbers, commencing with 19.

*17.* In selling a Persian rug a dealer numbers tickets from 1 to 200, and obtains 200 persons to draw each a ticket. Each person pays for his ticket as many cents as the number on the ticket he draws, and the owner is then determined by lot. How much does he receive for his rug?

# GEOMETRICAL PROGRESSION

**970.** A **geometrical series** is one in which each number after the first is derived by multiplying or dividing the preceding one by a given number.

**971.** The multiplier or the divisor is called the **ratio.**

**972.** When the numbers increase from left to right, the series is **ascending,** as 2, 6, 18, 54; when they decrease, it is **descending,** as 108, 36, 12, 4.

**973.** The quantities considered are the **first term** (a), the **ratio** (r), the **number of terms** (n), the **last term** (l), and the **sum of the terms** (S).

**974. To find the last term of a geometrical progression.**

EXAMPLE.—What is the tenth term of the series 2, 10, 50, etc.?

EXPLANATION.—First term = 2; second term = $2\times5$; third term = $2\times5\times5$; fourth term = $2\times5\times5\times5$; then the tenth term = $2\times5^9$ or 3906250, for in any term the ratio is used one time less than the number of that term. By substituting letters for the terms used, the following formula is deduced:

$$l = r^{n-1}\times a$$

From this formula the following formulas are deduced:

$$a = \frac{l}{r^{n-1}}$$

$$r = \sqrt[n-1]{\frac{l}{a}}$$

NOTE.—The n may be found by observing how many times the ratio may be divided successively into $\frac{rl}{a}$.

**975. To find the sum of a geometrical series.**

EXAMPLE.—What is the sum of 7 terms of the series 2, 8, 32, etc.?

EXPLANATION.—The sum (S) = 2+8+32+128+512+2048+8192 (1)

4 (ratio) sum (4S) = 8+32+128+512+2048+8192+32768 (2)

Subtracting (1) from (2), 3 S = 32768 − 2, or 32766, and S equals ⅓ of 32766, or 10922.

32766 is the last term, 4 is the ratio, and 3 (the divisor) is one less than the ratio. Hence the formula:

$$S = \frac{l\times r - a}{r-1}$$

Substituting figures for the letters used, $l = \frac{8192\times4-2}{4-1}$, or 10922

**976.** RULE.—*The last term equals the first term multiplied by the ratio raised to a power one less than the number of terms.*

*The sum of a geometrical series equals the last term times the ratio and this product divided by the ratio less one.*

### WRITTEN PROBLEMS

**977.** *1.* Given a = 6, r = 6, n = 5; find last term.

*2.* Given a = 4, n = 7, l = 2916; find the ratio.

*3.* Given l = 640, n = 8, r = 2; find the first term.

*4.* Given l = 6576, r = 6, a = 6; find the number of terms.

*5.* Given a = 8, r = 4, n = 4; find the last term and the sum of all the terms.

*6.* If $\frac{1}{4}$ of the contents of a vat evaporated each day for 8 days, what part of the original contents remains?

*7.* A milk dealer received $\frac{1}{2}$ cent for the first pint of milk sold, and trebled the price on each pint he sold. What did he receive for a gallon sold by the pint?

*8.* A man having $1000 to pay at the end of a year, began by paying $5 the first month, $10 the second month, $20 the third month and so on. At the end of the year has he overpaid or does he still owe and how much?

*9.* A debt of $93410 was canceled by paying $10 the first year, and trebling each following yearly payment; how many years did it require to pay the debt?

*10.* The cost of the 20th cow was $7864.32. If the first cow cost $1\frac{1}{2}$ cents and the prices were in geometrical progression, what was the ratio?

*11.* A merchant who began business with $6000 capital, increased the capital annually by $\frac{1}{4}$ of itself. What was his capital at the end of the sixth year?

*12.* A man deposited $1000 in a savings bank that pays 4% compounded semi-annually. If the depositor withdrew no money from the bank for six years, what was the amount he had on deposit?

*13.* If a rubber ball, falling upon a marble floor, rebounds to $\frac{1}{3}$ the height from which it has fallen, then falls again and rebounds $\frac{1}{3}$, etc., how far will it move before coming to rest, if tossed to a height of 50 feet?

NOTE.—Last term = 0.

# MENSURATION

**978. Mensuration** is the process of finding the lengths of lines, the areas of surfaces and the volumes of solids.

**979.** A **line** has *length* without breadth or thickness.

**980.** A **surface** has *length* and *breadth*, without thickness. A plane surface, or plane figure, is one that is flat or level.

**981.** A **solid** has *length*, *breadth* and *thickness*, or depth.

**982.** Lines are either *straight* or *curved*.

**983.** A **straight line** is the shortest distance between points.

**984.** A **curved line** is one that changes its direction at every point.

**985. Parallel lines** have the same direction and are equidistant.

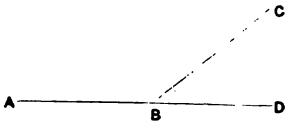
**986.** A line is **perpendicular** to another when the two angles thus formed are equal.

**987.** An **angle** is the difference in direction of two intersecting lines, or the opening between them.

**988.** When one straight line intersects another so as to make the four angles so formed equal, these angles are called **right angles.**

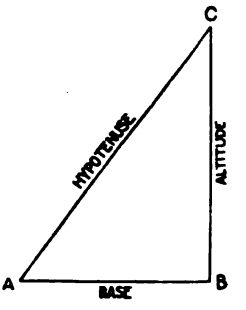
**989.** An **acute angle** is an angle that is less than a right angle; as, CBD.

**990.** An **obtuse angle** is one that is greater than a right angle; as, ABC. Acute or obtuse angles may be called **oblique.**




### Right-angled Triangles

**991.** A **triangle** is a plane surface bounded by three straight lines. The following are the various kinds: right-angled, acute-angled, obtuse-angled and equi-angular; or, if named according to the sides, equilateral, isosceles and scalene.

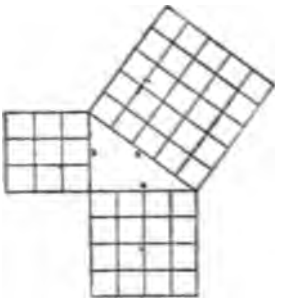



**992.** A **right-angled triangle** is a triangle which has one right angle.

**993.** The **base** of a triangle is the line upon which it rests; as, AB. Any side may become the base.

**994.** The **altitude** of a triangle is its height perpendicular to the base; as, BC.

**995.** The **hypotenuse** of a *right-angled* triangle is the side opposite the right angle; as, AC.



$3^2+4^2=5^2$
$9+16=25$

**996.** In every right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the base and altitude.

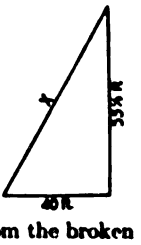
**997.** The hypotenuse of a right-angled triangle is equal to the square root of the sum of the squares of the other two sides.

**998.** The base of a right-angled triangle is equal to the square root of the difference of the squares of the hypotenuse and altitude.

**999.** The altitude of a right-angled triangle is equal to the square root of the difference of the squares of the hypotenuse and base.

**1000. To find any side of a right-angled triangle.**

EXAMPLE.—A tree is broken in such a manner that the top rests on the ground 40 feet from the foot of the tree; the part remaining standing is 53¼ feet; what is the length of the broken part?




$40^2=1600$
$53\frac{1}{4}^2=2844\frac{1}{16}$
$4444.44$

$\sqrt{4444.44}=66.6$, or $66\frac{2}{3}$

EXPLANATION.—The broken part of the tree, the upright part and the ground from the broken end to the foot of tree form the three sides of a right-angled triangle, the base and altitude being known.

NOTE.—Always draw the geometrical figure.

**1001.** RULE.—*To find the hypotenuse, extract the square root of the sum of the squares of the other two sides.*

*To find the base or the altitude, extract the square root of the difference between the squares of the other two sides.*

### WRITTEN PROBLEMS

**1002.** *1.* A ladder leaning against a house reaches 84 feet, its base being 38 feet from the house. What is the length of the ladder?

*2.* Two rafters, each 38 feet long, meet at the ridge of a roof 18 feet above the attic floor. What is the width of the house?

*3.* Two ships start at the same point. One goes due north 120 miles, the other due east 160 miles. How far are they apart?

*4.* A tree was broken 56 feet from the top and fell so that the end struck 42 feet from the foot of the tree. Find the height of the tree.

*5.* Find the distance between a lower corner at one end and the opposite upper corner at the other end of a room 40 feet long, 36 feet wide, 18 feet high.

*6.* Find the hypotenuse of a right-angled triangle, the base of which is 260 feet and altitude 82 feet.

*7.* Find the hypotenuse of a right-angled triangle, the base of which is 1760 yards and whose altitude is 880 yards.

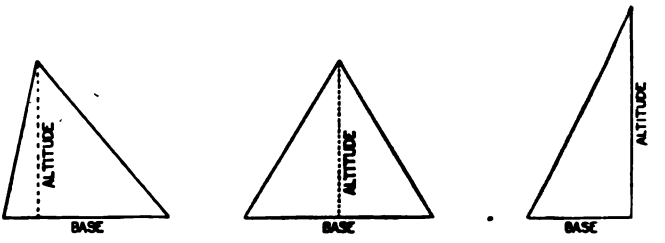
*8.* Find the hypotenuse of a right-angled triangle, the base of which is 4 feet and altitude 3 feet.

*9.* Find the diagonal of a square which is 12 inches on a side.

*10.* Find the diagonal of a rectangle whose length is 4 rods and width is 3 rods.

### Surfaces of Triangles

**1003. To find the area of a triangle.**




EXAMPLE 1.—What is the area of a triangle whose base is 35 yards and whose altitude is 49 yards?

$$35 \times \tfrac{1}{2} \text{ of } 49 = 35 \times 24\tfrac{1}{2} = 857\tfrac{1}{2} \text{ sq. yd. area}$$

EXAMPLE 2.—What is the area of a triangle whose sides are respectively 30, 32 and 40 feet?

$$\frac{30+32+40}{2} = 51 - \begin{cases} 30 = 21 \\ 32 = 19 \\ 40 = 11 \end{cases}$$

$$\sqrt{51 \times 21 \times 19 \times 11} = 473.116 \text{ sq. ft.}$$

**1004.** RULE.—*Multiply the base by one-half the altitude.* Or,

*When the three sides are given and not the altitude, take half the sum of the three sides, from it subtract each side separately, multiply the half sum and these remainders together and extract the square root.*

### WRITTEN PROBLEMS

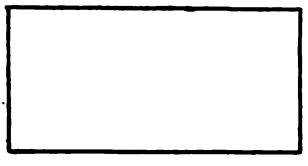
**1005.** *1.* What is the area of a triangle whose base is 36 feet and altitude 15 feet?

*2.* What is the area of a triangular field whose altitude is 50 yards and whose base is 70 yards?

*3.* What is the area of a triangular field whose altitude is 25 chains and whose base is 20 chains?

*4.* What is the area of a triangle whose sides are 150 yards, 150 yards and 250 yards?

### Surfaces of Quadrilaterals

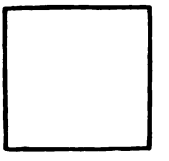


RECTANGLE

**1006.** A **parallelogram** is a plane surface bounded by four straight lines, its opposite sides being parallel.

**1007.** A **rectangle** is a parallelogram whose angles are all right angles.

**1006.** A **square** is a rectangle whose four sides are equal in length.



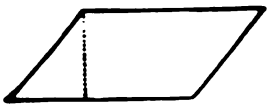
SQUARE

**1009. To find the area of a rectangle.**

EXAMPLE.—What is the area of a rectangular field whose length is 20 chains and whose width is 15 chains?

$$20 \times 15 = 300 \text{ sq. ch., or } 30 \text{ A.}$$

**1010.** RULE.—*Multiply the length by the breadth.*



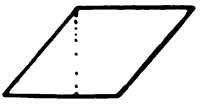
RHOMBOID

**1011.** A **rhomboid** is a parallelogram, the adjoining sides of which are not equal to each other, and which contains no right angle.

**1012. To find the area of a rhomboid.**

**1013.** RULE.—*Multiply the base by the altitude.*

**1014.** A **rhomb** is an equilateral parallelogram having oblique angles.



RHOMB

**1015. To find the area of a rhomb.**

EXAMPLE.—What is the area of a rhomboidal figure 6 inches long, whose altitude is 3 inches?

$$6 \times 3 = 18 \text{ sq. in.}$$

**1016.** RULE.—*Multiply the base by the altitude.*

## WRITTEN PROBLEMS

**1017.** *1.* Find the area of a rectangular field 12 chains 80 links long, 10 chains wide.

*2.* Find the area of a rectangular field 100 yards long, 87 yards wide.

*3.* Find the area of a field 36 rods square.

*4.* Find the surface of a blackboard 8 yards long and 4 feet wide.

*5.* What is the area of a rhomboid whose base is 12 chains and altitude 8 chains?

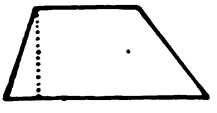
*6.* What is the area of a rhomboid whose base is 200 yards and altitude 70 yards?

*7.* Find the altitude of a rhomboid whose area is 1 acre and base 95 yards.

*8.* Find the altitude of a rectangle whose area is 1 square mile, base 6 chains.

*9.* Find the altitude of a rhomboid of 200 acres, whose base is 100 rods.

*10.* If a road 3 miles long contains 12 acres, how wide is it?



TRAPEZOID

**1018.** A **trapezoid** is a four-sided plane figure, of which two sides are parallel and the other two are not parallel.

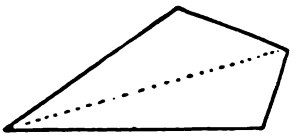
**1019. To find the area of a trapezoid.**

EXAMPLE.—The parallel sides of a trapezoid are respectively 20 and 26 inches and the altitude is 18 inches; what is the area?

$$\frac{20+26}{2} = 23 \qquad 23 \times 18 = 414 \text{ sq. in.}$$

**1020.** RULE.—*Multiply ½ the sum of the parallel sides by the altitude.*

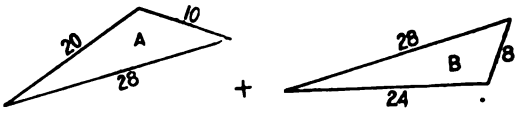
**1021.** A **trapezium** is a four-sided plane figure of which no two sides are parallel.



TRAPEZIUM

**1022. To find the area of a trapezium.**

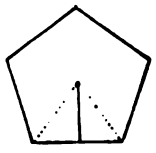
EXAMPLE.—The four sides of a trapezium are respectively 20, 10, 8, 24 ft.; the figure is divided into two triangles by a line 28 ft. long. What is the area of the figure?



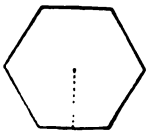

Area of A = 70.42 sq. ft.
Area of B = 89. sq. ft.
Area of figure = 159.42 sq. ft.

**1023.** RULE.—*Separate the trapezium into two triangles, find the area of each, and take the sum.*

### Surfaces of Polygons



PENTAGON



HEXAGON

**1024.** A **polygon** is a figure bounded by straight lines or arcs, especially more than four; a figure having many angles.

**1025.** A **regular polygon** is one whose sides and angles are all equal.

**1026. To find the area of a regular polygon.**

EXAMPLE.—The sides of the hexagonal base of a statue are 10 feet each and the distance from the center of the base to the middle of each side is 8.66 feet; what is the area of the figure?

$$\frac{60 \times 8.66}{2} = 259.8 \text{ sq. ft.}$$

**1027.** RULE.—*Multiply the distance around the polygon (perimeter) by ½ the distance from the middle of one of the sides to the center of the polygon.*

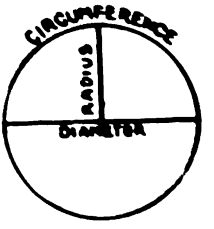
### WRITTEN PROBLEMS

**1028.** *1.* A hexagon is inscribed in a circle whose diameter is 8 feet. What is the area of the parts bounded by the circle and the hexagon?

NOTE.—A side of the hexagon equals the radius of the circle.

*2.* An octagonal summer house has one side 12 feet long and the distance from one corner to the centre is 20 feet. What will it cost to put down a new floor of 2-inch planks costing $38 per M?

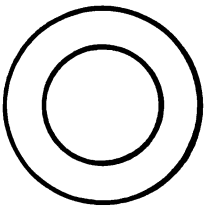
### The Circle



CIRCLE

**1029.** A **circle** is a plane surface bounded by a curved line called its *circumference*, every point in which is equally distant from a point within, called its *center*.

**1030.** **Concentric circles** are circles having the same center and unequal radii.


CONCENTRIC CIRCLES

**1031.** The **diameter** of a circle is a straight line passing through its center, beginning and ending in the circumference, and dividing the circle into two equal parts, called *semicircles*.

**1032.** The **radius** of a circle is a straight line from its center to its circumference. It is ½ the length of the diameter.

**1033. To find the circumference of a circle.**

**1034.** RULE.—*Multiply the diameter by 3.1416.*

**1035. To find the diameter of a circle.**

**1036.** RULE.—*Divide the circumference by 3.1416, or multiply the circumference by .31831.*

**1037. To find the area of a circle.**

**1038.** RULE.—*Multiply the circumference by ½ the radius, or square the diameter and multiply by .7854, or square the circumference and multiply by .07958.*

### WRITTEN PROBLEMS

**1039.** *1.* Find the circumference of a circle whose diameter is 3.5 yards.

*2.* Find the circumference of a circle whose diameter is 12.3 meters.

*3.* What is the circumference of a circle whose radius is 28 inches?

*4.* The circumference of a circle is 100 yards; find the radius.

*5.* The diameter of a wheel is 28 inches; what is its circumference?

*6.* What is the circumference of a circle whose diameter is 100 rods?

*7.* Find the area of a circle whose diameter is 1 chain.

*8.* Find the area of a circle whose diameter is 1 foot.

*9.* Find the distance traversed in an hour by a point in the circumference of a water-wheel 17 feet in diameter, if the wheel makes 1400 revolutions in an hour.

*10.* The circumference of a circle is one-half of a mile. Find its diameter.

*11.* The diameter of a circle is 290 feet. Find its circumference.

*12.* Of two concentric circles, the diameter of the inner is 25 rods, and that of the outer one is 28 rods. Find the area of the space included between the two circumferences.

**1040. To find the circumference or the diameter of a circle, the area being given.**

EXAMPLE.—Find the diameter of a circle whose area is 200 sq. ft. Find circumference.

$200 \div .7854 = \sqrt{254.64} = 15.95$ ft. Diameter
$200 \div .07958 = \sqrt{2513.2} = 50.13$ ft. Circumference

**1041.** RULE.—*Divide area by .7854 and extract the square root of the quotient to find diameter. Divide area by .07958 and extract the square root of the quotient to find circumference.*

### WRITTEN PROBLEMS

**1042.** *1.* The area of a circle is 10 acres; what is the diameter?

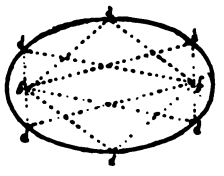
*2.* The area of a circle is 1 square mile; what is the radius?

*3.* The area of a circle is 100 square feet; what is the circumference?

*4.* The area of a square inscribed in a circle is a square rod; what is the diameter of the circle?

*5.* A square whose side is 10 feet is inscribed in a circle; what is the area of the circle?

### The Ellipse

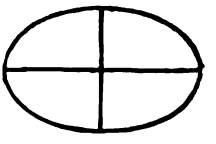


ELLIPSE
$f$, $g$, foci; $pf+pg$, constant

**1043.** An **ellipse** is a plane figure bounded by a curved line, such that the sum of the distances from any point of the curve to two fixed points (the *foci*) is constant.

**1044.** The **transverse axis** of an ellipse is the diameter drawn through the foci.

**1045.** The **conjugate axis** is the diameter at right angles to the transverse axis.



**1046. To find the area of an ellipse.**

**1047.** RULE.—*Multiply ½ of the transverse axis by ½ of the conjugate axis, and the resulting product by 3.1416.*

### WRITTEN PROBLEMS

*1.* Required the area of an elliptical board 8 feet in length, 4.5 feet in width.

*2.* Find the difference in area of an elliptical surface, the axes of which are 75 and 45 inches, and a circle having a diameter equal to the conjugate axis.

Solids.

TRIANGULAR PRISM

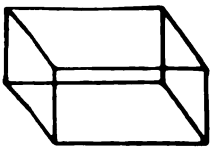
SQUARE PRISM

PENTAGONAL PRISM

**1048.** The principal regular solids are the *prism*, *cube*, *pyramid*, *cone* and *sphere*.

**1049.** A **Prism** is a solid whose bases, or ends, are any similar equal and parallel plane figures, and whose sides are parallelograms.

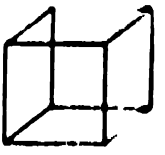
**1050. Prisms** are *triangular*, *square*, *pentagonal*, etc., according as the figures of their ends are triangles, squares, pentagons, etc.



PARALLELEPIPED

**1051.** A prism whose bases are parallelograms is called a **parallelepiped.**

**1052.** A **cube** is a solid bounded by six equal squares.



CUBE

**1053. To find the entire surface of a prism.**

**1054.** RULE.—*Multiply the distance around the base by the altitude, and to the product add the areas of the top and bottom.*

**1055. To find the volume or contents of any prism.**

**1056.** RULE.—*Multiply the area of the base by the altitude of the prism.*

**1057. To find the volume or contents of a parallelepiped.**

**1058.** RULE.—*Multiply the length by breadth and height.*

CYLINDER

**1059.** A **cylinder** is a round prism of uniform diameter with circles for its ends.

**1060.** The **altitude** of a cylinder is the distance from the center of one base to the center of the other.

**1061. To find the entire surface of a cylinder.**

**1062.** RULE.—*Multiply the circumference of the base by the altitude and to the product add the areas of the two ends.*

**1063. To find the volume or contents of a cylinder.**

**1064.** RULE.—*Multiply the area of the base by the altitude.*

### WRITTEN PROBLEMS

**1065.** *1.* Find the entire surface of a parallelepiped whose length is 6 feet and sides of the base 4 feet.

*2.* Find the volume of a granite block 6 feet high, 2 feet wide, 1 foot thick.

*3.* Find the volume of a triangular prism whose area is 3 square feet and whose length is 10 feet.

*4.* Find the entire surface of a cylinder 18 inches in diameter, 6 feet long.

**1066.** A **pyramid** is a solid bounded by a plane polygon for its base, and by triangular planes meeting in a point called the *vertex*.

PYRAMID

**1067. To find the entire surface of a pyramid.**

**1066.** RULE.—*Multiply the perimeter of the base by ½ the slant height, and to the product add the area of the base.*

**1069. To find the volume or contents of a pyramid.**

**1070.** RULE.—*Multiply the area of the base by ⅓ of the altitude.*

CONE

**1071.** A **cone** is a solid that tapers uniformly from a circular base to a point called the *vertex*.

**1072. To find the entire surface of a cone.**

**1073.** RULE.—*Multiply the circumference of the base by ½ the slant height, and to the product add the area of the base.*

**1074. To find the volume or contents of a cone.**

**1075.** RULE.—*Multiply the area of the base by ⅓ of the altitude.*

**1076.** A **sphere** is a solid, every point of whose surface is equally distant from an interior point, called the *center* of the sphere.

SPHERE

**1077.** The **diameter,** or axis, of a sphere is a straight line passing through the center and terminated both ways by the surface.

**1078.** The **radius** of a sphere is a straight line passing from the center to any point on the surface. Its length is ½ of the diameter.

**1079. To find the surface of a sphere.**

**1080.** RULE.—*Multiply the circumference by the diameter, or multiply the square of the diameter by 3.1416.*

**1081. To find the volume or contents of a sphere.**

**1082.** RULE. *Multiply the surface by $\frac{1}{6}$ of the diameter, or multiply the cube of the diameter by .5236.*

SPHEROID

**1083.** A **spheroid** is a solid formed by the revolution of an ellipse about one of its axes.

**1084. To find the volume, or contents, of a spheroid.**

**1085.** RULE. *Multiply the square of the revolving axis by the fixed axis, and the resulting product by .5236.*

## WRITTEN PROBLEMS.

**1086.** 1. Find the whole surface of a pyramid a side of the square base being 6 feet and the slant height 8 feet.

2. Find the whole surface of a pyramid having a rectangular base 5 feet by 4 feet and a slant height of 4 feet 6 inches.

3. Find the volume of an octagonal pyramid, the area of the base being 2 square feet and the altitude being 3 feet.

4. Find the entire surface of a cone, the radius of the base being 2 feet and the slant height 5 feet 6 inches.

5. Find the volume of a cone whose diameter is 12 inches and altitude 3 feet.

6. Find the surface of a sphere 4 feet in diameter.

7. Find the volume of a sphere whose circumference is 3.1416 feet.

## REVIEW PROBLEMS.

**1087.** 1. What is the area of a triangle whose base is 10 feet, altitude 16 feet?

2. What is the area of an isosceles triangle the equal sides of which are 170 yards and the base 140 yards?

3. What is the area of a triangle the sides of which are 30 feet, 25 feet and 29 feet?

4. If the base of a right angled triangle is 42 feet and its altitude 40 feet, what is the hypotenuse?

5. A rectangular field is 280 feet long, 272 feet wide. Find the distance between the opposite corners.

6. A ladder placed against a wall reaches to a height of 48 feet. The bottom of the ladder is 32 feet from the foot of the wall. Find the length of the ladder.

7. If 32 cubic yards of soil are put in a box 4 feet in length, 3 feet in width, how deep will it be?

8. What is the weight of a beam 1 foot 3 inches in diameter, if one weighs 631 pounds to the cubic foot?

9. A wagon 7 feet long, 4 feet 6 inches wide, is built to hold 3 tons of coal. What is its depth, there being 36 pounds to a cubic foot of coal?

*10.* A mow 28 feet long, 24 feet wide, and 12 feet high is filled with timothy hay. If the weight of a cubic foot is 7 pounds, what is the value of the hay in this mow at $15 per ton of 2000 pounds?

*11.* A bin 12 feet long, 7 feet wide, is built to hold 500 bushels of oats. Find its depth.

*12.* What is the weight of a conical piece of metal 22 inches in diameter, 15 inches in slant, if the metal of which it is composed weighs 420 pounds to the cubic foot?

*13.* A field in the form of a rectangle contains 3 acres of ground. Its length is three times its width. What will it cost at 48 cents a running yard to put a fence around it?

*14.* Which costs the more and how much, a fence around a 10-acre field whose length is twice its width, at 62 cents a running yard, or the same kind of fence around a square field containing the same area?

*15.* A chandelier is suspended from the center of the ceiling of a room. The room is 15 feet high, 24 feet long, 22 feet wide. If the bottom of the chandelier is 9 feet from the floor, what is the distance from a lower corner of the room to the central point of the bottom of the chandelier?

*16.* A reservoir in the form of a circle is 9 feet deep and 48 feet in diameter. When full, how long will the water contained in it last a town of 800 people, if to each inhabitant there are allowed 50 gallons every 24 hours?

*17.* The rain which falls on a roof 90 feet long and 85 feet wide, in 48 hours, when the rainfall averages $\frac{3}{4}$ of an inch every two hours, will exactly fill a cylindrical tank 22 feet in diameter. Find the depth of the tank.

*18.* A ball made of iron, which weighs 450 pounds to the cubic foot, has a diameter of 9 inches. How many balls whose diameters are 1 inch would be required to weigh as much?

*19.* A field is 126 chains long and 105 chains wide. What is it worth at $490 an acre?

*20.* The base of a triangle is 90 feet. Its area is 2242 square feet. Find its altitude.

*21.* A rectangular field contains 9 acres of ground. One of its dimensions is 42 chains. Find the other dimension in rods.

*22.* The dimensions of a dumb-bell are as follows: the handle is 5 inches long and 1 inch thick; the ends are spheres 3 inches in diameter. If the dumb-bell is made of iron weighing 450 pounds to the cubic foot, what is its weight?

*23.* What will it cost, at 92 cents per square yard, to macadamize a section of street $2\frac{1}{2}$ miles long, 80 feet wide?

*24.* If the diameter of a circle is 320 rods, find its circumference.

*25.* If the circumference of a circle is a quarter of a mile, find its area.

*26.* If a pipe 3 inches in diameter will supply a certain amount of water in a given time, how many pipes ½-inch in diameter will be required to supply the same amount of water in the same time?

*27.* At 8½ cents a pound, what is the cost of a grindstone, the diameter of which is 18 inches and thickness 4 inches, if in the center there is a hole 1 inch square? A cubic foot of this kind of stone weighs 185 pounds.

*28.* A cabinet maker has a piece of mahogany 18 feet long, 7 feet 6 inches wide. How large a square table-top could be made from this, no allowance for waste?

*29.* A platform, circular in form, holds 850 people. Allowing 2 square feet for each person, what is the diameter of the platform?

*30.* A right-angled triangle, base 5 feet and altitude 4 feet, is made to turn round on the longer side; find the volume of the cone thus generated.

*31.* The surface of a certain solid is 3 times as great as the surface of a similar solid; what is the ratio of their volumes?

*32.* A rectangular field is 300 rods long and 200 yards wide; what is the length of its diagonal?

*33.* A lot is 1 rod square; how many cubic yards of earth will it require to raise the surface of the lot 2 feet?

*34.* Find the weight of a spherical iron ball whose diameter is 8 inches, a cubic foot of iron weighing 450 pounds.

*35.* If a sphere 4 inches in diameter weighs 4 pounds, what will be the weight of a sphere of the same material 1 foot in diameter?

*36.* Find the weight of a spherical shell 1 inch thick, outside diameter 1 foot, the metal weighing 450 pounds to the cubic foot.

*37.* The sides of a trapezium taken in order are 12, 12, 9 and 7, and the diagonal which cuts off the first two sides is 14 rods. What is the area of the trapezium?

*38.* How many gallons of water will a tank hold that is 8 feet square at the base and 9 feet deep?

*39.* A room is 24 feet long, 15 feet high, 18 feet wide. Find the diagonal of the room.

# LATITUDE AND LONGITUDE

**1088. Latitude,** distance north or south of the equator, is measured in degrees, minutes and seconds; no latitude can exceed 90 degrees, because the distance from the equator to either pole is one-fourth of 360°, or 90°.

**1089.** Places north of the equator are in *north latitude;* those south, in *south latitude.* Places on the equator have no latitude.

**1090. Longitude,** distance east or west from a selected or prime meridian (usually Greenwich), is measured in degrees, minutes and seconds; no longitude can be greater than 180 degrees, because the distance from a given meridian to that meridian is 360°; then one-half of 360° is *east* and one-half is *west.*

NOTE.—In the United States the meridian of Washington is sometimes selected, and maps show longitude from Greenwich at the top and from Washington at the bottom.

**1091.** Places east of the selected meridian are in *east longitude;* those west, in *west longitude.*

**1092. To find the difference in latitude or longitude.**

EXAMPLES.—(*a*) The latitude of Philadelphia is 39° 56′ 39″ N., and of Montreal 45° 24′ 28″ N.; what is the difference in latitude? (*b*) The longitude of Berlin is 13° 24′ 28″ E., and of New York is 74° 3′ W.; what is the difference in longitude?

EXPLANATION.—(*a*) Since Montreal and Philadelphia are both north of the equator, their difference of latitude must be the distance that Montreal is farther north than Philadelphia; this distance is found by subtraction.

| | |
|---|---|
| 45° 24′ 28″ | N. |
| 39 56 39 | N. |
| 5° 27′ 49″ | |

| | |
|---|---|
| 13° 24′ 28″ | E. |
| 74 3 00 | W. |
| 87° 27′ 28″ | |

(*b*) From the meridian of Berlin to the prime meridian is 13° 24′ 28″, and from the prime meridian to the meridian of New York is 74° 3′; the distance from Berlin to New York, or the difference of longitude, is found by addition.

**1093.** RULE.—*The difference of latitude or longitude is found by subtraction when both places are in like latitudes or longitudes; by addition, when the places are in different latitudes or longitudes.*

### WRITTEN PROBLEMS

**1094.** *1.* The latitude of New York is 40° 24′ 40″ N., of Cape Horn 55° 58′ 30″ S., and of Washington 38° 53′ 39″ N. What is the difference in latitude between New York and Cape Horn? between Washington and Cape Horn? between New York and Washington?

*2.* The longitude of Detroit is 82° 58′ W., of San Francisco 122° 26′ 15″ W., and of Rome 12° 27′ E. What is the difference in longitude between Detroit and San Francisco? between Detroit and Rome? between San Francisco and Rome?

3. The latitude of Philadelphia is 39° 56′ 39″ N., of Callao 11° 45′ 20″ S., and of Boston 42° 21′ 27″ N. What is the difference in latitude between Philadelphia and Callao? between Boston and Callao? between Philadelphia and Boston?

4. The longitude of Philadelphia is 75° 9′ 5″ W., of New Orleans 90° W., and of Calcutta 88° 19′ 2″ E. What is the difference in longitude between Philadelphia and Calcutta? between New Orleans and Calcutta? between Philadelphia and New Orleans?

5. The longitude of Canton is 113° 15′ 33″ E., of St. Paul 93° 4′ 55″ W., of Omaha 95° 55′ 15″ W. What is the difference in longitude between Canton and St. Paul? between Canton and Omaha?

NOTE.—If the difference in longitude exceeds 180 degrees, subtract this difference from 360 degrees to find the real difference in longitude.

## LONGITUDE AND TIME

**1095.** The rotation of the earth on its axis from west to east (360° in 24 hours) causes midday or noon, or sunrise, or any given time, to pass westward 15° in one hour; 15′ in one minute; 15″ in one second.

NOTE.—When it is sunrise, 6 A. M., in Philadelphia, places west of Philadelphia, as Chicago, will not have seen the sunrise, hence the hour will be before sunrise, or earlier. When Chicago sees the sunrise, Philadelphia will see the sun above the horizon, and the time indicated by clocks will be past 6 or later than Chicago. From this we conclude that a place *east* of a given place has *later time* than the given place; places *west*, *earlier time*. Places on the same meridian have the same time.

**1096. Solar time** is the sun's time as indicated by a sun dial.

**1097. Standard time** is time established by the leading railroad companies of the United States. For convenience they have divided the United States into four time-belts each fifteen degrees wide, and have adopted the time of a single meridian in that belt as the standard time for the entire belt.

These belts are:

1. **Eastern,** whose time is the local or solar time of the 75th meridian, which is 9′ east of Philadelphia.

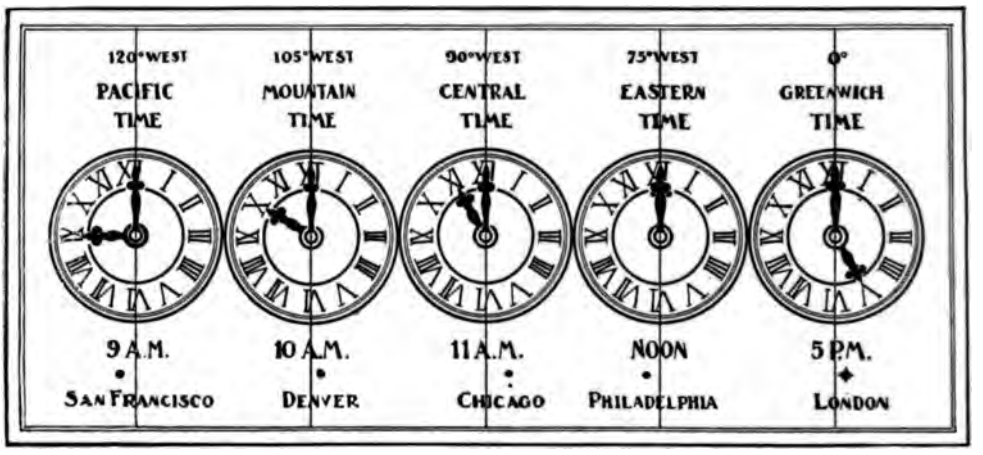
2. **Central,** whose time is the local or solar time of the 90th meridian, which is 2° 15′ west of Chicago.

3. **Mountain,** whose time is the local or solar time of the 105th meridian, which is 44′ west of Denver.

4. **Pacific,** whose time is the local or solar time of the 120th meridian, which is 2° 26′ east of San Francisco.

NOTE.—An outline map showing time-belts and official time may be secured from a railroad company.

**Dials Exhibiting Comparison of Solar and Standard Time**




**1098. To find the difference in time between places differing in longitude.**

EXAMPLE.—(*a*) When it is 11 A. M., solar time, at Philadelphia, 75° 9′ 5″ W., what time is it at Denver, 104° 44′ 20″ W.? (*b*) What is the standard time at Denver when it is 11 A. M. standard time at Philadelphia?

EXPLANATION.—(*a*) The difference in longitude between the two places is 29° 35′ 15″. Divide this difference by 15, since every 15° = one hr.; 15′ = one min.; 15″ = one sec. This gives 1 hr. 58 min. 21 sec., the difference in time between the two places. Denver, being west of Philadelphia, has earlier time; subtracting 1 hr. 58 min. 21 sec. from 11 A. M. leaves 9 hr. 1 min. 39 sec.; hence the time at Denver is 1 min. 39 sec. after 9 A. M.

(*b*) The standard time at Denver (mountain time) is 2 hr. earlier than Philadelphia (eastern time), hence subtract 2 hr. from 11 A. M. 9 A. M. is the standard time at Denver.

| | | | | |
|---|---|---|---|---|
| | 104° | 44′ | 20″ | W. |
| | 75 | 9 | 5 | W. |
| 15) | 29 | 35 | 15 | |
| | 1 | 58 | 21 | |
| | hrs. | min. | sec. | |
| | 11 | 00 | 00 | |
| | 1 | 58 | 21 | |
| | 9 | 1 | 39 | |

**1099.** RULE 1.—*Divide the number of degrees, minutes and seconds (the difference in longitude between two places) by 15; the quotient will be the difference of time in hours, minutes and seconds.*

RULE 2.—*Multiply the hours, minutes and seconds (the difference in time between two places) by 15; the product will be the difference in degrees, minutes and seconds between two places.*

### WRITTEN PROBLEMS

**1100.** *1.* The longitude of Pekin is 118° E. and that of Berlin 13° 24′ 28″ E. What is the difference in solar time? When it is 3 P. M. at Berlin, what time is it at Pekin?

*2.* The longitude of Paris is 2° 20′ 17″ E., and that of Philadelphia 75° 9′ 5″ W. What is the difference in time? When it is half-past two o'clock P. M. in Philadelphia, what time is it in Paris? in Berlin?

*3.* What is the standard time in New York when the standard time in Portland, Oregon, is 25 min. 18 sec. past 3 o'clock P. M.?

*4.* The longitude of Washington is 77° 3′ W. If the difference in time between Washington and Edinburg is 4 hr. 53 min, 28 sec., what is the longitude of Edinburg? When it is 20 min. past 3 o'clock P. M. at Edinburg, what time is it in Washington?

*5.* The captain of a ship observed that when the sun crossed the meridian, the solar time, shown by his chronometer set to Greenwich time, was 22 min. 4 sec. after 3 o'clock P. M. In what longitude was the ship?

*6.* St. Paul, whose longitude is 93° 4′ 55″ W., is situated in the Central belt. What is the difference between the *local* or *sun time* and the *standard time* at St. Paul?

*7.* In traveling from Philadelphia to Seattle, Washington, my watch, which shows Philadelphia time, must be changed on arriving at Seattle. How much must I change it and which way (standard time)?

*8.* The longitude of Manila is 121° 15″ E., and of San Juan 66° 5′ W.; what is the difference in longitude? When it is 5 P. M. Tuesday at San Juan, what is the time at Manila?

*9.* The longitude of Berlin is 13° 24′ 28″ E., and of Philadelphia 75° 9′ 5″ W. A cablegram sent from Berlin at 12.30 noon requires 30 minutes in transit. At what hour will it be received in Philadelphia?

*10.* Sydney is in longitude 151° 38′ 42″ E., and San Francisco is 122° 26′ 15″ W.; when it is 4 P. M. July 4th at San Francisco, what time is it at Sydney?

*11.* A and B met, and on comparing watches found that A's indicated 10.30 A. M. and B's showed 5.10 P. M. The longitude of A's place was 82° 58′; what was the longitude of B's place?

*12.* Two persons are so situated that they are 14° 42′ 48″ apart, yet are in the same time belt. The first is 6° 14′ 27″ East of the standard meridian, and the other is 8° 28′ 21″ West. Will the local time of each be too fast or too slow, and how much?

## MENTAL REVIEW PROBLEMS

**1101.** *1.* A man's age increased by $\frac{1}{3}$ of his age equals 40 years; what is his age?

*2.* What number is that which if its $\frac{1}{4}$ be taken away, the remainder will be 33?

*3.* Three times a number increased by its $\frac{1}{5}$ equals 32; what is the number?

*4.* What number is that which being increased by its $\frac{1}{4}$ and $\frac{1}{10}$ equals 27?

*5.* A man having spent $\frac{1}{4}$ of his money had 45 cents remaining; what sum had he at first?

*6.* Margaret spent $\frac{1}{2}$ of her money in one store and $\frac{1}{5}$ of her money in another store and had 21 cents remaining; how much had she at first?

*7.* One-half of the length of a pole is in the air, $\frac{1}{5}$ in the water and 18 feet in the ground; required the length of the pole.

*8.* Two-thirds of Adam's money diminished by 2 dollars equals $\frac{1}{2}$ of his money; what was his sum of money?

*9.* What number is that which diminished by its $\frac{1}{3}$ and increased by its $\frac{1}{6}$ will equal 50?

*10.* What is Benjamin's age, if $\frac{3}{4}$ of his age increased by 6 years equals 36 years?

*11.* The cost of a horse diminished by $\frac{1}{4}$ of its cost and five dollars equals seventy dollars; what did the horse cost?

*12.* Abram's money increased by twenty dollars equals $\frac{4}{3}$ of his money; what is his money?

*13.* Bella's age increased by 32 years equals 3 times her age; what is her age?

*14.* Four times a number diminished by once the number equals 75; what is the number?

*15.* If the height of a tree be increased by its $\frac{3}{4}$ and 20 feet, the sum will be twice the height; what is the height of the tree?

*16.* The cost of a horse diminished by $\frac{1}{5}$ of its cost and \$30 equals $\frac{1}{2}$ of its cost; what did the horse cost?

*17.* I sold $\frac{1}{3}$ of my farm and then bought 30 acres and now have 80 acres; how much land had I at first?

*18.* If the height of a monument be increased by its $\frac{1}{3}$ and that sum diminished by its $\frac{1}{2}$, the height will equal 75 feet; what is its height?

*19.* A father and his son earned in one week \$24; how much did each earn if the father earned three times as much as his son?

*20.* A pole 60 feet in length was sawed into two unequal parts so that $\frac{1}{2}$ of the longer piece equaled the shorter; what was the length of each piece?

*21.* X and Y had equal sums of money. X lost $10, Y earned $20 and they then together had $110; how much had each at first?

*22.* A cow and a horse cost $130. If the cow cost ⅖ as much as the horse, minus $10, required the cost of each.

*23.* A wheelman rode 130 miles in 3 days. He rode 10 miles farther the second day than the first and 10 miles less the third day than the second; how far did he ride each day?

*24.* M and N together have $40; how many dollars has each, if three times M's money equals N's?

*25.* P and Q can build ⅖ of a boat in a day and Q does ⅓ as much as P, what part can each complete in a day?

*26.* Divide $66 between two persons so that the first has $2 as often as the second shall have $3½.

*27.* A's money added to ⅔ of B's money equals $70; how much money has each if B's money is to A's as 3 to 2?

*28.* If ⅔ of a yard of cloth cost ⅖ of a dollar, what will ⅓ of a yard cost?

*29.* If 5 men earn $40 in a certain time, how much will 4 men earn in twice the time?

*30.* Two men hire a pasture for $5. One turns in 3 horses for 7 days and the other 7 horses for 2 days; how much should each pay?

*31.* Two men had equal sums of money; one lost $7 and the other lost $4 and they then together had $19. How much did each have at first?

*32.* Jones and Black can build ⅖ of a boat in a day, and Black builds ⅔ as much as Jones; how much can each build in a day?

*33.* If 5 men can build 40 rods of wall in 16 days, how many men can build half as much wall in 4 days?

*34.* If a five-cent loaf of bread weighs 8 ounces when flour is worth $4 a barrel, how much should it weigh when flour is worth $5 a barrel?

*35.* Three men hired a horse for 30 days at the rate of $2 a day. The first used it 8 days, the second 10 days and the third 12 days; how much should each pay?

*36.* A and Z contract to do a piece of work in 90 days; A sends 9 men and Z 15 boys. What part of the price should each receive, supposing 5 boys to do as much as 3 men?

*37.* X, Y and Z dig a ditch for $72; X get $1 a day, Y $1½ a day and Z $1¾ a day. How many days were they employed and what did each receive?

*38.* Two men can mow a field of grass in 4 hours; the first can mow it alone in 9 hours; how long would it require the second alone to mow it?

*39.* Brown can dig a ditch in 6 days and Black can dig it in 8 days; in what time can they dig it working together?

*40.* A cistern can be filled by an inlet pipe in 5 hours and emptied by an outlet pipe in 4 hours; if the cistern is full and both pipes are opened, in what time will the tank be emptied?

*41.* A cistern can be filled by one pipe alone in 6 hours and by a second alone in 9 hours; how long will it require both to fill it?

*42.* A cistern can be filled by an inlet pipe in 4 hours and emptied by an outlet pipe in 5 hours; if the cistern is empty and both pipes are opened, how long will be required to fill it?

*43.* Brown, Baker and Broad lunch together, Brown having 3 loaves, Baker 5 loaves and Broad 24 cents to divide between them. If the bread be divided equally among them, how should Brown and Baker divide the money?

*44.* What is the time of day, if the time since noon equals $\frac{1}{8}$ of the time since midnight?

*45.* At what time after 3 o'clock are the hour and minute hands together?

*46.* What is the time of day, if the time till midnight equals $\frac{3}{5}$ of the time since midnight?

*47.* At what time after 3 o'clock are the hour and minute hands opposite?

*48.* A pole whose length was 64 feet was broken into two lengths so that if 4 feet be taken from the longer and added to the shorter, the pieces will be equal; what are the lengths of the broken pieces?

*49.* A lady having two watches bought a chain worth \$20. If the chain be put upon the silver watch, they together will be worth $\frac{2}{3}$ as much as the gold watch; if the chain be placed upon the gold watch, they together will be worth 4 times as much as the silver watch. What is each watch worth?

*50.* A boat which sails at the rate of 6 miles an hour moves down a river whose current is 3 miles an hour; how far can it go and return in 8 hours?

*51.* A philanthropist gave $\frac{3}{4}$ of his money, less \$12, to a poor family and then had \$21 left; how much did he give away?

*52.* A toy balloon in the air fell $\frac{1}{3}$ of the distance to the ground and then arose $\frac{1}{2}$ the distance it was from the ground; what part of the first distance is it now from the ground?

*53.* If $\frac{1}{2}$ of my weight be added to my weight and from the sum 55 pounds be taken, the result will be 200 pounds; what is my weight?

*54.* If the height of a monument be increased by its $\frac{1}{2}$ and from this sum be taken the difference between its $\frac{1}{2}$ and $\frac{1}{3}$, the result will be 40 feet; what is its height?

*55.* The sum of three numbers is 55; the second is $\frac{1}{2}$ greater than the first and the third is twice the second; find the numbers.

*56.* A man walks 120 miles in three days, walking each day 10 miles more than the preceding day; how many miles did he walk the second day?

57. Two pipes fill a cistern of 300 barrels; $\frac{2}{3}$ of what flows through one pipe being equal to $\frac{4}{5}$ of what flows through the other; how much is carried in by each pipe?

58. Hunter's money is to Fisher's money as 2 to 3; how much money has each, if Hunter's money and $\frac{1}{2}$ of Fisher's money is equal to $330?

59. If 8 men can do a piece of work in 6 days, how many days will be needed to complete the work, if 4 men be added when the work is $\frac{1}{2}$ done?

60. Jones and Smith rent a pasture for $40. Jones puts in 12 horses and Smith 8 cows. How much should each pay if 2 cows eat as much as 3 horses?

61. X, Y and Z build a wall, working an equal number of days; X gets $3 a day; Y, $2 a day; and Z, $1 a day. They receive $60; what is each one's share?

62. Brown can build a shed in 10 days and Black in 12 days; how many days would it take to build it if they work together?

63. White and Grey working together can build a boat in 4$\frac{4}{5}$ days. White working alone will build it in 8 days; in how many days would Grey working alone build it?

64. A piece of work can be done by 4 men or 6 boys in 10 days; how long would it require 2 men and 10 boys to do it?

65. Wood can cut a cord of wood in $\frac{1}{2}$ of a day and Black can cut it in $\frac{2}{3}$ of a day; working together, how many cords can they cut in a week of 6 days?

66. William receives $2 a day and pays 50 cents a day board; at the end of 40 days he has saved $40; how many days was he idle?

67. James had 3 loaves of bread and David had 5 loaves, which they shared with Benjamin, who gave them 16 cents; how shall they share the money?

68. Two men mow a field of grass for $40; the first mows twice as much as the second, less 4 acres, and is paid $24; how many acres does each mow?

69. What is the asking price of cloth, if by dropping 25% and selling it at $2.40 a yard, a gain of 20% is made?

70. A sum of money put on interest at a certain rate for 3 years amounts to $280, and the same sum at the same rate for 10 years amounts to $320; find the sum and rate?

71. If 5 dollars be taken from $\frac{1}{3}$ of a sum of money there will remain the original sum; what was the sum?

72. A tank can be filled by an inlet pipe in 4 hours and emptied when full by an outlet pipe in 1$\frac{1}{2}$ hours; if the tank be full and both pipes opened, in what time will the tank be empty?

73. A working alone can build a boat in 7 days, B in 8 days, and C in 6 days; in what time can they, working together, build it?

74. A cyclist makes a journey of 300 miles in 4 days, riding each day 10 miles farther than on the preceding day; how many miles did he ride each day?

75. If 5 men can do a piece of work in a certain time, how many men would be required to do double the amount of work in half the time?

## GENERAL REVIEW PROBLEMS

**1102.** *1.* The net proceeds of a sale are \$940.10. The total expenses are: 3% commission, \$14.10 storage, \$16.50 drayage. Find the amount of the sale.

*2.* I sold a house for \$2450 and gained 14%. I sold another for the same price and lost 14%. Find my net loss or gain.

*3.* I buy a piano for \$250. I want to sell it so as to gain 15% after throwing off 10%. What price shall I ask for it?

*4.* Change £142 19s. 7d. to French money.

*5.* What is the cost in U. S. money of 900 tons of steel, invoiced at £3 8s. 10d. a ton?

*6.* I sold a lot of goods for \$760.25, and thereby lost 12%. At what price should the goods have been marked to gain 10% and allow a trade discount of 3%?

*7.* The net proceeds of a sale of hardware are \$210.50, the commission 5%, the other expenses \$7.42. Find the sales and the amount of the commission.

*8.* An insurance company insures a house valued at \$42000 for $\frac{3}{4}$ of its value at $1\frac{7}{8}\%$ premium. This company reinsures $\frac{3}{4}$ of its risk at $\frac{7}{8}\%$. What rate per cent. is the first company getting on its risk?

*9.* A pipe can fill a cistern in 24 hours; another pipe in 30 hours. There is a delivery pipe running from the cistern that will empty it in 20 hours. Suppose the cistern to be empty, and the three pipes opened at once; in what time will it be filled?

*10.* A can carry a ton of coal to the third story of a building in $5\frac{1}{4}$ hours, B can do the same in $6\frac{1}{2}$ hours, C in $7\frac{3}{4}$ hours. In what time could the three together carry $2\frac{1}{2}$ tons?

*11.* A rectangular field produces 1726 bushels of wheat, at the rate of 82 bushels to the acre. If one side of the field is 726 feet, what is the distance around the field?

*12.* A can do a piece of work in 12 days, B in 15 days; if C works with them, they can do the work in 4 days. In what time can C do the work alone?

*13.* If it requires $3\frac{1}{2}$ bushels of seed to sow an acre, how many quarts will be required to sow a lot 110 feet square?

*14.* A cistern 8 feet in diameter, 7 feet deep, has how many gallons of water in it when the water is within 3 inches of the top?

*15.* The hypotenuse of a right-angled triangle is 82 feet, the base 62 feet. Find the altitude.

*16.* What will it cost, at 48 cents a square yard, to paint the dome of a hall, the dome being in the form of a hemisphere 40 feet in diameter?

*17.* How many bricks, 8 inches long and 4 inches wide, are required to lay a pavement 176 feet long, 9 feet wide? What will they cost at \$13.50 a thousand?

*18.* A square field containing 12 acres is surrounded by a tight board fence 8 feet high. What did the boards cost at \$22 per M?

*19.* I buy muslin at 6¾ cents a yard and mark it to gain 20%, but on account of a defect I conclude to sell it at 85% of the marked price. What is my loss or gain on 1000 yards?

*20.* A miller bought a ton of wheat at 68 cents a bushel, which he manufactured into flour. If each bushel yielded 36¼ pounds and he received $4.10 a barrel, what is his per cent. of gain or loss?

*21.* A bin is 9 feet 5 inches long, 4 feet 3 inches wide, 5 feet 8 inches deep, and is ⅞ full of wheat. How many bushels does it contain and what is the wheat worth at 68 cents a bushel?

*22.* How deep must a circular cistern 4 feet in diameter be to hold 40 barrels of water?

*23.* The fore wheel of a carriage is 3 feet 6 inches in diameter; the hind wheel 4 feet 8 inches. How many revolutions will the smaller one make while the larger one makes 900?

*24.* An agent sold goods for $1395 and lost ⅛ of the cost. For how much should he have sold the goods to gain 20%?

*25.* A man lost $920, which was 25% of what he had remaining. What per cent. of his money did he lose?

*26.* I go to bank to-day with two notes, one dated seven days ago for $2200, at 3 months; the second is my own note dated to-day, at 90 days, for a sum sufficient to make my balance large enough to enable me to give a check for $4500 and have a balance of $300 in bank. If my present balance is $290, what is the face of the note?

*27.* A coal dealer buys a car load of coal, 35000 pounds, at $4.15 a long ton, which he retails at $5.60 a long ton. What per cent. does he gain if there is a loss of 3% in handling the coal?

*28.* The minute-hand of a clock is 5 inches long, the hour-hand 3½ inches. Over how much more area does the minute-hand pass in 5 hours than the hour-hand?

*29.* At what per cent. above the manufacturer's price must a wholesale merchant mark goods so that he can allow a retailer a discount of 30% and 5% and still make a profit of 25%?

*30.* An article is marked to gain 33⅓%, but the seller throws off ⅛ and the collector is afterward paid 15% for collecting the debt. What is the per cent. of loss?

*31.* A's money is 5% less than B's, and B's is 15% more than C's. If they together have $19360.72, how much does each have?

*32.* A book jobber buys books at 40% and 10% off list and marks his stock at an advance of 15% on list prices. He makes a sale to a private library at a discount of 15% from his marked price and makes a profit of $2500. How much did the books cost the jobber?

*33.* I ordered an agent to buy flour for me which I afterward sold at 20% profit and gained $1.50 on a barrel. If my agent's commission was 3½%, and his total commission $25, how many barrels did he buy?

*34.* A dairyman sold a quantity of milk, butter and cheese, receiving for all \$115.25. He gained 15% on the milk, 18% on the cheese, 22% on the butter. If the amount received for each was equal, find the cost of each.

*35.* A grocer wishes to mix coffees worth 38, 42, 45, 50 and 55 cents a pound so as to produce a mixture worth 45 cents a pound. Find the number of pounds of each kind he must take.

*36.* A clerk is required to mark goods so as to make 25% after throwing off 10%. By selling a lot of these goods for \$78 net, there was a gain of but 3%. How should he have marked the goods?

*37.* I go to bank to-day with four notes: one dated to-day, at 90 days, for \$1500; the second dated 7 days ago, at three months, for \$920; the third dated to-day, at 60 days, for \$1720; the fourth is my own note dated to-day, at 90 days, for enough to raise my bank balance to \$5000. What was the face of my own note, my balance previous to having these notes discounted being \$290.62?

*38.* I buy in Paris 328 meters of silk at 27.42 francs per meter. If exchange is at 5.19, what is the cost of a draft that will pay for the invoice?

*39.* A river is moving at the rate of 4 miles an hour. It is 119 feet wide and has an average depth of $10\frac{1}{2}$ feet. How many tons of water will pass a given point on the river's bank in 12 minutes, water weighing $62\frac{1}{2}$ pounds to the cubic foot?

*40.* On May 12, 1915, I asked my broker to purchase for me 320 shares Central Railroad stock and left with him as margin \$3000. He bought on May 27 at $106\frac{1}{4}$; on June 2 he sold 150 shares at $107\frac{3}{8}$, and on June 18 he sold the remainder at $107\frac{1}{2}$. Our account was settled on June 30. How much was due me on that day, and what was my gain?

*41.* Two concentric circles have diameters of 350 feet and 390 feet, respectively. What is the area of the space enclosed between the circumferences?

*42.* A wall $5\frac{1}{4}$ feet high, $2\frac{1}{2}$ feet thick, cost for building \$2006.90, at the rate of \$1.18 a perch. Find the length of the wall.

*43.* A can do a piece of work in 27 days; B in 36 days; C in 42 days. They all work together until $\frac{7}{8}$ of the work is done, when A retires, leaving B and C to finish it. In what time is the whole work done? How long do B and C work?

*44.* A ditch 426 feet long, $2\frac{1}{2}$ feet wide, $5\frac{1}{4}$ feet deep, cost for digging \$1522.65. How wide should a ditch be that is 294 feet long, $4\frac{3}{4}$ feet deep, to cost \$2420?

*45.* A cylinder is 7 feet in diameter, 38 feet long. Find its entire surface.

*46.* A cylindrical tank is 7 feet in diameter and 15 feet deep. It has in it 560 gallons of water. What per cent. is this of its capacity?

*47.* A man bought a house and lot, paying five times as much for the house as for the lot. If the house had cost 25% less than it did, it would have cost \$3800. Find the cost of each.

*48.* A dealer spent equal sums in hats, coats and shoes, and sold at a profit of 20% on the hats, 22% on the coats, and 15% on the shoes. The entire sales amounted to \$22500. Find the cost of each.

*49.* A invested a certain amount in real estate, and B invested 4 times as much. A lost 15% and B gained 20%. The difference between the amounts received was $1250. How much did each invest?

*50.* A company engaged an agent to sell for them on a commission of 5% and shipped him merchandise amounting to $5500. The agent purchased additional merchandise to the amount of $1200, and the firm sent him $500 cash. At the end of a certain period the agent had sold goods to the amount of $4920, and the goods on hand amounted to $3006.78. Find the loss or gain. Find also the amount due the firm by the agent at this time.

*51.* A circular piece of ground, containing 5 acres, is enclosed by a tight board fence 7 feet high. What was the cost, at $13 per M, of 1-inch lumber required to make it?

*52.* A and B form a partnership on January 1, 1915, with the understanding that the gains and losses are to be divided in proportion to their average net investments. On January 1 A invests $3000 and B invests $2500. A on March 1 invests $1500, on July 1 $2200, and withdraws on September 1 $2000. B on April 1 invests $1800 and on September 1 $2000, and withdraws on November 1 $3300. A statement of their resources and liabilities at time of settlement is as follows:

| | | | | | |
|---|---|---|---|---|---|
| Resources: | Cash, | $2200 | Liabilities: | Bills, | $1920 |
| | Mdse., | 6850 | | Sundry Accounts, | 2600 |
| | Bills, | 3290 | | | |
| | Book Accounts, | 2600 | | | |

Close each partner's account.

*53.* If a wall 520 feet long, 4½ feet high, 2½ feet thick, requires the work of 175 men, 212 days of 10 hours each in building, how high should a wall be that is 248 feet long, 2½ feet thick, to require the work of 98 men, 217 days of 9 hours each?

*54.* Sold for account of consignor on September 8, 1914, $5240.26 on 4 months' credit; September 29, $538 cash; October 16, $2750.85, 60 days; on September 15, paid freight amounting to $95.23; my commission for selling is 5%. What is due to consignor, and when are the proceeds due by equation?

*55.* A merchant bought 22 pieces of cloth, each containing 25 yards, at $4.50, on a credit of 6 months, and sold them at $6.25 on a credit of 3 months. What was his cash gain at the end of 6 months, money being worth 6%?

*56.* Bought a check on a bank which had suspended, at 65% of its face, and invested it for 5% railroad bonds at 82. What rate of interest do I receive on my investment?

*57.* Bought, at 8% discount, a 6% mortgage (face $3800), with two years to run. What interest do I get on the money invested, if the mortgage is paid at maturity?

*58.* A bath tub will hold 200 gallons of water. It is filled by a faucet discharging 50 gallons in 4 minutes, and emptied by a waste pipe discharging 48 gallons in 4 minutes. If both the pipes are opened at once and the waste pipe closed one hour afterward, in what time will the tub be full?

*59.* A retail merchant sold a quantity of silks for $1250, thereby gaining 20%. The wholesale merchant from whom he bought them made a profit of 15%, and the importer who sold them gained 20%. What did they cost the importer?

*60.* A merchant in Philadelphia made the following importation from Belfast, Ireland:

| | | | |
|---|---|---|---|
| 1720 | yds. | linen at | 2s. 6d. |
| 1850 | " | " | 3s. 3d. |
| 512 | " | " | 5s. 2d. |
| 1680 | " | " | 3s. 4d. |

Duty 22% ad valorem and 8 cents a yard specific. Find the duty. Find also the cost of a draft in payment of the invoice, exchange at 4.87.

*61.* A manufacturer carried on business for four years. The first year he gained 20%, which at the close of that year he put into the business. The second year he lost 15%. The third year he gained 25% of what he had at the beginning of that year, and put it into the business. The fourth year he gained 16⅔%, and then his total capital was $40250.85. How much had he gained in the four years?

*62.* If the sales in a certain merchandise account are $256742.90, the cost of the merchandise $275420.82, and the inventory $45923.75, what per cent. is the profit?

*63.* The list price of a sewing machine is $60; an agent buys at 25% off and sells for $55. What is his per cent. of profit?

*64.* A merchant sells goods at retail 35% above cost, and at wholesale 15% less than the retail price. What is his gain per cent. at wholesale?

*65.* If I buy at list less 40%, will I gain or lose if I add 50% to the list and sell at ¾ off?

*66.* What is the balance due June 15, 1915, on a note of $1350, dated June 15, 1914, and showing the following indorsements: August 18, 1914, $50; September 23, 1914, $75; October 20, 1914, $90; November 24, 1914, $75; January 14, 1915, $5; March 25, 1915, $60; April 21, 1915, $350? Work by the United States rule.

*67.* Find the cost of 300 shares Reading Railroad stock (par $50) at 11⅞, brokerage ⅛%.

*68.* I bought 290 shares Philadelphia Traction at 72⅜, brokerage ¼%. What did they cost me?

*69.* I bought a bill of exchange on London, and paid $2892.50 for it. What was the face of the bill, exchange at 4.87½?

*70.* What is the face of a bill of exchange on Paris which can be bought for $1590.62 at 5.15?

*71.* What is the duty on 9000 yards of 28-inch dress goods at 42 cents per square yard?

*72.* What is the duty on 720 gallons of brandy at $1.25 per gallon, the allowance for leakage being 2%?

*73.* A firm in Philadelphia imports from Paris 9200 yards of 22-inch silk, the marked price of which at the time of purchase was 3.25 francs per meter. What was the duty at 10 cents per square yard and 35% ad valorem?

*74.* What is the duty on a consignment of iron invoiced at £2265 18s. 6d. at 42%?

*75.* A man's personal property is assessed at $5000, his real estate at $13000. Find his total tax at the rate of $1.32 per $100.

*76.* The expenses of a town for a year, not including the tax collector's commission of 3%, are as follows: for schools, $3500; for roads, bridges, etc., $1700; for incidental expenses, $800. If the total valuation of the town is $2450000, what should be A's tax whose property is assessed at $3500?

*77.* A lumber dealer purchased two piles of wood; one pile was 66 feet long, 7 feet high, 12 feet wide, and contained $43\frac{5}{8}$ cords. The other pile was 29 feet long, 16 feet wide and of sufficient height to contain $54\frac{3}{4}$ cords. Find the height of the latter pile.

*78.* A contractor agreed to dig two cellars at the same price per cubic yard. He received $450 for one 75 feet long, 28 feet wide, 9 feet deep. What should he have received for the other, if its length was 88 feet, width 33 feet, depth 8 feet?

*79.* The owner of a mill had it insured at a rate of $1\frac{1}{4}$%. He afterward introduced steam power, and the company took an additional risk of $1500. They, however, raised the rate $\frac{3}{4}$%. The extra premium amounted to $65. For what amount was the mill first insured?

*80.* A note of $5000, dated July 7, 1914, with interest at $5\frac{1}{2}$%, has the following indorsements: September 15, 1914, $10; October 28, 1914, $25; December 23, 1914, $1000; February 25, 1915, $1500; April 28, 1915, $90; May 26, 1915, $1300. What is due July 7, 1915? Use United States and Mercantile rules.

*81.* I bought 357 shares Lehigh Navigation (par $50) at $41\frac{1}{8}$, brokerage $\frac{1}{4}$%. Find the cost.

*82.* A legacy of $15000 is invested as follows: 50 shares P. R. R. (par $50) at $53\frac{3}{4}$; 92 shares Central at $82\frac{1}{2}$; 36 Lehigh Valley (par $50) at $27\frac{1}{8}$; and the balance in Union Pacific at $41\frac{1}{8}$. Brokerage at regular rates. Find the number of shares of Union Pacific purchased and the balance unspent (no fraction of a share being purchased).

*83.* A fire insurance company had a risk of $58000 at $\frac{1}{8}$% premium, and reinsured $\frac{1}{2}$ of the risk in another company at $\frac{1}{4}$% premium and $\frac{1}{3}$ of it in another at $\frac{1}{8}$% premium. What rate per cent. is the first company making on its risk?

*84.* If the premium paid for insuring a property was $45.20, and the rate of insurance was sixty cents on $100, for what sum was it insured?

*85.* If from 220 reams of paper 3000 copies of a book containing 380 pages can be printed, how many reams will be required to print 8000 copies of a book containing 480 pages?

*86.* A grocer wishes to mix 500 pounds of coffee which he can sell at 75 cents a pound and gain 25%, by using coffees worth 38, 40, 45, 65 and 70 cents a pound. How many pounds of each should he take?

*87.* What is the amount of $998.65 from January 16, 1914, to September 14, 1914?

*88.* What is the cost of a circular piece of ground 228 feet in diameter, at $300 an acre?

*89.* A legacy of $22000 is invested as follows: $2500 of 4% Government Bonds at $111\frac{3}{4}$; $3200 of 6% railroad bonds at $104\frac{1}{2}$; $1600 of 5% railroad bonds at $98\frac{3}{8}$; and the balance in U. S. 5s at $128\frac{3}{4}$. Find the total income from the investment. Find also the balance unspent (no bond being of smaller denomination than $100), brokerage at regular rates.

*90.* If I buy 4% stock at $97\frac{3}{4}$, brokerage $\frac{1}{4}$%, what per cent. income do I receive on my investment?

*91.* If I buy 6% bonds at $97\frac{5}{8}$, brokerage $\frac{1}{4}$%, what per cent. income do I receive on my investment?

*92.* I bought a 5% bond (par $1000) at such a price as to yield me $4\frac{1}{4}$% on my investment. Find the cost of the bond.

*93.* Bank stock paying 5% is selling at $112\frac{1}{2}$. Find the per cent. income on investment.

*94.* A stone wall 3 feet thick, $4\frac{1}{2}$ feet high, cost for its construction $1590, at 96 cents a perch. Find the length of the wall.

*95.* A contractor agrees to build a cellar which shall be 18 feet long, 16 feet wide and 9 feet deep, inside measure, when finished. His contract price is 62 cents per cubic yard for making the excavation, and $1.25 a perch for the wall. If the wall is 2 feet thick, find his total contract price.

*96.* At $18.50 per M, what is the cost of sufficient lumber 2 inches thick to make 50 boxes, each 4×4 feet, 3 feet deep, inside measure?

*97.* The expenses of a town for a year, not including the tax collector's commission of 3%, are as follows: for schools, $2500; for roads, bridges, etc., $1200; for incidental expenses, $900. If the total valuation of the town is $1420000, what should be A's tax whose property is assessed at $5500?

*98.* A fire insurance company had a risk of $85000 at $\frac{7}{8}$% premium, and reinsured $\frac{1}{8}$ of the risk in another company at $\frac{5}{8}$% premium and $\frac{1}{2}$ of it in another at $\frac{3}{4}$% premium. What rate per cent. is the first company getting on its risk?

*99.* The stock of a wholesale house is insured in a number of companies for $175000, and is damaged by water to the amount of $57641.73. What per cent. of its risk should be paid by each company, and what amount should a company pay that had a risk of $2500 upon it?

*100.* A roof is 42 feet long, 28 feet from eaves to ridge. How many slates are required to cover it, if the slates used are 10 inches wide, showing 12 inches to the weather, and what will they cost at $10.90 a square (100 sq. ft.)?

*101.* Find the entire surface of a cylinder 9 feet in diameter, 15 feet long.

*102.* A garden 125 feet long, 86 feet wide, inside measure, has on its border a stone wall 3 feet high, 18 inches thick. What is the cost of the wall at 92 cents a perch?

*103.* The stock of a wholesale house is insured in a number of companies for $15000, and is damaged by water to the amount of $900. What per cent. of its risk should be paid by each company, and what amount should a company pay that had a risk of $6500 upon it?

*104.* The value of a ship's cargo is $20750, and the owner desires to insure it for a sum which will cover the value of the cargo and the cost of insurance. If the rate of insurance is $1\frac{1}{4}\%$, what should be the amount of the policy?

*105.* A lumber dealer purchased two piles of wood: one pile was 72 feet long, 10 feet wide, and high enough to contain 45 cords; the other pile was 18 feet wide, 16 feet high, and long enough to contain $87\frac{1}{2}$ cords. Find the height of the first pile and the length of the second pile.

*106.* A contractor agreed to dig two cellars at the same price per cubic yard. He received $350 for one 70 feet long, 25 feet wide, 8 feet deep. What should he have received for the other, if its length was 96 feet, width 32 feet, depth 9 feet?

*107.* $85\%$ of a meter is what per cent. of a yard?

*108.* I have in bank to-day $311.72. I send to bank H. R. Reed's note dated to-day for $726 at 3 months, and my own note dated to-day at 90 days for a sum sufficient to enable me to give a check for $1150 and have left in bank $125.10. Find the face of my note.

*109.* If 19 men can do a piece of work in 12 days, how many days should 26 men require to do the same work?

*110.* If 14 men or 19 boys can do a piece of work in 42 days, in what time can 12 men and 22 boys do the same work?

*111.* A can do a piece of work in 9 days, B in $9\frac{1}{2}$ days, and C in $10\frac{1}{4}$ days. In what time could they do it, working together?

*112.* I send to bank to-day William Smith's note dated to-day, in my favor, for $950 at three months, and my own note at 60 days, dated to-day, for a sum sufficient with the proceeds of Smith's note to allow me to draw a check for $1450 and leave in bank a balance of $375.50. If my balance before sending these notes to bank for discount was $250.10, what is the face of my own note?

*113.* I send an agent $1190 to invest in apples at $1.10 a barrel, allowing him a commission of $4\%$. He pays 17 cents a piece for barrels and 22 cents a barrel transportation. How many barrels of apples does he buy? What is the surplus?

*114.* On a lot of merchandise which I sold for $962.50, I lost $18\%$. The cost price of the remaining merchandise is $2642.50. At what per cent. advance must I sell this to recover my loss?

*115.* A can do a piece of work in $10\frac{1}{2}$ days, B in $11\frac{1}{4}$ days, C in $12\frac{1}{2}$ days. In what time could they do the work together?

*116.* An importer buys in France 426 meters of silk at 26.14 francs per meter. He pays 40% duty. What should the entire quantity be marked in order to gain 15% and throw off 5%?

*117.* I buy two houses for $9670, giving 14% more for one than for the other. How should I sell the cheaper house to gain 20%?

*118.* What is the duty at 42% on 456 dozen pocket-knives invoiced at 17s. 4d. a dozen?

*119.* The net proceeds of a consignment are $1496.72. The rate of commission is 4%. The other expenses are: $14.62 drayage; $26.75 storage; $32.96 sundry expenses; what were the total sales?

*120.* At what price per yard must I sell silk that cost me 5.42 francs a meter, in order to gain 30%?

*121.* Certain cloth shrinks 5% of its length in sponging. How many yards of the cloth did I have before sponging, if after sponging there are 672½ yards?

*122.* I purchased a quantity of green coffee. In roasting it there is a loss of 10% of its weight. What was the gross cost of the coffee, purchased at 18 cents a pound green, if after roasting I sell the entire quantity for $288 at 24 cents a pound?

*123.* I import from Sheffield, Eng., the following: 120 dozen knives at 18s. 3d.; 215 dozen knives at £1 1s. 1d.; 295 dozen knives at £1 3s. 2d.; 82 dozen knives at £1 7s. 8d. I pay 40% duty, £1 19s. 2d. consul fees, 15s. 3d. drayage, 3% commission for buying, and $7.90 drayage in Philadelphia. Find the total cost, exchange at 4.86½.

*124.* If 220 men working 9 hours a day can build a wall 550 feet long, 3½ feet high, 2¼ feet thick in 268 days, how long a wall should 185 men build, working 10 hours a day, if the wall is 7½ feet high, 2½ feet thick, and the men work 275 days?

*125.* On June 19, 1914, I give a note payable on demand for $2800 with interest at 4%. I pay $500 each quarter. What amount will take up the note one year from date, there having been three payments made? Use both United States and Mercantile rules.

*126.* A can do a piece of work in 15 days, B in 18 days, C in 20 days. The three work together until $\frac{5}{11}$ of the work is done, when A retires, leaving B and C to finish it. In what time will the work be done?

*127.* A man invests $25000 as follows: 72 shares of 5% stock at 143¼; 27 shares of 4% stock at 105½; 22 shares of 5% stock at 109⅜; and the balance in stock paying 6% at 115¼. What is his total income, brokerage ¼%?

*128.* What is the weight of an iron pipe 18 feet long, the inside diameter being 10 inches and the outside diameter 12 inches, if a cubic foot of iron weighs 450 pounds?

*129.* A field contains 10 acres; it is in the form of a square. Find the length of the diagonal of the field.

190. A ball 9 inches in diameter weighs 750 pounds. How many balls of the same metal, 3 inches in diameter, will be required to weigh as much?

191. In the following statement, when is the balance due by equation? What amount would settle it September 28, 1914?

*To* John D. Bantram *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| Mar. | 5 | Balance | $492.50 | Mar. | 16 | Cash | $200.00 |
| | 24 | Mdse., 60 days | 450.92 | Apr. | 22 | Note, 4 months | 600.00 |
| Apr. | 15 | 2 mos. | 490.15 | May | 27 | Draft, 30 days | 300.00 |
| | 22 | 10 days | 690.15 | Sept. | 22 | Cash | 200.00 |
| May | 18 | 1 mo. | 212.42 | | | | |

192. On June 22, 1914, I gave William Haller a note for $1800 and agreed to pay $1.50 every day thereafter until the 30th of June, inclusive, when I would give a new note for the balance due on that day. If the interest is at 6%, what will be the face of the new note? United States rule.

193. What is the value of a shaft of iron 18 inches in diameter and 22 feet long, at $16.50 a ton (2240 lb.), a cubic foot of iron weighing 450 pounds?

194. A and B form a partnership each putting in $3000. At the end of 4 months A draws out $1200 and B $600; at the end of 6 months A draws out $900 and B $300; and at the end of 9 months A draws out $750 and B $450. At the end of the year they dissolve partnership, having a capital of $5250. How must they divide it?

195. A has a circular garden, and B a square one; each contains 2 acres. A walk 6 feet wide is made around each garden. Which walk contains the greater area, and how much? How much greater would the area of each walk be if it corresponded the 2 acres?

196. A man bought four houses for $12,000 and sold them as follows: on the first he gained 20%, on the second he gained 15%, on the third he lost 15%, and on the fourth he lost 4%. If he received the same amount of money for each house, what was the cost of each?

197. A retail dealer's private mark is Hia. Smith. How can he mark goods which he buys for $2.40 and sells at a gain of 25%?

NOTE. The letters of the private mark in their respective order correspond to the digits 1 2 3 4 5 6 7 8 9 and 0.

198. A retail dealer's private mark is Charleston. He has marked a lot of goods at $8.15 per yard. Find the cost of the goods if he gain is 30%.

199. On June 1, 1914, I purchased through a broker 100 shares of Lake Shore R. R. at 84½ and deposited $3000 as margin. On June 18 he sold 75 shares at 87½ and on June 30, he sold the balance for 88½. On June 15 a dividend of 4% was declared. What is due me on June 30? What is my profit?

*140.* John R. Lamper and R. E. Butterwick are masons and form a partnership to do job-work. They agree to share equally. Each partner keeps an account of the labor he has performed, the expenses he has paid and the collections he has made. Lamper's labor amounted to $2850.62. Butterwick's labor amounted to $3002.65. Lamper paid for expenses $196.15, but lost 10% of his labor accounts. Butterwick paid for expenses $212.62, and failed to collect $302.25. Adjust each partner's account.

*141.* John Harper begins a business January 1, 1914, which is to consist of preparing building lumber. He rents a factory for $300 a year. He buys raw lumber amounting to $2550 and sells during the year finished lumber to the amount of $1475.22. He has paid $217.32 during the year for necessary expenses, and for labor $725.38. Find his loss or gain for the year, if the material on hand inventories $2175. 142.48 loss.

*142.* In what time, at simple interest, will $1796.14 amount to $2496.12 at 4½%?

*143.* What amount will settle the following account November 11, 1914?

*Dr.* H. M. KENNEDY *Cr.*

| 1914 | | | | | 1914 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| June | 6 | Balance | 403 | 19 | July | 13 | Cash | 475 | 90 |
| " | 22 | Mdse., 60 days | 1126 | 92 | Sept. | 18 | Note, 10 days | 500 | 00 |
| July | 27 | " 2 mos. | 472 | 85 | Nov. | 4 | Draft, 30 " | 375 | 00 |
| Aug. | 19 | " 1 mo. | 396 | 12 | | | | | |

*144.* A and B united in the purchase of two car-loads of coal to be divided between them. The invoice was as follows:

| DATE | CAR NO. | EGG | | STOVE | | PRICE | | TOTAL | |
|---|---|---|---|---|---|---|---|---|---|
| | | Tons | cwt. | Tons | cwt. | $ | cts. | $ | cts. |
| July 9 | 44303 | 20 | 00 | | | 2 | 65 | 53 | 00 |
| " | 47363 | | | 20 | 00 | 2 | 75 | 55 | 00 |
| | | | | | | | | 108 | 00 |

The freight was $1.70 a ton on 40 tons. The coal was weighed by the wagon-load as delivered. A received 23625 lbs. of the egg coal and 23375 lbs. of the stove coal; B received 22000 lbs. of the egg coal and 21900 lbs. of the stove coal. In settling for the coal and freight, A paid $91 and B paid $85; was this settlement correct?

*145.* If 15% of a certain number is 2232 more than 3% of three times the number, what is the number?

*146.* Received an invoice of goods, 14% of which was unsalable; at what per cent. above cost must I sell the remainder in order to clear 30% on the whole invoice?

*147.* Find the net amount of the following invoice of glassware:

25 doz. goblets @ 60 cents, 40% and 10% off
40 doz. goblets @ 55 cents, 35% and 5% off
50 doz. dishes @ 85 cents, 50% and 15% off
60 doz. dishes @ $1.25, 30% and 3% off.

*148.* A Georgia planter shipped 400 bales of cotton, averaging 510 lb. each, to a Philadelphia broker who sold it for him at 8½ cents a pound, charging 3% commission and $73.40 for other expenses. The broker remitted the proceeds by draft, purchased at ⅛% premium; what was the face of the draft?

*149.* The International Navigation Company insured a vessel and cargo for ⅔ of their value with the London Marine Insurance Company at 3½%. The London Marine reinsured ½ of the risk with the Transatlantic Insurance Company at 2¾%. The vessel was lost at sea, and the loss of the Transatlantic Company was $1200 more than that of the London Marine. What amount did the International Navigation Company lose?

*150.* A merchant bought goods as follows: Jan. 1, 1914, $2234.76; Feb. 2, 1914, $1362.84; Mar. 16, 1914, $6327; May 15, 1914, $7415.80; July 1, 1914, $4225.66. On each of these bills he was entitled to a credit of four months. What does he owe December 1, 1914?

*151.* William Hendrickson bought of Levi Morris merchandise amounting to $6324.78, terms 30 days, and gave in payment his note at 90 days; what was the face of the note?

*152.* A note for $5000, with interest at 4%, dated Jan. 6, 1914, has the following indorsements: Apr. 1, 1914, $1000; June 1, 1914, $1000; Aug. 12, 1914, $1000; Oct. 1, 1914, $1000. What is due December 1, 1914, by the Mercantile rule?

*153.* A can do a piece of work in 14 days, B can do it in 20 days. If C works with them, all three can do the work in $6\frac{4}{11}$ days. A commences the work and continues for 3 days; B then relieves him, and works 5 days; B and C work together for the next two days. How long does it then take A and B to finish the work?

*154.* On January 8, 1912, I bought $12000 worth of 6% railroad bonds at 116¼, brokerage ¼%. On May 1 and November 1 of each year I cashed the coupons, and on September 8, 1914, I sold the bonds at 114⅞, brokerage ¼%. How much less did I receive than if I had loaned my money on mortgage at 4½%?

*155.* A steamboat that can run 18 miles an hour with the current, and 14 miles an hour against it, requires 16 hours to go to a certain point and return. What is the distance?

*156.* A Philadelphia firm of exporters and importers, having received an account sales from its agents in London showing net proceeds £784 14s. 10d. to its credit, directs the agents to remit a draft for 10248.75 francs to one of the firm's creditors in Paris, and forward the balance by draft on Philadelphia. If Paris exchange is 25.18 at London, and Philadelphia exchange 4.89, find the face of each draft.

*157.* If it requires 125 reams of paper to print 5000 copies of a book of 280 pages, each page being 5¼×8⅛ inches, how many reams will be required to print 10000 copies of a book of 176 pages, each page 6½×9¼ inches?

*158.* A and B are equal partners in business, and at the time of settlement it is found that A has to his credit $4250.75, while B has overdrawn his account $1920.65. There are no other resources or liabilities. How shall they settle?

*159.* A merchant added $1700 to his capital the first year; during the second year he further increased it by a sum equal to 10% of his original capital; during the third year he lost 40% of what he had at the end of the second year, and found that he then had just what he began with. What was his original capital?

*160.* Robert F. Allison and Peter J. Crosby form a copartnership, with the understanding that each is to share in gains or losses in proportion to his net investment. Allison invests $10000, January 1, $3000, March 10; $4000, June 1; and $2000, September 1. He withdraws $475, February 1; $350, April 1; $275, May 1; $500, July 1; and $675, October 1. Crosby invests $8000, January 1, $2000, April 1; $1000, July 1; and $1500, August 1. He withdraws $700, March 1; $300, June 1; $200, August 15; and $150, September 1. They gain during the year $3200; how should this be divided?

*161.* Six per cent. interest is charged and allowed on each item in the following account. What is the balance due December 31, 1914?

*Dr.* ARTHUR C. BENNETT *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| Jan. | 14 | Mdse., net | 3418 20 | Feb. | 19 | Cash | 2000 00 |
| Mar. | 4 | " 60 days | 1725 44 | May | 20 | Note, 30 days | 1500 00 |
| Apr. | 21 | " 30 " | 378 19 | July | 15 | Cash | 1000 00 |
| Aug. | 10 | " 60 " | 2621 80 | Sept. | 24 | Cash | 2500 00 |

*162.* What is the value of a wedge of fine gold 8 inches long, 5 inches wide and 4 inches thick at the butt (the other end being a sharp edge)? The specific gravity (the ratio of its weight to the weight of an equal volume of water) of gold is 19.26; a cubic foot of water weighs 1000 ounces avoirdupois; one pound avoirdupois equals 7000 grains Troy; and 23.22 grains of fine gold are worth $1.

*163.* The duty of 40% on a quantity of silk imported from France, invoiced at 6 francs a yard, was $520.80; how many yards were there in the importation?

*164.* A farmer bought one hundred animals for $100, consisting of calves at $10, sheep at $3, and hens at 50 cents each. How many of each did he buy?

*165.* What will it cost, at 34 cents a cubic yard, to make the excavation for a circular pond, 50 feet in diameter, 2 feet deep at the circumference and 5 feet deep at the center, the bottom having a uniform slope from the circumference to a point at the center in the form of an inverted cone?

*166.* If the pond in Problem 165 be filled within six inches of the top, what per cent. of its entire capacity will it contain?

*167.* An importer buys an invoice of Italian goods amounting to 23724 lire, pays the duty at 30%, and marks the goods at 25% above cost. He sells one-half

the invoice at this price, but is obliged to sell the remainder at 10% below his arked price. Find his gain on the entire invoice.

*168.* A stock of goods worth $24000 is insured in four different companies as llows: in the first for $5000; in the second for $5000; in the third for $4000, and in e fourth for $3000. Each of the policies contains the 80% coinsurance clause. If e goods were to be damaged to the extent of $2368, how much would each company ıy?

*169.* What is the interest at 5% on £582 12s. 6d. from January 14, 1914, to ptember 22, 1915?

*170.* A note for $3246.27, with interest at 4%, dated May 14, 1914, has the llowing indorsements: August 20, 1914, $46.27; November 12, 1914, $100; April 16, 15, $100; July 23, 1915, $100; June 10, 1916, $50; August 10, 1916, $25; March 14, 17, $200; June 7, 1917, $500; December 3, 1917, $150; April 4, 1918, $300. How uch will be due August 1, 1918 (United States rule)?

*171.* If it takes 4 men and 7 boys 2 days to do a certain piece of work, or 3 men and boys $3\frac{3}{11}$ days to do the same work, how long would it take 2 men and 1 boy to do it?

*172.* Which is the better investment, and how much more annual income will yield—to invest $32000 in 5% bonds at $92\frac{1}{8}$%, brokerage $\frac{1}{4}$%, or in 6% bonds at $7\frac{1}{8}$, brokerage $\frac{1}{4}$%, reckoning $1000 as the smallest denomination purchased, and e balance to be deposited in a saving fund at 3% interest?

*173.* An invoice of merchandise from Amsterdam, amounting to 12000 guilders, e duty on which was 40% and freight $432.50, cost altogether $7372.10; at what te of exchange was the draft in settlement purchased?

*174.* How many gallons each of wines worth $1.15, $1.25, $1.40 and $1.70 a llon, respectively, should be added to 120 gallons of wine worth $1.45 a gallon to rm a mixture worth $1.35 a gallon?

*175.* A and B form a partnership, agreeing to invest equal amounts and share ually in gains or losses, and further agreeing that neither withdraw any money om the business for a period of five years, and that in case the yearly statement ow a loss at the end of any year, they will make additional equal investments fficient to make up the loss. At the end of the first year their books show a net in of $12\frac{1}{2}$%; at the end of the second year a net loss of 8% on the capital at the ginning of this year, which they make good by additional investments; at the end the third year, a net gain of 20% on this year's capital; at the end of the fourth ar a net loss of 5%, which they make good; at the end of the fifth year, a net gain of %. Each partner now has $12744 to his credit. What amount did each invest on mmencing business, and what was his average profit per cent. per annum on his iginal investment?

*176.* If a cistern 16 feet long, 8 feet wide and 12 feet deep can be filled in 8 hours 2 pipes, each 1 inch in diameter, how long will it take 3 pipes, each $\frac{1}{16}$ of an inch diameter, discharging twice as fast, to fill a cistern 18 feet long, 7 feet wide and 9 feet ep?

*177.* What is the balance due on the following account June 30, 1915, interest at 6%?

*Dr.* B. F. DODGE *in account with* KELLY & Co. *Cr.*

| 1915 | | | DAYS | INTEREST | AMOUNTS | 1915 | | | DAYS | INTEREST | AMOUNTS |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Jan. | 4 | Mdse., net | *** | ** ** | 2312 18 | Jan. | 29 | Cash | *** | ** ** | 2000 |
| Feb. | 11 | " 30 d. | *** | * ** | 467 50 | Feb. | 26 | " | *** | * ** | 350 |
| Mar. | 15 | " 60 d. | *** | ** ** | 712 84 | Mar. | 1 | " | *** | * ** | 200 |
| Apr. | 21 | " 30 d. | ** | * ** | 327 48 | Apr. | 26 | " | ** | * ** | 500 |
| May | 19 | " net | ** | * ** | 674 32 | May | 31 | " | ** | * ** | 500 |
| June | 3 | " net | ** | * ** | 925 75 | June | 30 | Interest bal. | | ** ** | |
| " | 30 | Interest bal. | | | ** ** | " | 30 | Balance | | | **** ** |
| | | | | *** ** | **** ** | | | | | *** ** | **** ** |
| June | 30 | Balance | | | **** ** | | | | | | |

*178.* The specific gravity of iron is 7.84; how much will a hollow globe of iron weigh, whose diameter is 8 inches, and the thickness of the metal $\frac{3}{4}$ of an inch?

*179.* How many feet of inch lumber will be required for the floor of a platform 30 feet long and 20 feet wide (from front to back), and for a roof of the same material, the roof to project one foot beyond the edge of the platform on all sides, and to be supported by posts 16 feet high above the floor in front and 12 feet above it at the back? What will the lumber cost at $22 per M?

*180.* A certain excavation consists partly of clay and partly of rock. If it were all clay, A could do the work in 20 days; if it were all rock, he could do it in 36 days. If it were all rock, B could do the work in 32 days; while if it were all clay he could do it in 18 days. A and B work together on the job and finish it in $10\frac{4}{7}\frac{4}{3}$ days; what per cent. of the work is clay, and what per cent. rock?

*181.* How many $1000 U. S. 4s at $122\frac{1}{4}$, brokerage $\frac{1}{8}$%, can be bought with $32500, and what will be the balance remaining?

*182.* If a man purchases $12000 U. S. 4s at $123\frac{5}{8}$, $5000 railroad 5s at $102\frac{1}{8}$ and $8000 railroad 6s at $110\frac{7}{8}$, through a broker, what rate of income does he receive on the total investment?

*183.* How many thousand ordinary bricks will be required for the walls of a round tower 12 feet in diameter, inside measurement, and 50 feet high, the walls being 2 feet thick for the first 15 feet, $1\frac{1}{2}$ feet thick for the next 15 feet, and the remainder of the wall 1 foot thick?

*184.* A man purchased $\frac{5}{8}$ interest in a factory, and afterward sold 18% of his share at a loss of 7% for $1699.95; what was the value of the whole factory at the rate at which he purchased his share?

*185.* 8 lbs. of tea and 5 lbs. of coffee of a certain grade are worth \$6.10, and 3 lbs. of tea and 7 lbs. of coffee of the same grade are worth \$3.62; what is one pound of each worth?

*186.* A grocer sells a blend of coffee at 34 cents a pound, which he claims to be $\frac{3}{4}$ Java at 33 cents, and $\frac{1}{4}$ Mocha at 36 cents; the Java coffee costs him 30 cents a pound and the Mocha 32 cents, so that he is apparently making a profit of $10\frac{1}{2}\%$. But the mixture actually contains $\frac{1}{4}$ Rio coffee, which costs the grocer only 18 cents a pound; what per cent. of profit does he really make?

*187.* A commission merchant received a consignment of cotton, consisting of 260 bales averaging 510 lbs. to the bale, which he sold, and, after deducting his commission at $3\%$, remitted \$10128.98 proceeds; at what price per pound did he sell the cotton?

*188.* The premium paid for insuring a stock of merchandise at $2\frac{1}{2}\%$ for $\frac{3}{4}$ of its value was \$689.87; what was the value of the merchandise?

*189.* A borrowed from B \$1000 at $6\%$ interest, agreeing to pay \$100 at the end of each year, on account of principal and interest, until the whole obligation was canceled; how many yearly payments did he make and what was the amount of his last payment?

*190.* Exchange being quoted at 5.14, what will be the cost of a draft in settlement of an importation of 6 dozen French clocks, invoiced at 22.50 francs each; for how much apiece must they be sold in Philadelphia to gain $20\%$, if the duty is $25\%$ and other charges in Philadelphia amount to \$37.40?

*191.* If 14 men can build a wall 60 feet long, 5 feet high, $1\frac{1}{2}$ feet thick in 20 days, how many men will be required to build a wall 340 feet long, $4\frac{1}{2}$ feet high, $2\frac{1}{2}$ feet thick in 50 days?

*192.* If a lot of goods is bought for \$327.42, and $\frac{2}{3}$ of it is sold at a gain of $30\%$, and $\frac{1}{4}$ of the remainder proves unsalable, what should the remainder be marked so that a discount of $10\%$ may be allowed and yet enough gained to net $25\%$ profit on the whole purchase?

*193.* A piece of ground in the form of a circle has a diameter of $16\frac{1}{4}$ rods; what will it cost, at 20 cents a cubic yard, to dig a ditch around it, outside, $2\frac{1}{2}$ feet wide and $3\frac{1}{2}$ feet deep?

*194.* What per cent. is gained by buying coal at \$3.75 a long ton and selling at \$5 a short ton?

*195.* A boy's father deposits \$300 in a savings bank when the boy is 16 years old. If this bank allows $1\%$ interest quarterly, and balances its books quarterly, but reckons no interest on fractions of a dollar, how much will there be to his credit when he reaches the age of 21?

*196.* A commission merchant in New York sells for me 1200 baskets of potatoes at 48 cents a basket; he pays freight and other charges \$112.80; deducts his commission of $3\%$; and invests the proceeds in flour, charging $2\%$ for purchasing. If he buys 100 barrels of flour, and remits a balance of \$7.18, at what price does he purchase?

*197.* If an insurance company paid \$3920, under its policy of \$20000 with "average clause," on a loss of \$6272, what was the value of the property insured?

*198.* A works 3 days on a certain piece of work, then B works 4 days, after which A finishes the work. If it takes A alone 21 days to do the work, or A and B together 12 days, how long will it take A to finish it?

*199.* Sold \$15000 U. S. 4s at $121\frac{5}{8}$, and invested the proceeds in railroad stock at $48\frac{3}{4}$, which was afterward sold at $52\frac{1}{8}$. What was the gain on the stock, if brokerage was paid?

*200.* The assessed valuation of the property in a certain town is \$2347896. The amount to be raised by taxation is \$48394.50. What is the tax rate? What is the tax on \$100?

*201.* What is the duty at \$1 per pound on an importation of partly manufactured silk from France, weighing 874 pounds and invoiced at 9098.45 francs, with the proviso that "in no case shall the duty be less than fifty per centum ad valorem of the invoiced cost" of the goods?

*202.* What would be the cost, exchange at 5.14, of a draft to pay for the importation in problem 201?

*203.* How many pounds each of teas worth 42, 48, 54, 60 and 75 cents per pound, respectively, should be taken to form a mixture of 80 lbs. at 55 cents?

*204.* If it takes 22 men, working 8 hours a day and 5 days a week, 4 weeks to do a certain piece of work, how many weeks will it take 40 boys, working 10 hours a day and 4 days a week, to do half as much, 9 boys doing as much as 7 men?

*205.* Find the contents of a triangular prism 4 feet high, the sides of the base being 14, 19 and 26 inches, respectively.

*206.* Bought a car-load of wheat, 34263 lbs., at 83 cents a bushel, and a car-load of corn, 32796 lbs., at 56 cents a bushel. Sold the wheat at a profit of 20% and the corn at a profit of $16\frac{2}{3}\%$. What was the total gain?

*207.* A, B and C form a partnership, agreeing to divide gains or losses equally, under the following conditions: B is to be general manager at a salary of \$2000 per annum, and C bookkeeper at \$1500 per annum, their salaries to be credited quarterly at the end of each quarter. Interest is to be charged and allowed at 6% per annum. A is to furnish $\frac{1}{2}$ of the capital, and B and C each $\frac{1}{4}$. A invests January 1, \$12000; B \$6000, and C \$6000. A withdraws March 1, \$675; June 1, \$500, and September 1, \$900. B withdraws February 1, \$300; May 1, \$500; August 1, \$400, and October 1, \$375. C withdraws April 1, \$500; July 1, \$500; September 1, \$400, and November 1, \$400. A invests \$2000 additional on July 1. The net gain for the year, exclusive of salary and interest items, is \$4760.83; what is the balance of each partner's account?

*208.* A pile of coal in the corner of a cellar reaches within 1 foot of the flooring above, which is 8 feet above the cellar floor, and extends 10 feet along each wall at the bottom of the pile, the pile having a plane slope (so that it forms $\frac{1}{4}$ of a pyramid). How much coal, long tons, is there in the pile, reckoning 80 lbs. to the bushel of 2150.4 cu. in.?

*209.* What amount is due on the following account January 1, 1914, interest at 6%?

*Dr.* AMBROSE C. HEADLEY *Cr.*

| 1914 | | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|---|
| Jan. | 4 | Mdse., net | | 757 14 | Mar. | 11 | Cash | 1500 00 |
| Feb. | 10 | " | 30 days | 1213 19 | Apr. | 15 | " | 800 00 |
| Mar. | 18 | " | 60 days | 472 18 | May | 1 | Note, 30 days | 215 00 |
| Apr. | 15 | " | 30 days | 327 65 | June | 15 | Cash | 500 00 |
| June | 9 | " | net | 782 80 | July | 1 | " | 300 00 |
| July | 22 | " | net | 485 25 | Aug. | 10 | " | 200 00 |
| Aug. | 19 | " | 60 days | 379 92 | Sept. | 1 | Note, 30 days | 300 00 |
| Sept. | 30 | " | 3 mos. | 915 20 | Oct. | 15 | Cash | 750 00 |
| Nov. | 3 | " | 30 days | 455 70 | | | | |
| Dec. | 15 | " | 60 days | 1114 22 | | | | |

*210.* A certain country paid 4½% interest on its debt. A war increased the debt 25%. During peace which followed, the debt was diminished $25000000, and the rate was reduced to 4%. The annual interest was then the same as at first. What was the indebtedness of the country before the war?

*211.* A piece of gold, alloyed with silver, is 14 carats fine and weighs 72 pennyweights. How much gold must be added to make it 18 carats fine?

*212.* A man drew out 50% of the amount he had to his credit in bank, and gave 16⅔% of the money withdrawn for a house, for which he paid $4000. He had the house insured for $3000 at ¾%. He invested the balance of the money withdrawn from bank in wheat, which he sold for cash, ½ at an advance of 30%, ⅓ at an advance of 18%, and ⅛ at a loss of 7½%. The house was destroyed by fire and the insurance company paid the amount of their policy, which the man deposited in bank, together with all his other receipts. How much had he in bank originally, what was the final balance of his bank account, and what was his per cent. of loss or gain?

*213.* A Philadelphia commission merchant received a consignment of wheat to sell, the total weight of which was 423632 lbs. He paid freight and other charges amounting to $1080.50; sold at 98 cents per bushel; charged 2½% commission; and remitted the proceeds by bank draft on Chicago, purchased at ¼% premium. How much did the consignor receive?

*214.* A and B have 15 acres to plow. At the end of 1½ days A leaves, and B finishes in 3¾ days. If B had left instead of A, it would have taken A 2½ days to finish. How long would it take each to plow the field alone?

*215.* If I buy 4% bonds maturing in 28 years, at 125¼, how much per cent. greater or less is my annual income than if I buy 4% bonds maturing in 10 years, at 112? (Brokerage ⅛% in each case.)

*216.* A walks at the rate of 3¾ miles an hour and starts 18 minutes before B. At what rate per hour must B walk to overtake A at the ninth mile-stone?

*217.* Divide 175 into four parts such that the first plus 2, the second minus 3, the third multiplied by 4, and the fourth divided by 5, shall be equal to each other.

*218.* The specific gravity of silver being 10.4, what will a ball of silver weigh, the diameter of which is 7 inches?

*219.* If a draft on London for £2348 13s. 7d. cost $11482.11, what was the rate of exchange?

*220.* A man borrowed enough money at 6 per cent. to pay for a house and also for repairs amounting to 2% of the purchase money. The house was vacant for a year, and during that time he had to pay $34.60 taxes. At the end of the year he sold the house for $4600 and found his net loss to be $6\frac{2}{3}\%$ of the purchase price. What did the house cost?

*221.* How many square yards are there in the surface of the sidewalk around a city square 244 feet × 480 feet on the building line, if the sidewalk is 18 feet wide?

*222.* A and B have equal incomes. A's expenses are $16\frac{2}{3}\%$ more than his income, while B lives on 75% of his. At the end of three years, B lends A enough money to pay his debts, and has $150 left. What is the income of each?

*223.* What per cent. is gained by buying silk at 4.95 francs a meter, and selling it at $\$1.87\frac{1}{2}$ a yard, if the duty is 50% and exchange $5.14\frac{1}{2}$?

*224.* How many gallons of water will a cylindrical boiler hold, 12 feet long and 5 feet in diameter, inside measurement, if there are 160 flues, each 3 inches outside diameter, passing through it lengthwise?

*225.* A merchant increased his capital the first year by 10% of itself; the second year he gained 20%; the third year he lost 25%. He then had $100 less than at first. What was his original capital?

*226.* At his death, A's property was valued at $42360; $27360 being in real estate and the remainder in personal property. His heirs were the widow, three sons and two daughters. The will provided that the widow should receive $\frac{1}{3}$ of the real estate and 20% of the personal property and that the remainder should be divided among the sons and daughters, each daughter receiving 25% less than a son. What was the share of each?

*227.* What per cent. of a kilogram is a pound?

*228.* How much must I invest in U. S. 4s at $122\frac{1}{2}$, brokerage $\frac{1}{8}\%$, to secure a quarterly dividend of $460?

*229.* Sent 12000 bushels of wheat to my agent, which he sold at 58 cents per bushel. He paid expenses $214.52 and deducted his commission of $2\frac{1}{2}\%$. He then invested the proceeds in sugar at $5\frac{1}{8}$ cents per pound, commission for buying 2%. How many pounds did he purchase?

*230.* A merchant's sales increased the second year 20% over the first year; the third year 25% over the second, and the fourth year 40% over the third. During the four years he sold $131250 worth of goods. What was the amount of his sales the first year?

*231.* James Farley has $352 in bank and a note for $850 coming due to-day, which has been made payable at his bank. For how much must he give his note at four months, dated to-day, that when discounted the bank may pay his $850 note and leave a balance to his credit at $55.25?

*232.* In the following account find the balance due February 1, 1916, reckoning by "daily balances," charging 6% interest on debit balances and allowing 4% interest on credit balances:

*Dr.* R. J. MELROSE *Cr.*

| 1916 | | DAYS | INTEREST | AMOUNTS | 1916 | | DAYS | INTEREST | AMOUNTS |
|---|---|---|---|---|---|---|---|---|---|
| Jan. | 4 500 Penna. | | | 26187 50 | Jan. | 1 Balance | | | 42128 17 |
| " | 7 1500 Readg. | | | 18177 50 | " | 4 Interest | • | •• •• | |
| " | 12 400 Erie pfd. | | | 28900 | " | 7 " | • | • •• | |
| " | 12 Interest | • | • •• | | " | 18 1500 Readg. | | | 18875 |
| " | 18 " | • | •• •• | | " | 22 400 Erie pfd. | | | 29500 |
| " | 22 " | • | • •• | | " | 26 500 Penna. | | | 26750 |
| Feb. | 1 Interest bal. | | •• •• | | " | 26 Interest | • | • •• | |
| " | 1 Balance | | | ••••••• •• | Feb. | 1 " | • | •• •• | |
| | | | | | " | 1 Interest bal. | | | •• •• |
| | | | | ••••••• •• | | | | •• •• | ••••••• •• |
| | | | | | 1916 | | | | |
| | | | | | Feb. | 1 Balance | | | |

*233.* A, B and C enter into partnership on March 10, 1914. A furnishing ½ of the capital and each of the others ¼: they agree to share gains or losses in this proportion, each partner being charged 6% interest on any sums he may withdraw during the year. A invests $10000, B and C each $5000. A withdraws during the year as follows: May 6, 1914, $212.50; July 23, 1914, $67.20; October 15, 1914, $132.70; January 20, 1915, $74.75; February 3, 1915, $65. B withdraws: April 16, 1914, $84.30; July 2, 1914, $105; September 28, 1914, $38.75; December 24, 1914, $165.80. C withdraws: June 10, 1914, $90; August 31, 1914, $123.40; November 27, 1914, $218.50. On March 11, 1915, before the interest on withdrawals has been computed, the books of the firm show a net loss of $1837.92; what is the balance of each partner's account after the books have been closed for the year?

*234.* A commission merchant in Philadelphia sold 84723 lbs. of corn at 48¾ cents a bushel, paying $32.40 charges, and receiving a commission of 2½%. He then purchased merchandise according to the consignor's directions, the invoice amounting to $324.30, on which he received a commission of 1½%. The balance due the consignor was remitted by draft at 30 days after date, purchased at ¾% discount; what was the face of the draft?

*235.* What amount will be required on September 17, 1917, to pay the balance due on a note for \$720, with interest at 4%, dated September 16, 1916, upon which the following payments were made: October 9, 1916, \$125; January 15, 1917, \$15; February 23, 1917, \$333; March 1, 1917, \$225 (U. S. Rule)?

*236.* A merchant sold a lot of goods at 12½% off list and another lot of the same value at 20% and 10% off list. He then allowed, on the whole invoice, a discount of 3% for cash. What was the value of each lot of goods at list price, if the cash paid was \$3841.20?

*237.* If the interest on \$510 at 6% for 4 years and 9 months is \$145.35, what will be the interest on \$1350 for the same time at the same rate?

*238.* An importation of 1312 meters of silk from Lyons, France, was invoiced at 3.5 francs per meter. The importer purchased a bill of exchange to pay for it at 5.15. The duty was 50% ,and other expenses \$42.50. At what price per yard must the silk be sold to gain 25%?

*239.* Suppose a globe to be inclosed in a cylinder that will exactly contain it, and the cylinder is to be inclosed in a cube that will exactly contain it; what decimal part of the volume of the cube is the volume of the cylinder, what part of the volume of the cylinder is the volume of the sphere, and what part of the volume of the cube is the volume of the sphere?

*240.* If the diameter of the base of a cone and the diameter of a hemisphere are equal, and the altitude of the cone is equal to the radius of the hemisphere, what is the ratio of the volume of the cone to the volume of the hemisphere?

*241.* Find the cost of a sheet-iron smoke-stack 40 feet high and 2 feet in diameter at 15 cents per square foot.

*242.* Merchandise weighing 8743 lbs. is transported 1472 miles at a through rate of \$3.24 per 100 pounds. The first road carried it 329 miles, the second 658 miles, the third 219 miles, and the fourth forwarded it to its destination. How shall the freight be apportioned, the fourth road receiving a 2-cent terminal?

*243.* An importer received an invoice of 350.5 meters of silk at 5.85 francs per meter. He paid duty at 50%, and other charges \$25.60. Exchange for his remittance cost him at the rate of 5.13½. What price per yard must he receive for the silk in order to gain 25%?

*244.* Bought goods at five dollars a gross, 20% off, and sold them at fifty cents a dozen, 20% and 10% off. Is there a profit or a loss, and what is the rate per cent.?

*245.* When is the balance of the following account due by equation?

*Dr.* FRANCIS E. HOYT *Cr.*

| 1914 | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|
| May | 8 | Mdse., 30 days | 800 00 | May | 20 | Cash | 150 00 |
| June | 20 | " 60 days | 463 29 | July | 1 | Note, 1 mo. | 412 50 |
| Aug. | 28 | " net | 377 89 | Sept. | 2 | Cash | 200 00 |

*246.* At 63 cents a square foot, what is the cost of a building lot having two parallel sides, respectively 140 feet and 116 feet long, and 83 feet apart?

*247.* If a draft on Philadelphia, payable 60 days after date, was bought in St. Louis for $3723.29 at $\frac{1}{2}\%$ premium, what was its face?

*248.* Find the height and area of a triangle whose base is 42 feet long, the other sides being each 35 feet long.

NOTE.—Observe that this triangle is equal to two *right-angled* triangles, each having a hypotenuse of 35 feet and a base of 21 feet.

*249.* A man sold two properties for $8522.50; on the first he lost $12\frac{1}{2}\%$, and on the second he gained 30%. What did he gain on the whole transaction, if $\frac{2}{3}$ of what he gave for the first property was equal to $\frac{1}{2}$ of what he gave for the second?

*250.* Having sent a New Orleans agent $1836.46 to be invested in sugar, after allowing $3\%$ on the investment for his commission, I received 32400 pounds of sugar. At what price per pound did the agent purchase?

*251.* A buys a bill of goods amounting to $6776.40, 20% and 10% off on the following terms: "Four months or less 3% for cash." He accepts the latter, and borrows the money at 6% to pay the bill. How much does he gain? 6-1.72

*252.* What is the weight of a pint of alcohol, if its specific gravity is .792?

*253.* A draft on Philadelphia for $9375.15 was purchased in Chicago at $\frac{1}{4}\%$ discount. If the draft cost $9300.16, at how many days after date was it drawn?

*254.* A and B together can do a certain piece of work in $11\frac{1}{2}$ days; A and C together can do it in $10\frac{1}{2}$ days; B and C together can do it in $14\frac{1}{4}$ days. If A and B work together for 2 days, and then A continues alone for 3 days, how long would it take C to finish the work alone?

*255.* A commission merchant in Chicago sells for me 12 bales brown sheeting, each bale containing 800 yards, at 7 cents per yard; pays transportation and other charges amounting to $72; and invests the proceeds in flour. If he charges $2\frac{1}{2}\%$ for selling and $1\frac{1}{4}\%$ for purchasing, and sends me 120 barrels of flour, at what price does he buy? 4.80

*256.* What is the balance due on the following account November 1, 1915, interest at $6\%$?

WALTER G. FARNSWORTH

| *Dr.* | | | | | *Cr.* | | | |
|---|---|---|---|---|---|---|---|---|
| 1915 | | | | | 1915 | | | |
| Mar. | 27 | Mdse., | 30 days | 2437 18 | Apr. | 7 | Cash | 2500 |
| " | 30 | " | net | 517 60 | June | 1 | " | 3000 |
| May | 12 | " | 30 days | 3224 71 | July | 1 | Draft, 10 days | 2500 |
| " | 28 | " | net | 327 14 | Aug. | 19 | Cash | 4000 |
| June | 10 | " | 30 days | 1895 36 | Sept. | 17 | " | 1800 |
| July | 21 | " | 30 " | 5243 07 | | | | |
| Sept. | 8 | " | 30 " | 678 25 | | | | |
| Oct. | 15 | " | 30 " | 1324 27 | | | | |

*257.* A merchant sold a bill of goods at 20%, 10% and 5% off list price, and allowed a discount of 3% for cash. What was the list price, if the cash paid was $3481.20?

*258.* Since the volumes of similar solids are to each other as the cubes of their corresponding dimensions, how many steel balls $\frac{1}{4}$ of an inch in diameter will weigh as much as one 2 inches in diameter?

*259.* A column 10 inches in diameter and 18 feet high is to be gilded with a spiral stripe $2\frac{1}{2}$ inches wide, winding around it so that the turns are 7 inches apart. What will it cost for the gold leaf needed, at $3.20 per book of 100 leaves $3 \times 4$ inches, estimating that one-sixth of the material will be wasted?

*260.* An exporter shipped 400 cases of canned goods to Liverpool, invoiced at 14s. 10d. per case, and drew on the consignees for the amount of the invoice, selling the draft at 4.87. He shipped a similar lot, 400 cases, to Havre, invoiced at 18.70 francs per case, and sold his draft on consignees at 5.15. For which draft did he receive the more money, and how much was the difference?

*261.* How many cubic inches of lead will weigh as much as 12 bushels of wheat? (The specific gravity of lead is 11.35.)

*262.* A note for $327.50, with interest at 5%, dated March 21, 1914, has the following indorsements: June 1, 1914, $25; July 15, 1914, $10; September 3, 1914, $50; October 20, 1914, $100; November 10, 1914, $20; December 8, 1914, $30. How much is due February 17, 1915?

*263.* If I pay $9.48 interest on $555 for 123 days, what is the rate per cent.?

*264.* Loaned $2400 at 6% simple interest until it amounted to $2998.80. For what time was the loan made?

*265.* Two clerks have 2000 circulars to address. One quits at the end of 4 hours, and it takes the other 10 hours to finish. If the one who left had remained 2 hours longer, the other could have finished in 5 hours. How many can each address in 1 hour?

*266.* Bought in Manchester, England, 8 gross of razors at £3 13s. 6d. a dozen, less 10% and 5%; what sum in United States money is equivalent to the net amount of the invoice?

*267.* A commission merchant sold 83748 lbs. of cotton at $7\frac{7}{8}$ cents a pound, paid transportation $368.72, and cartage $8, and charged 3% commission. What per cent. of profit did the consignor make if the cotton cost him $5600?

*268.* A shipment of 300 cases of merchandise, valued at $87.50 a case, was insured for $20000, the policy containing the "average clause." In consequence of a fire at the railroad depot, 42 cases of the goods were partly damaged; and the proceeds of the damaged lot, when sold by auction, amounted to $843.50. What amount was recovered on the policy?

*269.* A can do $\frac{3}{8}$ of a certain piece of work in 4 days; B can do $\frac{4}{9}$ of it in 3 days; C can do $\frac{4}{9}$ of it in 7 days. If A begins the work alone and works for 3 days, and then B and C relieve him, working together for $2\frac{3}{4}$ days, how long would it take A and C together to finish the work?

*270.* Received \$1436.84 on September 10, 1914, as the amount of a loan at 4½%, made June 14, 1913; what was the principal?

*271.* A speculator deposited \$5000 with his broker, who 10 days later purchased for him 500 shares of stock at 92⅛ (brokerage ¼%). The stock was sold 23 days afterward at 94¼ (brokerage ¼%), and settlement made the same day. How much did the speculator receive from the broker (interest at 6%)?

*272.* The tax on a certain property, at \$18.93 on \$1000, amounts to \$460.32; what is the assessed valuation of the property?

*273.* What is the duty on 680 cubic feet of marble, invoiced at 21465 lire, at \$1 per cubic foot and 25% ad valorem?

*274.* A draft on San Francisco for \$6850.75, payable 60 days after date, was purchased at ⅜% discount; how much did it cost, money being worth 6%?

*275.* How much tea at 22 cents, 28 cents and 50 cents a pound must be mixed with 45 pounds at 64 cents a pound, so that the whole may be sold at 48 cents a pound and produce a gain of 20%?

*276.* X and Q are partners, sharing gains or losses in proportion to average net investment. On January 1 X invests \$18000, and Q invests \$15000. X draws out \$350 at the end of each month during the year, and Q draws out \$300 on the 15th of each month. At the end of the year they have a net gain of \$4350 to divide; how much does each receive?

*277.* What is the cash balance of the following account on January 1, 1915?

*Dr.* CALVIN THOMPSON *Cr.*

| 1914 | | | | | 1914 | | | |
|---|---|---|---|---|---|---|---|---|
| Jan. | 5 | Mdse., 60 days | | 389 26 | Apr. | 1 | Cash | 600 |
| Feb. | 4 | " 30 " | | 414 70 | June | 1 | " | 500 |
| Mar. | 12 | " 90 " | | 853 25 | Aug. | 1 | " | 500 |
| Apr. | 10 | " net | | 721 19 | Oct. | 1 | " | 1000 |
| June | 17 | " 90 days | | 436 20 | Dec. | 15 | " | 1500 |
| Aug. | 12 | " 30 " | | 525 00 | | | | |
| Sept. | 9 | " net | | 324 50 | | | | |
| Nov. | 10 | " 90 days | | 622 80 | | | | |
| Dec. | 22 | " 60 " | | 875 70 | | | | |

*278.* An ice company has its men at work on a lake cutting 10-inch ice. If ice weighs 58 lbs. to the cubic foot, and 20300 lbs. can be loaded in a car, how long a strip 100 feet wide will the men cut to fill 40 cars?

*279.* In digging a well 4 feet in diameter, 37 cubic yards of earth were taken out; what was the depth of the well?

*280.* At what rate will \$2468, in 1 yr. 1 mo. 13 da. amount to \$2744.28?

*281.* The net proceeds of a sale were \$3214.78; the charges \$212.50, and 4% commission. What were the gross proceeds?

*282.* Goods that were purchased for \$2319.75, less 30%, 10% and 4%, were marked 40% above net cost and sold at 20%, 12½% and 3% off the marked price; how much was the loss?

*283.* How many reams of paper, in sheets 24×38 inches, will be required to print 12000 copies of a book of 288 pages, each page measuring 6×9½ inches (untrimmed); and how much will this amount of paper cost, at 6 cents per pound, if it weighs 60 pounds to the ream?

*284.* What per cent. of the volume of a pound of lead is the volume of a pound of gold?

NOTE.—This is a problem in compound proportion. The specific gravity of lead is 11.35; that of gold, 19.26. A pound of gold weighs 5760 grains; a pound of lead, 7000 grains.

*285.* The polar diameter of the earth is 7899 miles; the equatorial diameter is 7925½ miles. How many cubic miles less is the volume of the earth, than if it were a perfect sphere 8000 miles in diameter?

*286.* A hexagonal prism is 5 feet 6 inches in height; the distance from the center of the base to the center of each side of the base is 17.32 inches; and the length of each side of the base is 20 inches. Give the contents of the prism in cubic feet.

*287.* A, B and C do a piece of work. The portions done by A and B together equal $\frac{7}{9}$ of the whole work; the portions done by B and C together equal $\frac{7}{11}$ of the whole. What part of the work was done by B?

*288.* The sides of two square fields are to each other as 3 to 5, and the total area of the two fields is 3¾ acres; what is the length of a side of each field in yards?

*289.* If the driving wheels of a locomotive are 4 feet 4 inches in diameter, at the rate of how many miles per hour is the locomotive running when the driving wheels are making 312 revolutions per minute?

*290.* Bought a consignment of wool in Buenos Ayres, the gross weight of which was 34675 lbs. @.15 pesos per lb.; an allowance of 8% of the gross weight was made for tare; freight charges were 9⅜ cents per cwt.; ad valorem duty 18¾%; specific duty 3⅞ cents per lb.; consul's fee 85 pesos. What was the cost in Philadelphia at ¾% premium, of a sight draft to cover all expenses in Buenos Ayres? At 2% commission what will my agent receive? At what price per lb. must it be sold to make a clear profit of 20%, after allowing 5% for damaged wool, and 10% of my sales for bad debts?

*291.* A Philadelphia soap manufacturer makes a cake of soap 3 inches long, 2 inches wide and ½ inch thick. He wishes to make a sample cake of the same soap, keeping the proportions of the larger cake, but using only one-half the material. Find the dimensions of the sample size.

*292.* Borrowed $18360 for one year at 6%. At the end of 3 months I invested in produce through an agent charging 2%; after keeping the goods 4 months I sold one-half of them at 18¾% profit, and the remainder at 15% profit, paying 2% commission for selling. I loaned the money I received at ¾% a month for the remainder of the year. What was my year's profit?

*293.* A legacy of $436.85 was left to a minor; it was invested at 4½% compound interest, and when the legatee was 21 years old it amounted to $750. How old was the child when the bequest was made?

296. A room is 27 ft. 4 in. long, 21 ft. 8 in. wide, 11 ft. 6 in. high. It has two doors, each 7 ft. by 3 ft., and a window 6 ft. by 3 ft. 6 in. What is the cost of painting the walls and ceiling at 30¢ a square yard, allowing for the doors and window? How many square yards of paper will be required to cover the walls, allowing 2 yd. for matching? How many rolls must be purchased if each roll contains 8 yd. and is 18 in. wide? What will be the cost of papering, at 25¢ a roll, without a border? What will be the cost of papering with a border costing 8¢ a yard, allowing 1 yd. for matching?

297. What will be the cost of painting a room 18 ft. 6 in. long, 15 ft. 4 in. wide, and 10 ft. 6 in. high, at 35¢ a yard, allowing 30 yards for doors and windows? How many rolls of paper 18 in. wide and 8 yd. long will be required to cover the walls, allowing 2 yd. for matching? What will be the cost of paper at 30¢ a roll, allowing 10% in addition to the number of rolls required for matching and 5% in addition to the length for shrinkage of the linen?

298. The assessed valuation of the real estate of a township is $521085; the personal property, $52107S. It has 3504 inhabitants subject to a road tax of $2.00. The year's expenses, not including the collector's commission of 2%, are for schools $4400, interest $3850, highways, $4580, salaries $3150, and for miscellaneous expenses $2075. If $585 is in the treasury and two hotels each pay a license of $250 into the treasury and the revenue from fairs amounts to $2500, what tax must be levied on a dollar to meet expenses, and provide a sinking fund of $3000? What would be the tax whose property is assessed at $7800 and who pays for two polls? Find the collector's commission.

299. Ten cylindrical tanks of oil, each 18 inches in diameter and 80 inches long inside measurement, and weighing 225 lbs. each, were imported from Liverpool. The invoice price was 10 crowns per gallon, commission 3½%, consul fee $3 crowns, freight 10c. per cwt., insurance 2%, an allowance of 11% was made for leakage. The ad valorem duty was 40%, and the specific duty was 10½ cents a gallon. A draft to cover all costs in Christiana was purchased in Philadelphia, the exchange being 97¼. At what price shall the oil be marked per gallon, to reduce the marked price 30%, and lose 10% of all in bad sales and still make a gain of 25%?

300. An importer of Philadelphia owed on foreign invoices as follows: Lee & Shepherd Sons, London, £1600 12s.; J. L. Von Buesche, Berlin, 1400 mark; Perrin Buzzell & Co., Paris, 3027 francs; F. Gonzalez, Mexico, 715 pesos. Exchange on London, 4.875; on Berlin, 97½; on Paris, 5.18¾; on Mexico, 78½. He purchased through his brokers, who charged him ½% brokerage, and issued one check to cover the total cost. What was the amount of the check? *$7,270.72*

301. A commission merchant of San Francisco bought for a Philadelphia firm a consignment of 120 chests of green tea, each containing 50 lbs. at 22½ cents a pound; 15 cases of Muscatel raisins, each containing 20 boxes of 50 lbs. each, at 18½ cents a pound; 10 cases of California prunes, each containing 20 boxes of 25 lbs. each, at 11½ cents a pound; charges 2% commission, 3%, guaranty, and 1½%, insurance. The firm in Philadelphia remitted a 30-day draft for the amount due at 7% per annum. What was its face and what did the firm pay for the draft, if the rate of interest was 6%?

*300.* Find the cost, at \$2.08 a yard, of carpeting a room 32½ feet long, 12½ feet wide and 12 feet high, the carpet (which is 22 inches wide) being laid the more economical way, and one foot being allowed to each strip for matching. The room has five windows, each 7½ feet by 4½ feet; three doors, each 9½ feet by 4½ feet; two fireplaces, each 5 feet by 6 feet, and a baseboard 10 inches high. Find the cost of papering the room at 50 cents a double roll, and putting on a border 15 inches wide, which costs 20 cents a yard. What must be the face of a 90-day note, dated and discounted to-day, to cancel the total cost?

*301.* I sent my agent in London \$10000 with instructions to purchase cloth at 15s. 6d. per yard, after paying consul fees of £15 8s. and charging a commission of 3%. What amount is unexpended, supposing he bought a whole number of yards? The duty was 20% ad valorem, 3c. a yard specific, the freight amounted to ½d. a yard, and other charges to \$125. After keeping the cloth three months and charging interest at 6% on total cost, I sell it at \$4.25 a yard and receive in payment two notes of equal sums, dated to-day, one at 30 days, the other at two months, both of which I have discounted immediately. What is my gain or loss on the investment?

*302.* A public square 200 yards on the building line has walks 20 feet wide around it, and two walks of the same width diagonally across the square. The cost of paving is \$1.50 per square yard. For what sum must a 60-day note be drawn to-day that when discounted will pay the debt?

*303.* What is the difference in volume between two wedges, each being 60 inches in altitude and 25 inches broad at the base, but the base of one being 70 inches long and its edge 50 inches, while the base of the other is 50 inches long and its edge 70 inches?

*304.* A room is 35 feet long, 28 feet wide and 12 feet high. What is the shortest distance a spider may crawl in a straight line from a lower corner at one end to the opposite upper corner at the other end?

*305.* Purchased an invoice of 356 yds. lace at 8s. 8d. per yd.; 546 yds. silk at 14s. 10d. per yd.; and 245 doz. spool silk at 3s. 8d. per doz. I ordered it shipped to my agent in San Francisco, who paid a duty of 45% on lace and 40% on spool silk. He sold the lace at \$3.25 a yd.; the silk at \$4.25 per yd., and the spool silk at 25c. per spool. He charged 3% commission for selling, and remitted to me the proceeds by bank draft at 15 days, bought at ⅜% discount. When I cashed the draft I paid the invoice and charges amounting to £3 4s. 10d. What was my gain or loss?

# APPENDIX

## THE METRIC SYSTEM

In 1875 a treaty was signed at Paris by seventeen of the principal nations of the world, the United States being among the number, which provided for the permanent organization of an International Bureau of Weights and Measures under the direction of an International Committee. The most important work of the International Committee was to provide for the construction of a sufficient number of platinum-iridium meters and kilograms to meet the demand of the interested nations. The comparison of all these standards with one another and with the original meter and kilogram was made at the International Bureau which had been established near Paris on neutral territory ceded to the International Committee by the French Government.

This work was completed in 1889, and after selecting a certain meter and a certain kilogram as the international prototypes, the others were distributed by lot to the different countries. The international meter and kilogram have values identical with the original meter and kilogram, are preserved in a special underground vault at the International Bureau, and are accessible only to the International Committee. The United States secured two meters and two kilograms, which are now preserved at the Bureau of Standards at Washington and serve as the fundamental standards of length and mass of the United States. It is the plan of the International Committee to intercompare all the national meters and kilograms with the international prototypes at regular intervals or whenever considered necessary.

At the present time the International Bureau of Weights and Measures is supported jointly by the following countries: The United States (its use "made lawful throughout the United States" by Act of Congress in 1866), Great Britain, Germany, Russia, France, Austria-Hungary, Belgium, Argentine Confederation, Spain, Italy, Mexico, Peru, Portugal, Roumania, Servia, Sweden, Norway, Switzerland, Venezuela, Japan, and Denmark.

The advantages claimed for the metric system are:

(1) The decimal relation between the units.

(2) The extremely simple relation of the units of length, area, volume and weight to one another.

(3) The uniform and self-defining names of units.

NOTE.—The facts contained in this appendix are based on a pamphlet issued by the Department of Commerce and Labor, entitled "The International System of Weights and Measures."

## SYNOPSIS OF THE SYSTEM

The fundamental unit of the metric system is the Meter—the unit of length. From this the units of capacity (Liter) and of weight (Gram) were derived. All other units are the decimal subdivisions or multiples of these. These three units are simply related; *e. g.*, for all practical purposes one **cubic decimeter** equals one **liter** and one **liter** of water weighs one **kilogram.** The metric tables are formed by combining the words **"meter,"** **"gram"** and **"liter"** with the six numerical prefixes, as in the following tables:

| PREFIXES | | MEANING | | UNITS |
|---|---|---|---|---|
| milli- | one thousandth | $\frac{1}{1000}$ | .001 | *meter*, for length |
| centi- | one hundredth | $\frac{1}{100}$ | .01 | |
| deci- | one tenth | $\frac{1}{10}$ | .1 | |
| Unit | one | | 1 | *gram*, for weight or mass |
| deka- | ten | $\frac{10}{1}$ | 10 | |
| hecto- | one hundred | $\frac{100}{1}$ | 100 | *liter*, for capacity |
| kilo- | one thousand | $\frac{1000}{1}$ | 1000 | |

## MEASURES OF LENGTH

The *meter* is the unit of length and is used for measuring dry goods, merchandise, engineering construction, building and other purposes where the yard and foot are used. The meter is 39.37 inches long, about a tenth longer than the yard.

| | |
|---|---|
| 10 millimeters (mm.) | = 1 centimeter (cm.) |
| 10 centimeters | = 1 decimeter (dm.) |
| 10 decimeters | = 1 meter (m.) |
| 10 meters | = 1 dekameter (dekam.) |
| 10 dekameters | = 1 hectometer (hm.) |
| 10 hectometers | = 1 kilometer (km.) |

The centimeter and millimeter are used in machine construction and similar work, instead of the inch and its fractions. The centimeter, as its name shows, is the hundredth of a meter. It is used in cabinet work, in expressing sizes of paper, books, and in many cases where the inch is used. The centimeter is about two-fifths of an inch and the millimeter about one-twenty-fifth of an inch. The millimeter is divided for finer work into tenths, hundredths and thousandths.

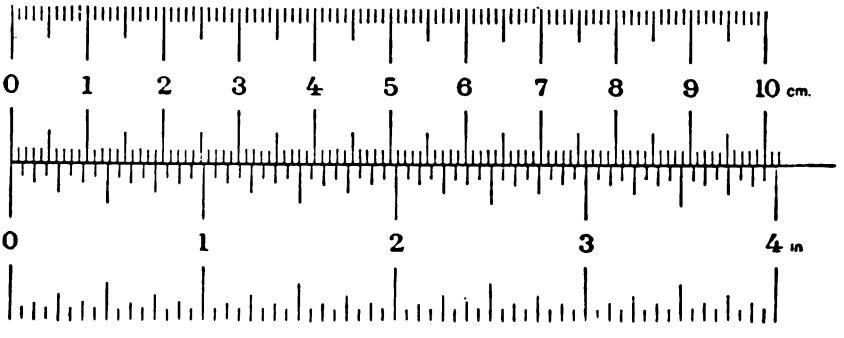



FIG. 1. COMPARISON SCALE: 10 CENTIMETERS AND 4 INCHES. (ACTUAL SIZE.

Where miles are used in England and the United States for measuring distances, the kilometer (1000 meters) is used in metric countries. The kilometer is about 5 furlongs. There are about 1600 meters in a statute mile, 20 meters in a chain, and 5 meters in a rod.

If a number of distances in millimeters, meters and kilometers are to be added, reduction is not necessary. They are added as dollars, dimes and cents are now added. For example, "1050.25 meters" is not read "1 kilometer, 5 dekameters and 5 centimeters," but "one thousand fifty meters, twenty-five centimeters," just as $1050.25" is read "one thousand fifty dollars and twenty-five cents."

## SURFACE MEASUREMENT

The table of areas is formed by squaring the length measures, as in our common system. For land measure 10 meters square is called an "Are" (meaning "area"). The side of one "are" is about 33 feet.

TABLE

| | |
|---|---|
| 10 milliares (ma.) | = 1 centiare (ca.) |
| 10 centiares | = 1 deciare (da.) |
| 10 deciares | = 1 are (A., or a.) |
| 10 ares | = 1 dekare |
| 10 dekares | = 1 hectare (ha.) |
| 10 hectares | = 1 kilare (ka.) |

The **Hectare** is 100 meters square, and, as its name indicates, is 100 ares, or about 2½ acres. An acre is about 0.4 hectare. A standard United States quarter section contains almost exactly 64 hectares. A square kilometer contains 100 hectares.

For smaller measures of surface the square meter is used. The square meter is about 20 per cent. larger than the square yard. For still smaller surfaces the square centimeter is used. A square inch contains about 6½ square centimeters.

## MEASURES OF VOLUMES

The cubic measures are the cubes of the linear units. The cubic meter (sometimes called the stère, meaning "solid") is the unit of volume.

It is used in place of the cubic yard and is about 30 per cent. larger. This is used for "cuts and fills" in grading land, measuring timber, expressing contents of tanks and reservoirs, flow of rivers, dimensions of stone, tonnage of ships, and other places where the cubic yard and foot are used.

TABLE

| | |
|---|---|
| 1000 cubic millimeters (mm.$^3$) | = 1 cubic centimeter (c.c. or cm.$^3$) |
| 1000 cubic centimeters | = 1 cubic decimeter (dm.$^3$) |
| 1000 cubic decimeters | = 1 cubic meter (m.$^3$ or stère) |

The thousandth part of the cubic meter (1 cubic decimeter) is called the **Liter.** (See table of capacity units.)

For very small volumes the cubic centimeter is used. This volume of water weighs a gram, which is the unit of weight or mass. There are about 16 cubic centi-

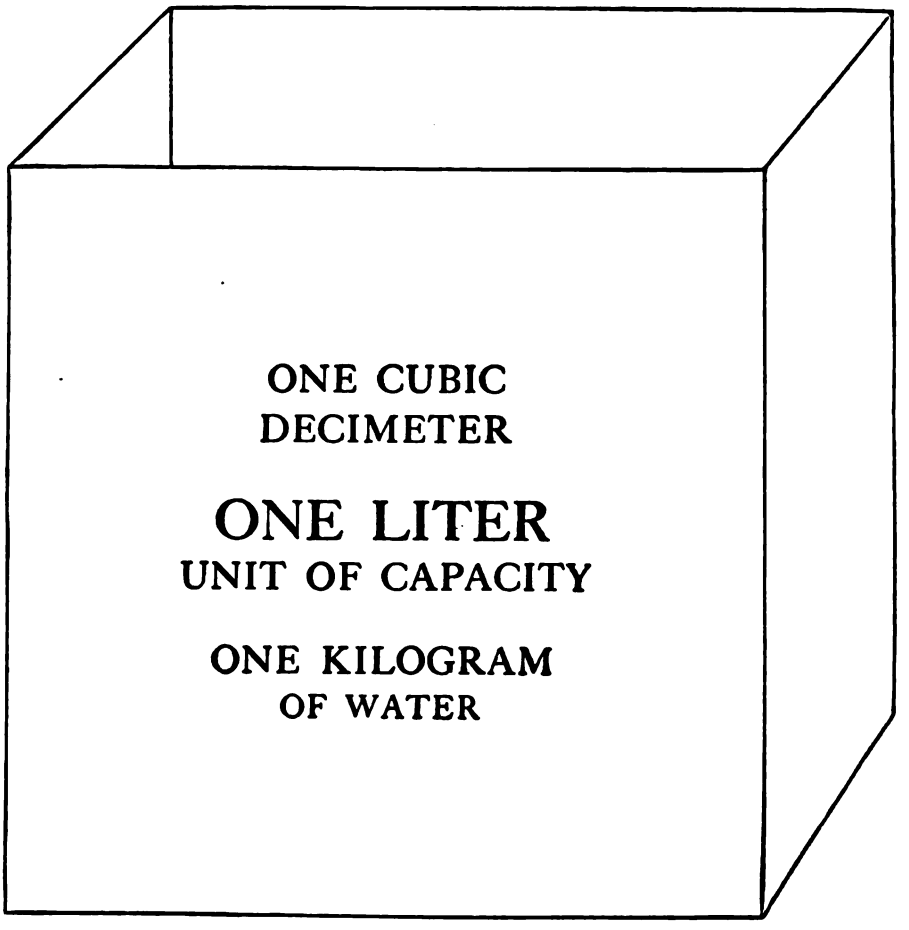



FIG. 2. CUBIC DECIMETER. (ACTUAL SIZE.)

meters in a cubic inch. The cubic centimeter is the unit of volume used by chemists as well as in pharmacy, medicine, surgery, and other technical work. One thousand cubic centimeters make 1 liter. A cubic meter of water weighs a metric ton and is equal to 1 kiloliter.

## MEASURES OF CAPACITY

The **Liter** is the unit of capacity. It equals a cubic decimeter.

The liter is used for measurements commonly given in the gallon, the liquid and dry quarts, a liter being 5 per cent. larger than our liquid quart and 10 per cent. smaller than the dry quart.

TABLE

| | | |
|---|---|---|
| 10 milliliters (ml.) | = | 1 centiliter (cl.) |
| 10 centiliters | = | 1 deciliter (dl.) |
| 10 deciliters | = | 1 liter (l.) |
| 10 liters | = | 1 dekaliter (dekal.) |
| 10 dekaliters | = | 1 hectoliter (hl.) |
| 10 hectoliters | = | 1 kiloliter (kl.) |

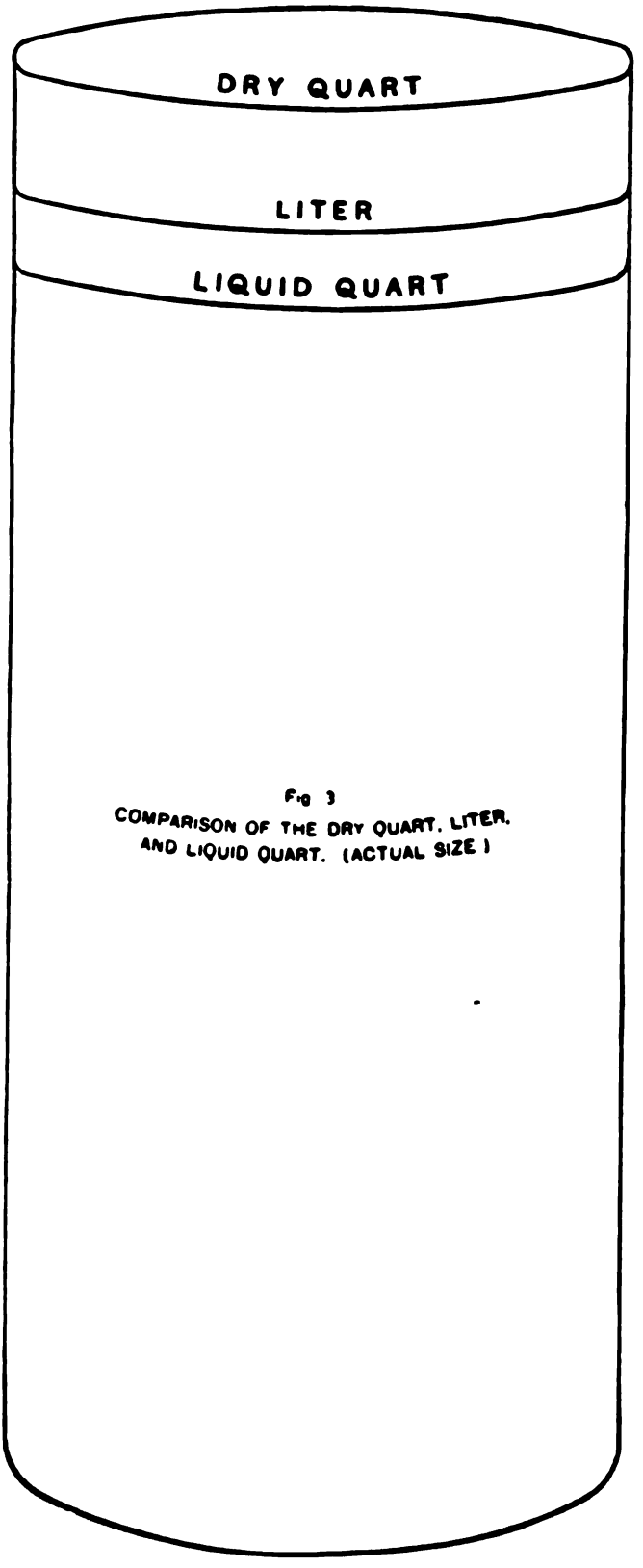



FIG 3
COMPARISON OF THE DRY QUART, LITER, AND LIQUID QUART. (ACTUAL SIZE)

The hectoliter (100 liters) serves the same purpose as the United States bushel (2150.4 cubic inches), and is equal to about 3 bushels, or a barrel. A peck is about 9 liters.

A liter of water weighs exactly a kilogram, *i. e.*, 1000 grams. A thousand liters of water weigh 1 metric ton.

## MEASURES OF WEIGHT

The **Gram** is the unit of weight. It is the weight of a cubic centimeter of distilled water at freezing temperature, and weighs 15.432 Troy grains.

TABLE

| | |
|---|---|
| 10 milligrams (mg.) | = 1 centigram (cg.) |
| 10 centigrams | = 1 decigram (dg.) |
| 10 decigrams | = 1 gram (g.) |
| 10 grams | = 1 dekagram (dekag.) |
| 10 dekagrams | = 1 hectogram (hg.) |
| 10 hectograms | = 1 kilogram (kg.) |

Measurements commonly expressed in gross tons or short tons are stated in metric tons (1000 kilograms). The metric ton comes between our long and short tons and serves the purpose of both.

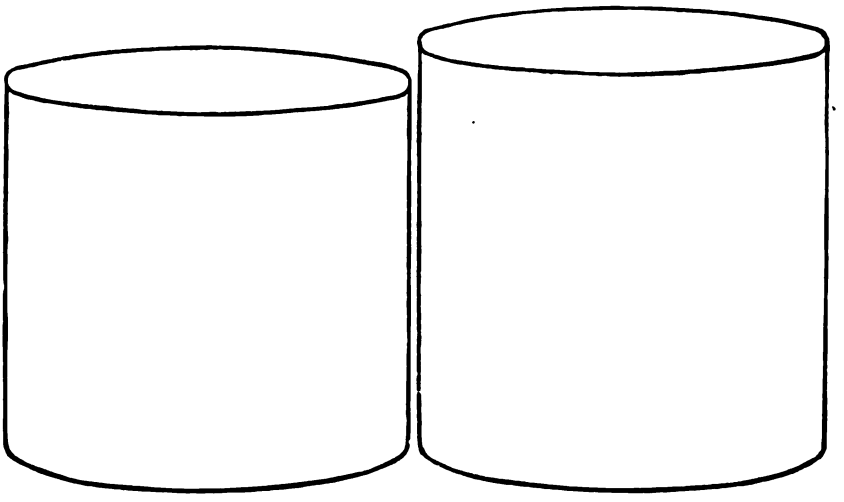


FIG. 4. RELATIVE SIZE OF 2-POUND AND 1-KILOGRAM (BRASS) WEIGHTS. (ACTUAL SIZE.)

The kilogram and "half kilo" serve for every-day trade, the latter being 10 per cent. larger than the pound. The kilogram is approximately 2.2 pounds.

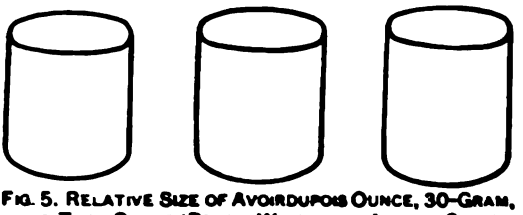


FIG. 5. RELATIVE SIZE OF AVOIRDUPOIS OUNCE, 30-GRAM, AND TROY OUNCE (BRASS) WEIGHTS. (ACTUAL SIZE.)

FIG. 6. RELATIVE SIZE OF GRAM AND SCRUPLE (BRASS) WEIGHTS. (ACTUAL SIZE.)

The gram and its multiples and divisions are used for the same purposes as ounces, pennyweights, drams, scruples and grains. For foreign postage, 30 grams is the legal equivalent of the avoirdupois ounce.